Fresnel Equations

- The fresnel effect is the observation that things get more reflective at grazing angles.
- Fresnel equations describe how much energy is reflected at a surface boundary.
- Remainder is absorbed as heat.
Fresnel Equations

\[ r_{\parallel}^2 = \frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)} \]

\[ r_{\perp}^2 = \frac{\tan^2(\theta_1 - \theta_2)}{\tan^2(\theta_1 + \theta_2)} \]

Fresnel equation for conductors

\begin{align*}
n_1 = 1.0, & \quad n_2 = 2.0 \\
n_1 = 2.0, & \quad n_2 = 1.0
\end{align*}
Fresnel Equations

Fresnel equation for conductors

\[ r_\parallel^2 = \frac{\sin^2 (\theta_1 - \theta_2)}{\sin^2 (\theta_1 + \theta_2)} \]

\[ r_\perp^2 = \frac{\tan^2 (\theta_1 - \theta_2)}{\tan^2 (\theta_1 + \theta_2)} \]

\[ F_r = \frac{1}{2} \left( r_\parallel^2 + r_\perp^2 \right) \]
Fresnel Equations

\[ r_{||}^2 = \frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)} \]

\[ r_{\perp}^2 = \frac{\tan^2(\theta_1 - \theta_2)}{\tan^2(\theta_1 + \theta_2)} \]

\[ F_r = \frac{1}{2} \left( r_{||}^2 + r_{\perp}^2 \right) \]

For \( \theta_1 = \theta_2 = 0 \):

\[ F_r = \frac{(\eta_1 - \eta_2)^2}{(\eta_1 + \eta_2)^2} \]

\[ \eta = \frac{1 + \sqrt{F_r}}{1 - \sqrt{F_r}} \]
Fresnel Equations

Schlick approximation:

\[ F_r \approx R_0 + (1 - R_0)(1 - \cos \theta_1)^5 \]

\[ R_0 = \left( \frac{\eta - 1}{\eta + 1} \right)^2 \]
Metal shading

Compute hit position ($\tilde{P} = \tilde{O} + t\tilde{V}$)
Call primitive to get normal ($\bar{N}$) (normalized)

$\cos\theta = \bar{N} \cdot \bar{V}$

if ($\cos\theta > 0$)
    normal = -normal
else
    $\cos\theta = -\cos\theta$

foreach light source
    compute phong term, just like Phong material
result = speclight * $R_0$

if depth of ray < maximum depth:

$$F_r = R_0 + (1 - R_0)(1 - cos\theta)^5$$

reflection direction = $\bar{V} + 2\cos\theta \bar{N}$
refl color = trace/shade ray(hitpos, reflection direction)
result += $F_r *$ refl color
Implementation tips

- Make sure the magnitude of your reflection direction == 1 (print it out)
- Scene now contains max ray depth
- Start with max ray depth==2
# Dielectric shading

<table>
<thead>
<tr>
<th></th>
<th>From light sources</th>
<th>From other surfaces</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diffuse reflection</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Specular reflection</td>
<td>Phong term</td>
<td>Fresnel reflection</td>
</tr>
<tr>
<td>Diffuse transmission</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Specular transmission</td>
<td>Phong term</td>
<td>Fresnel transmission</td>
</tr>
</tbody>
</table>
Fresnel equations

Fresnel equations for transparency

\[ r_{\parallel}^2 = \frac{\eta_2 \cos \theta_1 - \eta_1 \cos \theta_2}{\eta_2 \cos \theta_1 + \eta_1 \cos \theta_2} \]

\[ r_{\perp}^2 = \frac{\eta_1 \cos \theta_1 - \eta_2 \cos \theta_2}{\eta_2 \cos \theta_2 + \eta_2 \cos \theta_2} \]

\[ F_r = \frac{1}{2} \left( r_{\parallel}^2 + r_{\perp}^2 \right) \]

\[ F_t = 1 - F_r \]
Fresnel Equations

\[ r_{\parallel}^2 = \frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)} \]

\[ r_{\perp}^2 = \frac{\tan^2(\theta_1 - \theta_2)}{\tan^2(\theta_1 + \theta_2)} \]

\[ F_r = \frac{1}{2} \left( r_{\parallel}^2 + r_{\perp}^2 \right) \]

For \( \theta_1 = \theta_2 = 0 \):

\[ F_r = \frac{(\eta_1 - \eta_2)^2}{(\eta_1 + \eta_2)^2} \]

\[ \eta = \frac{1 + \sqrt{F_r}}{1 - \sqrt{F_r}} \]
Fresnel Equations

Schlick approximation:

\[ F_r \approx R_0 + (1 - R_0)(1 - \cos \theta_m)^5 \]

\[ \theta_m = \max(\theta_1, \theta_2) \]

\[ F_t = 1 - F_r \]

\[ R_0 = \left( \frac{\eta - 1}{\eta + 1} \right)^2 \]