What is “Perspective?”

- A mechanism for portraying 3D in 2D
- “True Perspective” corresponds to projection onto a plane
- “True Perspective” corresponds to an ideal camera image

Perspective (Mural) Games

M.C. Escher, Another World II (1947)

Perspective

M.C. Escher, Ascending and Descending (1960)

M. C. Esher

- Perspective is “local”
- Perspective consistency is not “transitive”
- Nonplanar (hyperbolic) projection

Nonplanar (Hyperbolic) Projection

M.C. Escher, Heaven and Hell
Nonplanar (Hyperbolic) Projection

M C Esher, *Heaven and Hell*

Curvilinear Projection

"True" Perspective in 2D

\[
\begin{align*}
\frac{h}{p} &= \frac{y}{x + p} \\
h &= \frac{py}{x + p}
\end{align*}
\]

"True" Perspective in 2D

\[
\begin{pmatrix}
x \\
y \\
1
\end{pmatrix}
\begin{bmatrix}
px \\
x + p \\
p
\end{bmatrix}
\Rightarrow
\begin{pmatrix}
x \\
y \\
\frac{py}{x + p}
\end{pmatrix}
\]

"True" Perspective in 2D

\[
\begin{pmatrix}
x \\
y \\
1
\end{pmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
\frac{1}{p} & 0 & 1
\end{bmatrix}
\begin{pmatrix}
x \\
y \\
\frac{py}{x + p} + 1
\end{pmatrix}
\Rightarrow
\begin{pmatrix}
x \\
y \\
\frac{px}{x + p}
\end{pmatrix}
\]
Geometry Same for Eye at Origin

What Happens to Special Points?

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
\frac{1}{h} & 0 & 1
\end{bmatrix}
\begin{bmatrix}
-p \\
0 \\
0
\end{bmatrix}
= 
\begin{bmatrix}
-p \\
0 \\
0
\end{bmatrix}
\]

What is this point??

Let's Look at a Limit

Observe,

\[
\lim_{n \to \infty}
\begin{bmatrix}
1 & 0 & n \\
0 & 1 & 1 \\
\frac{1}{n} & 0 & 1
\end{bmatrix}
= 
\begin{bmatrix}
n \\
0 \\
0
\end{bmatrix}
\]

We see that

\[
\begin{bmatrix}
n \\
0
\end{bmatrix}
\iff +\infty
\text{ on x-axis}
\]

Where does Eye Point Go?

• It gets sent to \(-\infty\) on x-axis
• Where does \(+\infty\) go on x-axis go?

What happens to \(+\infty\)?

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
\frac{1}{h} & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 \\
0 \\
\frac{1}{h}
\end{bmatrix}
= 
\begin{bmatrix}
p \\
0 \\
0
\end{bmatrix}
\iff 
\begin{bmatrix}
p \\
0
\end{bmatrix}
\]

It comes back to virtual eye point!

What Does This Mean?
“Pencil of Lines” Becomes Parallel

Parallel Lines Become a “Pencil of Lines”!

What Does This Mean?

“True” Perspective in 2D
Viewing Frustum

“True” Perspective in 2D

“True” Perspective in 2D

What happens for large \( p \)^

Projection Becomes Orthogonal: “Right Thing Happens”