1. (20%) Write a proof, from first principles (by building the $D$ function) for $\text{Halt}_{TM}$ being undecidable. $\text{Halt}_{TM} = \{\langle M, w \rangle \mid M \text{ is a TM that halts on } w \}$. Follow the structure of arguments given on Page 1 of http://www.eng.utah.edu/~cs3100/lectures/l27/notes27.pdf

* Suppose there is a decider "H" for $\text{Halt}_{TM}$.

* $H(M,w)$ accepts if $M$ will halt on $w$, and rejects otherwise.

* Derive a contradiction as follows. Define a machine $D$

* $D(M) =$
  
  if $H(M,M)$ ACCEPTS, -- i.e. $M$ halts on $M$
  then LOOP -- $D$ rejects $M$
  else ACCEPT -- $D$ accepts $M$ when $M$ does not halt on $M$

* Now does $D$ halt on $D$? I.e. does $D(D)$ halt or loop?

* $D(D) = H(D,D)$
  
  - if $H(D,D)$ accepts, i.e. $D$ halts on $D$, then $D$ loops on $D$
  
  - if $H(D,D)$ rejects, i.e. $D$ loops on $D$, then $D$ halts on $D$

* The contradiction is complete. Hence $H$ cannot exist.

2. (20%) Write a detailed mapping reduction proof from $\text{Halt}_{TM}$ to $\text{A}_{TM}$, showing details similar to those in Figure (a), Page 2, http://www.eng.utah.edu/~cs3100/lectures/l27/notes27.pdf

* Given an $M$ and $w$, make a copy of $M$ (calling it $M'$), and then change every accept state to a non-accept state, and vice versa.

* Now, given $A_{TM}$, build a decider for $\text{Halt}_{TM}$ as follows:

  - Feed $M,w$ to one copy of $A_{TM}$, calling this $A_{TM}$‘s outputs $a_1$ and $r_1$
  
  - Feed $M',w$ to another copy of $A_{TM}$, calling this $A_{TM}$‘s output $a_2$ and $r_2$

  - Now what truth-tables are possible?
3. Explain what the sets $A$ and $B$ of Figure (b) are for these proofs. Write out the “if and only if” style proof “punchline” (e.g. $x \in A$ if and only if $f(x) \in B$; hence a solver for $B$ would solve $A$) to make sure you understand what is going on. You can get ideas on how to write from Page 2.

- $(5\%) \ A_{TM} \leq m \ PCP$.
  We build the mapping reduction in such a way that
  
  $M,w \in A_{TM}$ if and only if $Puzzle_{\{M,w\}}$ has a solution.

- $(5\%) \ PCP \leq m \ CFG_{amb}.$ Here, $CFG_{amb}$ is the language of CFG encodings that are ambiguous.
  Here, we build a mapping reduction in such a way that
  
  $P \in Solvable_{PCPs}$ if and only if $CFG_{\{P\}}$ is unambiguous.

4. $(40\%)$ Encode the following Lewis Carroll puzzle using the DDCal tool and find a proof. You may have to strengthen the given conditions. Thoroughly explain how BDDs helped you solve this puzzle (one-page description).

I’ll help you by giving a template, below.

```
# A puzzle by Lewis Carroll :
#
# From the premises
#
#(a) None of the unnoticed things, met with at sea, are mermaids.
#
#(b) Things entered in the log, as met with at sea, are sure to be worth remembering.
#
#(c) I have never met with anything worth remembering, when on a voyage.
#
#(d) Things met with at sea, that are noticed, are sure to be recorded in the log.
#
# Prove that I have never met with a mermaid at sea
#
# N = it is noticed, M = it is a mermaid, L = entered in log,
# R = worth remembering, I = I have met with it at sea, T = met at sea
#
# First specify the desired variable ordering. DDCal can later reorder
```
\[ \text{var} = T \* N \* M \* L \* R \* I \]

#(a) None of the unnoticed things, met with at sea, are mermaids.
\[ A_1 = T \* \neg N \rightarrow \neg M \]

#(b) Things entered in the log, as met with at sea, are sure to be worth remembering.
\[ A_2 = T \* L \rightarrow R \]

#(c) I have never met with anything worth remembering, when on a voyage.
\[ A_3 = I \rightarrow \neg R \]

#(d) Things met with at sea, that are noticed, are sure to be recorded in the log.
\[ A_4 = T \* N \rightarrow L \]

# Prove that I have never met with a mermaid at sea
\[ \text{proofGoal} = M \rightarrow \neg I \]

# Negate proof-goal and add it in
\[ \text{contra1} = A_1 \* A_2 \* A_3 \* A_4 \* \neg \text{proofGoal} \]

# Oops, need frame axiom: not met at sea \rightarrow I have not met with it at sea
\[ \text{frame} = \neg T \rightarrow I \]

\[ \text{contra} = \text{contra1} \* \text{frame} \]

[contra1 contra]

5. (10%) Write a one-page writeup on NP-completeness. Read about NP-complete problems in [http://en.wikipedia.org/wiki/NP-complete](http://en.wikipedia.org/wiki/NP-complete). Mention some of the common NP-complete problems. Elaborate on some of the common misunderstandings about NP-complete problems that are listed there. You may survey other sources also - but please cite every source you survey!

Summarize well.