Procedurally for each type what do we do?

LECTURE 21 starts here pg. 1 (review)

1) DNS ⇒ have full velocity fields we can follow our procedures exactly. e.g. filter calc stats etc.

2) Reduced 2D data (PIV type) from windtunnel water tunnel etc.

- in these studies 2D data of u, v (regular) or u, v, w (stereoscopic) velocity are taken.
- DATA allows for 2D filtering (appropriate for BLs) see Higgins 2007
- Problem, what about Π? we need $\tilde{S}_{ij}$!

components of $\Pi = \tau_{11} S_{11} + \tau_{22} S_{22}\ \tau_{33} S_{33} + 2 S_{12} \tau_{12} + 2 \tau_{23} S_{23} + 2 \tau_{13} S_{13}$ ⇒ 6 terms!

common assumption: Carper Porte-Agel 2000, ~~Lin et al 99~~, Chen et a 2005) for boundary layers with 3D

Break it up!

horizontal planes:

$$\Pi^M = -\tau_{11} \tilde{S}_{11} - 2\tau_{12} S_{12} - \tau_{22} \tilde{S}_{22}$$

vertical:

$$\Pi^M = -\tau_{11} \tilde{S}_{11} - 2\tau_{13} \tilde{S}_{13} - \tau_{33} \tilde{S}_{33}$$

switch

what kind of assumptions? (Liu in grid turbulence uses) for 2D PIV (u, v only)

$$\langle \tau_{13} \tilde{S}_{13} \rangle = \langle \tau_{23} \tilde{S}_{23} \rangle = \langle \tau_{12} \tilde{S}_{12} \rangle$$

and

$$\langle \tau_{33} \tilde{S}_{33} \rangle = \langle \tfrac{1}{2} (\tau_{11} + \tau_{22}) \tilde{S}_{33} \rangle$$

how can we get $\tilde{S}_{33}$? ⇒ continuity by assuming incompressible!

- in general we have to work with the τ_{ij} and S_{ij} comp. we have.

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What about 1D data (point sensors)

Sonic / hotwire etc.

→ X ←

- only measures velocity at a point.
- How do we filter? Discuss

→ make an array (draw pick)

⇒ use Taylor's hypothesis ↗ (Taylor, 1938) (Pope pg. 224)

- assume the flow can be considered to be frozen

⇒ wind speed can be used to translate measurements in time to their position in space.

⇒ this is only an approximation. ⇒ only useful for scales where eddies evolve with timescales longer than the time it takes to advect past the sensor.

Simply the time for an eddy of size λ to advect past a sensor is

$$T = \lambda / |u_i|$$

⇒ usually the average or convection vel.

to satisfy this we need to have

$$\sigma_{|u_i|} < 0.5\,|u_i|$$

- So what if we only have 1 sensor?

Porte-Agel et al 1998 assume isotropy for resolved scales $\sim \Delta$

$$\Rightarrow \langle \hat{S}_{ij}\hat{S}_{ij} \rangle = \frac{15}{2}\langle \hat{S}_{11}^2 \rangle \quad \text{and} \quad |\hat{S}| = \sqrt{15}\left|\frac{\partial \tilde{u}_1}{\partial x_1}\right|$$

~~⇒ tensor alignment go to output!~~

~~⇒ conditional sampling~~

→ what type of basic results do we get? go to handout!

- Besides looking at the stats we've discussed, Researchers also explore other statistical properties of SGS models using a priori studies.

Ex:

Tensor alignment:

Recall)

Eddy-viscosity models : $\tau_{ij}^{m} = -2\nu_T \tilde{S}_{ij}$

Similarity : $\tau_{ij}^{m} = C_L L_{ij}$

non linear : $\tau_{ij}^{m} = C_{nl} \Delta^2 \left(\frac{\partial \tilde{u}_i}{\partial x_k} \frac{\partial \tilde{u}_j}{\partial x_k} - \frac{1}{3} \frac{\partial \tilde{u}_m}{\partial x_k} \frac{\partial \tilde{u}_m}{\partial x_k} \delta_{ij} \right)$

(see prev. lecture for details.)

what do all these imply?

all eddy v implies. $\Rightarrow$ Eddy v implies τ_{ij} is in direction of $\tilde{S}_{ij}$

sim " " τ_{ij} is in dirn of L_{ij}

non linear " " τ_{ij} " " of $\frac{\partial \tilde{u}_i}{\partial x_k} \frac{\partial \tilde{u}_j}{\partial x_k}$

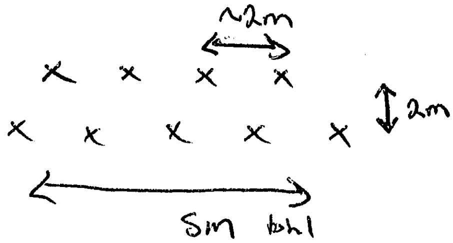
$\Rightarrow$ a few ~~several~~ Researchers have looked into this question (related to how a model ~~...~~ will reproduce flow structures)

$\Rightarrow$ why related to flow structures? $\Rightarrow$ ask class...

Studies : Tao et al 2002 (JFM), Higgins et al 2003 (BLM), Higgins et al 2009 JAOT.

How? Example lets look at Higgins et al 2003

used data from sonic arrays in ABL (Porte Agel et al 2001)



w/ this array we can calculate:
- 2D filtered values (Taylors in streamwise)
- all our gradients ($\tilde{S}_{ij}$)
- all comp. of τ_{ij}
- only approx 2D filtering which is common in BL studies.

- So how do ~~we~~ we look at alignment of 2 tensors (e.g. $\tau_{ij} \to \tilde{S}_{ij}$)? (see paper for details)

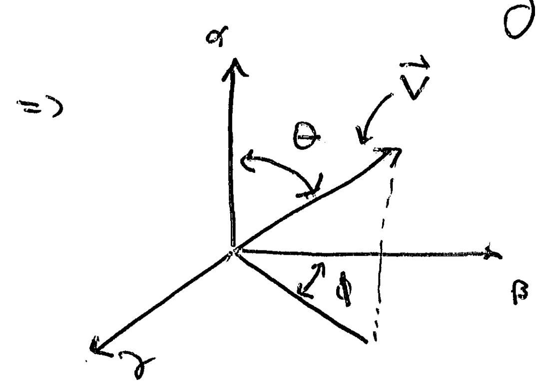
→ ~~the~~ Examine the eigen values => vectors of the two tensors.

→ why eigenvalues?

→ for symmetric tensor (what we have) eigenvectors form an orthonormal basis

if we look at alignment between a vector and a tensor described by ~~eigenvalues~~ eigenvectors α, γ, β

=>



ex: between $\vec{\alpha}$ and vorticity ω

$$\cos(\theta) = \frac{|\vec{\omega} \cdot \vec{\alpha}|}{|\vec{\omega}||\alpha|}$$

(see Higgins et al 2003 or Tao et al 2002 for details)

=> what did they find (go to handout later).

- another common way to examine the association of coherent structures and SGS models is using

conditional statistics : (Porté-Agel et al 2001, Carper and Porté-Agel 2004 etc.)

e.g.

$$\langle \Phi | c \rangle = \frac{1}{N} \sum_{n=1}^{N} \Phi(x_n + x', y, z)$$

where

Φ quantity of interest (temp, velocity, vorticity)

c condition satisfied

x_n points where c is true

$\frac{X}{2} \le x' \le \frac{X}{2}$ where X is the window length (for streamwise sample).

what types of things to sample on?

- could be anything but for SGS studies

$\Rightarrow$ Π (large neg. or positive)

$\chi \equiv$ scalar dissipation (" ")

these two most common

using this (Carper associated backscatter / forward scatter with coherent structures)