- 1st Short Review of Projects (very generic)
  - Reminder due Thursday → 1 paragraph statement of project topic: should describe: * Project 1 or 2 (ie I am doing a priori / a posteriori)
  - * I will use x and x SGS models
  - * my analysis will include (anything non standard you will work on)
  - * I ~~plan~~ will do x change to the project description (describe) (types of flows / Group eg Overholt and Pope 1998)
  - * my project is collaborative and I will work with ——

<u>Today</u>: A priori studies ...

outline: I) a priori vs a posteriori ~~(...)~~ (strategies)

II) goal of a priori (recap of last time) and what stats to calc.

III) types of a priori studies

1) DNS
2) wind tunnel
3) atmospheric field experiments.

IV) procedurally how to calculate values

1) full data available (DNS)
   what is the basic requirement for an a priori study.
2) Reduced data sets (2D eg. PIV)
3) Atmospheric and windtunnel (hotwire data)
   a) Taylors hypothesis
   b) common assumptions about $\tilde{S}_{ij}$ + $\tau_{ij}$

V) specific stats: (review are ✓)

- correlation coefficients ✓
- $\langle \tau_{ij} \rangle$ and instantaneous ✓
- $\Pi$ and instantaneous ✓
- model coefficients ✓
- transport terms (un common (cite Sullivan)

explain {
- tensor alignment.
- ~~Stat~~ association with flow phenomena.
}

**a priori study**: an offline examination of the physics associated with a SFS modeling strategy.
→ requires knowledge of the turbulent flow field at a high resolution (large dynamic range)

**a posteriori study**: examining simulation results using new and existing SGS models in different flow cases. This is the ultimate test of model performance and has much lower data requirements. ⊗ For Example crude mean values are enough are the minimum requirement (although specification of ICs and BCs usually complicate comparisons). ⊗ It is also important to note that theory can be used.

**Main Goal of a priori studies**: statistically determine the necessary (and hopefully sufficient) conditions under which SFS models produce accurate sol$^{n}$s ⊗ ie what properties must $\tau_{ij} = \widehat{u_i u_j} - \hat{u}_i \hat{u}_j$ share with $\tau_{ij}^M = f(\hat{u}_i)$?

**Example from last class**: if we take our filtered equations and average them we get:

$$\frac{\partial \langle \hat{u}_i \rangle}{\partial t} + \langle \hat{u}_j \rangle \frac{\partial \langle \hat{u}_i \rangle}{\partial x_j} = -\frac{\partial}{\partial x_i} \langle \hat{p} \rangle + \nu \frac{\partial^2 \langle \hat{u}_i \rangle}{\partial x_j^2}$$

$$-\frac{\partial}{\partial x_j} \langle \tau_{ij} \rangle - \boxed{\frac{\partial}{\partial x_j} \left[ \langle \hat{u}_i \hat{u}_j \rangle - \langle \hat{u}_i \rangle \langle \hat{u}_j \rangle \right]}$$

⇓ New term.

- if we compare this to the modeled version of the equation (i.e. $\tau_{ij} \to \tau_{ij}^M$) resulting in an equation for $u_i^M$ we find that to for the modeled equation to exhibit the correct mean and 2nd-order moments

(⊗) $\langle u_i^M \rangle = \langle \hat{u}_i \rangle$, $\langle p^M \rangle = \langle \hat{p} \rangle$, $\langle u_i^M u_j^M \rangle = \langle \hat{u}_i \hat{u}_j \rangle$

- for this to hold then $\langle \tau_{ij} \rangle = \langle \tau_{ij}^M \rangle$ to within a constant.

⇑

⇒ this condition yields a necessary condition for LES to yield **both** the correct mean and 2nd order moments.

(*) however it is **not** a sufficient condition

even a model that yields the correct mean stress could yield an erroneous velocity field.

⇒ To get ~~the~~ a sufficient condition on predicting the correct mean 1st ~~and~~ and 2nd-order stats. we need to look at the mean equation for the 2nd order moments:

$$\frac{\partial}{\partial t}\langle \hat{u}_i \hat{u}_j \rangle + \langle \hat{u}_k \rangle \frac{\partial}{\partial x_k}\langle \hat{u}_i \hat{u}_j \rangle = \nu \frac{\partial^2}{\partial x_k^2}\langle \hat{u}_i \hat{u}_j \rangle - \frac{\partial}{\partial x_k}\Theta_{kij} + \frac{2}{\rho}\langle \hat{p}\hat{S}_{ij} \rangle - \varepsilon_{ij} - \Pi_{ij}$$

- $\frac{\partial}{\partial x_k}\Theta_{kij}$: transport of stress by correlations between resolved velocity fluctuations, w/ resolved press. and transport due to unresolved (see Meneveau 94 for more details)
- $\frac{2}{\rho}\langle \hat{p}\hat{S}_{ij} \rangle$: Press-strain redistribution
- $\varepsilon_{ij}$: viscous dissipation of the resolved field
- $\Pi_{ij} = -[\langle \hat{S}_{ik}\tau_{kj} \rangle + \langle \hat{S}_{jk}\tau_{ki} \rangle]$ is the drain of resolved KE by SGS stress

~~(SGS transfer ...)~~

(*) Note the similarity of this equation with the filtered KE equations we looked at previously (*) in fact, if we take $i=j$ we have the **mean** resolved KE equation.

* from this equation we can deduce that if we want to properly predict the correct 2nd + 3rd order moments ie to ensure:

$$\langle u_i^m \rangle = \langle \hat{u}_i \rangle, \quad \langle u_i^m u_j^m \rangle = \langle \hat{u}_i \hat{u}_j \rangle, \quad \langle u_i^m u_j^m u_k^m \rangle = \langle \hat{u}_i \hat{u}_j \hat{u}_k \rangle$$

(*)

$$\langle p^m u_i^m \rangle = \langle \hat{p}\hat{u}_i \rangle, \quad \langle p^m S_{ij}^m \rangle = \langle \hat{p}\hat{S}_{ij} \rangle$$

and

$$\varepsilon_{ij}^m = \varepsilon_{ij}$$

* To get all these correct, with the requirement that we must seperately model transport and "dissipation" properly:

(*)

$$\langle S_{ik}^m \tau_{kj}^m \rangle + \langle S_{jk}^m \tau_{ki}^m \rangle = \langle \hat{S}_{ik}\tau_{kj} \rangle + \langle \hat{S}_{jk}\tau_{ki} \rangle$$

and

$$\langle u_i^m \tau_{jk}^m \rangle + \langle u_j^m \tau_{ik}^m \rangle = \langle \hat{u}_i \tau_{jk} \rangle + \langle \hat{u}_j \tau_{ik} \rangle + \underline{\text{Const}}$$

if we generalize this for high-Re isotropic flows we find that we can only get the correct resolved KE by having $\langle S_{ij}^m \tau_{ij}^m \rangle = \langle \hat{S}_{ij}\tau_{ij} \rangle$

* ie our model must extract energy at the correct Rate typically this is seen as the main task of an SGS model and most a priori studies focus on the study of $\langle \Pi^{m} \rangle = \langle \Pi \rangle$ (↳ real)

⇒ Discuss PDFs & eg we can make equivalent statements about the pdfs ⇒ to get pdf ($\Pi$) = pdf ($\Pi^m$) → pdf $\tau_{ij}$ etc.

⇒ So what are the key stats in most a priori studies?

1) $\langle \Pi^{m} \rangle$, $\langle \Pi \rangle$ average dissipation
   and PDF ($\Pi^m$), PDF ($\Pi$) + spatial distribution.
2) $\langle \tau_{ij}^{m} \rangle$, $\langle \tau_{ij} \rangle$ and PDFs and spatial distribution. (↳ depends on app and addresses transport issues)
3) model coefficients: $C_s$, $C_L$, $\beta^{CLN}$ etc.
4) many times all terms are compared by looking at correlation coeffs. (linear)

(add SGS force $\frac{\partial (\tau_{ij})}{\partial x_j}$)

⇒ What do we need to calculate these and how do we do it? (Go to project 2 description) circled stuff

⇒ What types of a priori studies exist? (see project 2 handout for description)

1) DNS (e.g. 3D)
2) wind tunnel / exp. in general (e.g. 2D)
3) atmospheric field Experiments (quasi 1D)

# LES LECTURE 20 (page 5)

Procedurally for each type what do we do? (LECTURE 21 starts here pg 1 (review))

1) DNS ⇒ have full velocity fields we can follow our procedures exactly. e.g. filter calc stats etc.

2) Reduced 2D data (PIV type) from wind tunnel / water tunnel etc.

- in these studies 2D data of $u, v$ (regular) or $u, v, w$ (stereoscopic) velocity are taken.
- DATA allows for 2D filtering (appropriate for BLs) see Higgins 2007
- Problem, what about $\Pi$? we need $\tilde{S}_{ij}$!

components of $\Pi = \tau_{11} S_{11} + \tau_{22} S_{22} \; \tau_{33} S_{33} + 2 S_{12} \tau_{12} + 2 \tau_{23} S_{23} + 2 \tau_{13} S_{13}$ ⇒ 6 terms!

common assumption: Carper Porte-Agel 2008, ~~Liu et al 94~~, (Chen et al 2005) for boundary layers with 3D

Break it up!

horizontal planes:

$$\Pi^M = -\tau_{11} \tilde{S}_{11} - 2\tau_{12} S_{12} - \tau_{22} \tilde{S}_{22}$$

vertical:

$$\Pi^M = -\tau_{11} \tilde{S}_{11} - 2\tau_{13} \tilde{S}_{13} - \tau_{33} \tilde{S}_{33}$$

switch

what kind of assumptions? (Liu in grid turbulence uses) for 2D PIV ($u, v$ only)

$$\langle \tau_{13} \tilde{S}_{13} \rangle = \langle \tau_{23} \tilde{S}_{23} \rangle = \langle \tau_{12} \tilde{S}_{12} \rangle$$

and

$$\langle \tau_{33} \tilde{S}_{33} \rangle = \langle \tfrac{1}{2} (\tau_{11} + \tau_{22}) \tilde{S}_{33} \rangle$$

how can we get $\tilde{S}_{33}$? ⇒ continuity by assuming incompressibility!

- in general we have to work with the $\tau_{ij}$ and $S_{ij}$ comp. we have.