LES of Turbulent Flows: Lecture 7
(ME EN 7960-008)

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Equations of Motion

Incompressible flow:

$$\frac{\partial u_i}{\partial x_i} = 0 \quad \text{Conservation of Mass}$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2} + F_i \quad \text{Conservation of Momentum}$$

$$\frac{\partial \theta}{\partial t} + \frac{\partial u_i \theta}{\partial x_j} = \nu_\theta \frac{\partial^2 \theta}{\partial x_j^2} + Q \quad \text{Conservation of scalar (temp, species, etc.)}$$

$$Sc \equiv \frac{\nu}{\nu_\theta} \Rightarrow \nu_\theta = \frac{\nu}{Sc} \text{ or } \frac{\nu}{\Pr}$$

Temperature (Pr=Prandtl #)

general scalar (Sc=Schmidt #)

Equations of Motion

- If we nondimensionalize these equations with a velocity scale and a length scale (for example the Freestream velocity and the BL height in a boundary layer)

- We get (where the * is a nondimensional quantity):

-Conservation of Mass: $$\frac{\partial u_i^*}{\partial x_i^*} = 0$$

- Conservation of Momentum:

$$\frac{\partial u_i^*}{\partial t^*} + \frac{\partial u_i^* u_j^*}{\partial x_j^*} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 u_i^*}{\partial x_j^{*2}} + F_i^*$$

where Re is based on our velocity and length scales => $Re = \frac{U_o \ell_o}{\nu}$

- For a general scalar quantity we have:

$$\frac{\partial \theta^*}{\partial t^*} + \frac{\partial u_j^* \theta^*}{\partial x_j^*} = \frac{1}{Sc\,\mathrm{Re}}\frac{\partial^2 \theta^*}{\partial x_j^{*2}} + Q^*$$

where Sc is the Schmidt number, the ratio of the diffusivity of momentum (viscosity) and the diffusivity of mass (for temperature we use the Prandtl number Pr). Sc is of order 1 (Pr for air ≈ 0.72)

Properties of the Navier-Stokes equations

• Reynolds number similarity: For a range of Re, the equations of motion can be considered invariant to transformations of scale.

• Time and space invariance: The equations are invariant to shifts in time or space. i.e., we can define the shifted space variable

$$\hat{x} = \bar{x}/L \text{ where } \bar{x} = x - X$$

$$\text{or } \hat{t} = (t - T)U/L$$

• Rotational and Reflection invariance: The equations are invariant to rotations and reflections about a fixed axis.

• Invariance to time reflections: The equations are invariant to reflections in time. They are the same going backwards or forwards in time =>

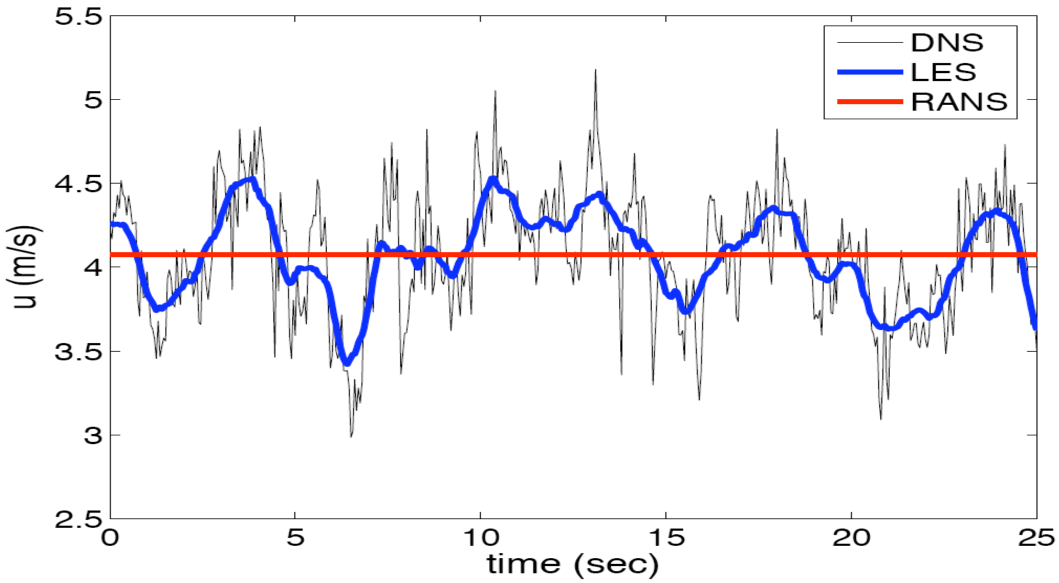
$$\hat{t} = -tU/L$$

• Galilean invariance: The equations are invariant to constant velocity translations.

$$\bar{x} = x - Vt$$

Approximating the equations of motion

- **In Numerical studies**, the equations of motion (incompressible, compressible or Boussinesq fluid) must be approximated on a computational grid
- **Three basic methodologies** are prevalent in turbulence application and research:
  - Direct Numerical Simulation (**DNS**)
    - resolve all eddies
  - Large-Eddy Simulation (**LES**)
    - resolve larger eddies, model smaller 'universal' ones
  - Reynolds-Averaged Navier-Stokes (RANS)
    - model just ensemble statistics



Some Pros and Cons of each Method

Direct Numerical Simulation (**DNS**):

- Pros
  - No turbulence model is required
  - Accuracy is only limited by computational capabilities
  - can provide reference data not available through experiments (i.e., unsteady 3D velocity and scalar fields)

- Cons
  - Restricted to low Re with relatively simple geometries
  - Very high cost in memory and computational time
  - typically "largest-possible" number of grid points is used without proper convergence evaluation.

Some Pros and Cons of each Method

Large-Eddy Simulation (**LES**):

- Pros
  - Only the small scales require modeling
  - Much cheaper computational cost than DNS
  - Unsteady predictions of flow are made => gain info about extreme events in addition to the mean
  - In principle, we can gain as much accuracy as desired by refining our numerical grid
- Cons
  - Basic assumption (small scales are universal) requires independence of small (unresolved) scales from boundary conditions (especially important for flow geometry).
  - Still very costly in practical engineering applications
  - Filtering and turbulence theory of small scales still needs development for complex geometry and highly anisotropic flows

Some Pros and Cons of each Method

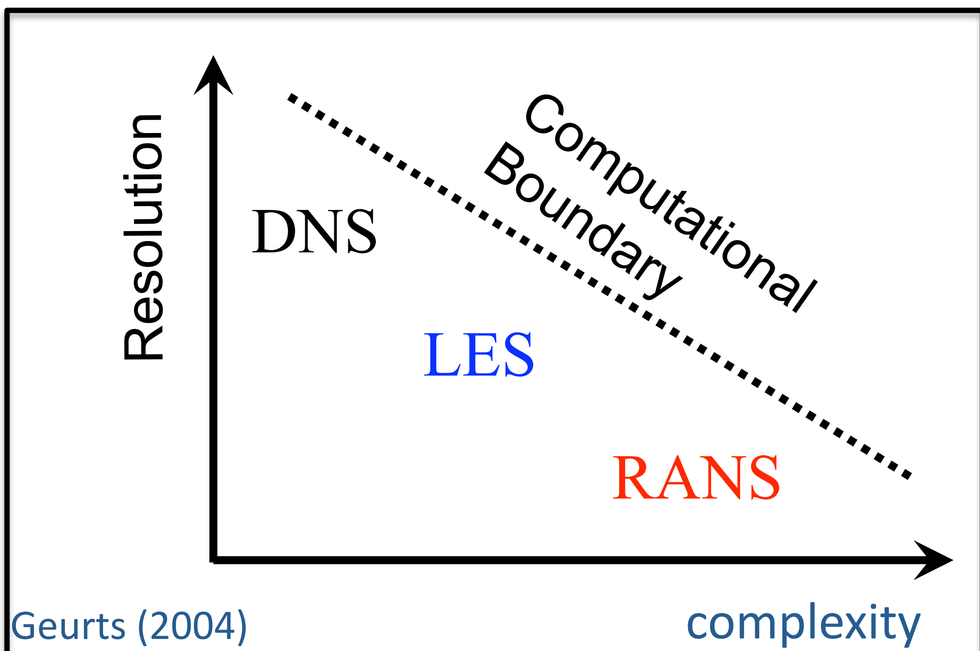
Reynolds Averaged Navier-Stokes (**RANS**):

• Pros

- Low computational demand (can obtain mean stats in short time)
- can be used in highly complex geometry
- When combined with empirical information, can be highly useful for engineering applications

•Cons

- Only steady flow phenomena are can be explored when taking full advantage of computational reduction
- Models are not universal => usually pragmatic “tuning” is required for specific applications
- More accurate turbulence models result in highly complex equation sets



Capabilities of different simulation methods