Curvature Based Point Stabilization for Compliant Framed Wheeled Modular Mobile Robots

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Abstract - **Posture stabilization of a compliant framed modular mobile robot is the subject of this paper. This is a new type of wheeled mobile robot that has advantages of a simple modular design that provides full suspension and steering capability without any additional components. Steering is achieved by coordinated control of the individual wheels in order to realize a desired trajectory. Due to the flexible nature of the robot, the kinematics is simplified by using an equivalent curvature based model, which increases mobility and decreases required traction forces for improved towing capacity. A time invariant control law is developed and extended to compensate for non-ideal initial conditions and drift. Simulation and experimental results are presented and show the proposed control law performs as expected.**

1. INTRODUCTION

A new breed of wheeled mobile robotic systems is the subject of this research: compliant framed modular mobile robots, Figure 1. The concept is unique in two ways. First, it uses a novel yet simple structure to provide suspension and highly controllable steering capability without adding any additional hardware to the system. This is accomplished by using flexible frame elements to couple rigid differentially steered axles. In this study, the frame element provides compliant roll and yaw between the axles. Relative roll provides suspension capability in order to accommodate uneven terrain, and yaw allows the axles to independently change heading for advanced steering capability. Steering and maneuvering of the system are thus accomplished via coordinated control of the axles. Since each axle can be steered independently, the system provides enhanced maneuverability in confined environments as well as the capability to control the shape of the frame.

A second unique aspect of the compliant framed mobile robot is its predisposition for modular mobile robotics. Reconfigurable modular robotic systems have been of keen interest to researchers during the last decade due to their improved ability to overcome obstacles and perform more tasks using a single hardware platform. Towards this goal, numerous researchers have devoted their efforts to investigating minimalist homogenous robotic modules. These systems have examined reconfigurable manipulation [1, 2], mobility [3-5], or a combination therein [6-8]. Homogeneity of the modules is argued to reduce maintenance, offer increased robustness through redundancy, provide compact and ordered storage, and increase the adaptability of the systems [9, 10]. The compliant frame allows this concept to be extended to wheeled mobile robots by allowing a number of different vehicle configurations to be formed from a set of uniform frame and axle modules.

The subject of this paper is an equivalent curvature based kinematic model developed for the purpose of simplified motion planning. Our robot is compared to existing forms of compliant mobile robots in Section 2 while also existing control strategies. The equivalent kinematic model is developed in Section 3 as well as the rational for its selection. A time invariant control law is developed in Section 4 for this model based upon the work of Indiveri [11]. Simulation and experimental results indicating the performance of the system are shown in Section 5. Future work and concluding remarks are discussed in Section 6.
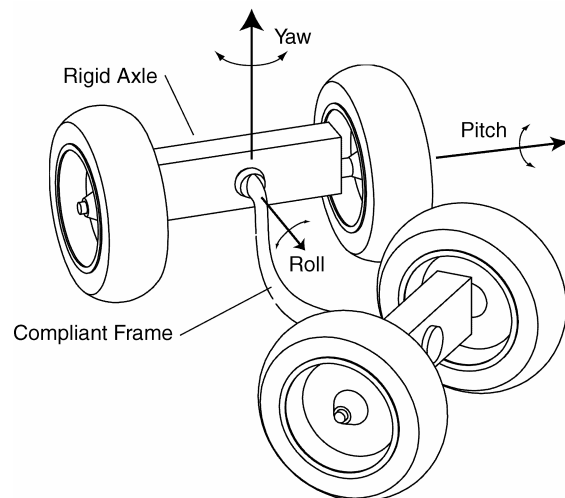

Figure 1 Compliant framed modular mobile robot.

2. BACKGROUND

A limited number of compliant vehicles can be found in the literature, and none possess a similar highly compliant frame whose deflection is controlled by coordinated actuation of the wheels. Earliest found reference to compliant vehicles is a system proposed for planetary exploration that uses compliant members to provide roll and pitch degrees of freedom for suspension capability between the axles [12]. This concept was later extended [13] in a design where the frame of a vehicle was composed of at least one helical spring, but hydraulic cylinders were then to be used to control the deflection. In each of these cases, compliance was introduced for accommodating terrain. More recent research has introduced compliance for accommodating measurement error and resulting wheel slip occurring between independently controlled axle units on a service robot [14]. This robot is similar in spirit to the compliant framed system in that it allows relative rotation between the axles, but this is provided by rotary joints connected to the ends of a frame with limited prismatic compliance. The system is intended for operation on flat surfaces in industrial service settings. As the author states, the system provides high levels of mobility, but since the axle units are coupled by a relatively non conforming rigid frame, its ability to maneuver in confined environments will be limited [15]. Other flexible robots use actuated articulated joints to provide similar relative motion between axles, as in the case of the Marsokhod rover [16] and other six wheeled research rovers with high relative DOF provided between axles modules [17]. The compliant frame mobile robotics system proposed here allows independent steering control of the axles with minimal slip and no additional hardware or actuators.

Control of mobile robots and nonholonomic systems has received a great deal of attention in recent years and many control alternatives have been proposed such as time-varying [18], adaptive [19], discontinuous [20], and neural network based [21] strategies. For a thorough survey of nonholonomic control techniques see the review in [22]. Many of the proposed techniques are well suited to unicycle type vehicles with the ability to perform a zero radius turn. While the kinematics of the compliant framed mobile robot are much more complex, we will show that they can be described in an equivalent coordinate frame that admits familiar steering algorithms. In particular, we extend controller dynamics discussed by Indiveri [11] and Aicardi et al. [23] to accommodate our non-ideal initial conditions. This is similar in spirit to the extension performed by Astolfi [24] in order to accommodate nonholonomic systems with drift.

3. KINEMATIC MODEL

The compliant framed mobile robot has much more complex steering kinematics than unicycle type vehicles since it possess independently steered axles with compliant coupling. Depending on the frame deflection imposed by the kinematics of the axles, variations in wheel forces can be modeled by considering the frame module as an Euler-Bernoulli beam [25]. This is represented in Figure 2 where the deflection is considered relative to the rear axle, which is denoted by the grounded connections on the beam for convenience. As the figure indicates, the axles can impose several fundamental shapes of the frame, each with their own variations in required traction forces. All axle configurations can be classified as some combination of these fundamental shapes. Case 1 would result when the axles are coordinated to maintain a pure bending, constant curvature condition which is the equivalent of $\psi_f = \psi_r$. This is equivalent to the first mode of bending for a simply supported beam. Cases 2 and 3 result when a moment and transverse force are applied to the beam. Case 2 is realized when the front axle is steered in a car like fashion with $\psi_r=0$. Case 3 may be produced when each axle is steered to the goal independently.

In order to evaluate which of these cases will require the least power and traction, the wheel traction forces due to the moments M for a given beam deflection angle Θ, can
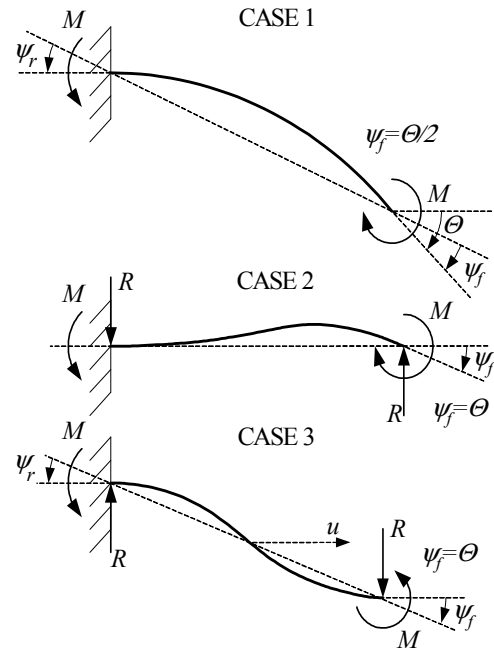

Figure 2 Euler-Bernoulli beam cases

be described by the equation,

$$F_m = \frac{\Theta EI}{2al} + \frac{Rl}{4a} \qquad (1)$$

where l is the length of the complaint link and a is the axle half width as shown in Figure 3, and ideal beam assumptions apply. The wheel traction forces due to the boundary conditions for the three cases are shown in Table 1 where Cases 2 and 3 would also have a lateral traction force due to the reaction force R, which produces a net wheel traction force determined by,

$$F_T^2 = F_m^2 + R^2 \qquad (2)$$

As inspection of the equations reveals, the minimum traction force required for steering the robot is realized when the system is deflected in the first mode of beam bending, Case 1. By minimizing the required traction force, the robot is able to exert larger forces for towing a load or accelerating the robot, and the probability of wheel slip is greatly reduced. As it turns out, Case 1 also results in the smallest Turning Radius, Table 1. Radius of curvature can be realized by considering the intersection of velocity normals relative to the center of the frame, as shown in Figure 3.

Thus, Case 1 was selected for our equivalent model since it requires minimum traction and provides maximum steering capability for the robot. Further illustration of the system in the Case 1 configuration is shown in Figure 3. Using this scenario to derive the kinematics, the net system position and orientation of the robot may be described by an equivalent posture attached to point O located at the center of a line drawn between the axle midpoints. Assuming that the front and rear axle's

steering angle ψ have equal absolute values and opposite signs, the familiar Cartesian kinematics for the center point are defined as,

$$\dot{x} = u \cos \phi$$
$$\dot{y} = u \sin \phi \qquad (3)$$
$$\dot{\phi} = \omega$$

where x and y are the Cartesian coordinates of a moving coordinate frame attached to the point, O, that describe the equivalent posture. The variable u represents the velocity of the coordinate frame moving in a heading ϕ relative to the global frame, and ω is the rate of change of ϕ. Eq (3) can then be derived in terms of the individual axle kinematics commonly found in the literature.

Before deriving the equivalent kinematic relationships, several desirable conditions must be described:

1. As opposed to the case of a unicycle type robot, the compliant framed mobile robot cannot turn with a zero radius of curvature. Thus the robot should proceed only on paths of bounded curvature.
2. For simplicity it is desired that the robot's motion proceed only in the forward direction.
3. The pure bending condition, $\psi_f = \psi_r$, should be maintained throughout all turning maneuvers to ensure Case 1 and also prevent wheel slip.

Table 1 Robot traction and steering performance, $K=a/l$.

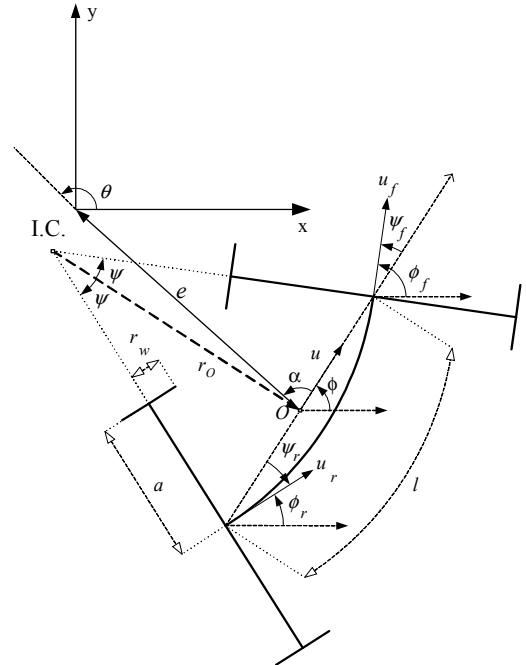| Case	Reaction Force, R	Traction Force, F_T	Turning Radius (approx. for small ψ_f)
1	0	$\dfrac{EI\psi_f}{Kl^2}$	$\approx \dfrac{l}{2\psi_f} = \dfrac{0.183}{\psi_f}$
2	$\dfrac{6EI\psi_f}{l^2}$	$2\dfrac{EI\psi_f}{Kl^2}\sqrt{1+9K^2}$	$\approx \dfrac{l}{2}\sqrt{1 + \dfrac{4}{\tan^2(\psi_f)}}$ $\approx \dfrac{0.366}{\psi_f}$
3	$\dfrac{12EI\psi_f}{l^2}$	$3\dfrac{EI\psi_f}{Kl^2}\sqrt{1+16K^2}$	∞

Figure 3 Compliant framed mobile robot kinematics

The point O can also be represented in polar like coordinates where

$$e = \sqrt{x^2 + y^2}$$
$$\theta = ATAN2(-y,-x) \qquad (4)$$
$$\alpha = \theta - \phi$$

with the system state equations thus defined as,

$$\dot{e} = -u\cos\alpha$$
$$\dot{\alpha} = -\omega + \frac{\sin\alpha}{e} \qquad (5)$$
$$\dot{\theta} = u\frac{\sin\alpha}{e}$$

The advantage of this polar like representation is that the state itself is not defined for $e = 0$, and therefore Brocketts Theorem does not hold and a smooth time invariant state feedback control law for global asymptotic stability is possible [24], where several controllers in the literature have been similarly implemented [11, 23, 24].

Using this transformation, the angular velocity of the equivalent posture attached to point O can be described as a function of the bounded curvature, or inverse turning radius, c.

$$\omega = \dot{\phi} = uc \qquad (6)$$

Substitution into the polar state equations (5) yields

$$\dot{e} = -u\cos\alpha$$
$$\dot{\alpha} = -u\left(c - \frac{\sin\alpha}{e}\right) \qquad (7)$$
$$\dot{\theta} = u\frac{\sin\alpha}{e}$$

The angular velocity of the robot center point can be described as a function of the steering angles ψ_f and ψ_r (Figure 3). Once again, it is assumed for simplicity and to ensure the pure bending condition that $\psi_f = \psi_r$. Expressions for the radius and curvature of the robot center point can be shown to be

$$r_O = \frac{l}{2\psi}\cos\psi \qquad c = \frac{1}{r_O} = \frac{2\psi}{l\cos\psi} \qquad (8)$$

The angular velocity of O then becomes

$$\dot{\phi} = uc = u\frac{2\psi}{l\cos\psi} \qquad (9)$$

and steering angle ψ may be solved for numerically using the equation,

$$\frac{\psi}{\cos\psi} = \frac{c\,l}{2} \qquad (10)$$

Referring again to Figure 3, the velocities u_f and u_r are shown to be

$$u_f = u_r = \frac{u}{\cos\psi} \qquad (11)$$

From Figure 3 it can then be deduced that,

$$\phi = \theta - \alpha$$
$$\phi_f = \theta - \alpha + \psi \quad \phi_r = \theta - \alpha - \psi \qquad (12)$$
$$\dot{\phi}_f = \dot{\theta} - \dot{\alpha} + \dot{\psi} \quad \dot{\phi}_r = \dot{\theta} - \dot{\alpha} - \dot{\psi}$$

Thus, the linear and rotational velocity of each axle may be found from u and c, the respective linear velocity and path curvature of the center posture O.

4.   TIME INVARIANT CONTROL LAW

Based upon the Lyapunov function,

$$V \equiv \frac{1}{2}\left(\alpha^2 + h\theta^2\right) \qquad (13)$$

Indiveri [11] suggest the use of the control law

$$u = \gamma e : \mathrm{sat}\left(\gamma\mathrm{e},\overline{u}\right)$$
$$c = \frac{\sin\alpha}{e} + h\frac{\theta}{e}\frac{\sin\alpha}{\alpha} + \beta\frac{\alpha}{e} \qquad (14)$$

which are smooth time functions that asymptotically drive the state (e, α, θ) towards the origin. The parameters $\gamma, \beta,$ *and* $h>0$ are constant gains and $\mathrm{sat}\left(\gamma\mathrm{e},\overline{u}\right)$ is a positive continuous saturation function that prevents the proportional control input u to grow larger than some upper bound $\overline{u}$. The controller meets conditions (*i*) and (*ii*) set forth in Section 3 and is of a simple yet effective design. Further details of this and other similar proofs can be found in [11, 23, 24].

In the case of the compliant framed mobile robot, the robot initial conditions will rarely match the initial conditions for u and c output by the controller. In addition, perturbations throughout the robot path resulting in error between the desired u and c and the actual u and c will result in error of the final stabilization point. This drift may be resolved with a change in the feedback where the controller dynamics are extended and written as a cascade system of the form

$$\dot{y} = f(e,\alpha,\theta,u,c)$$
$$\dot{u} = g(u,u_D) \tag{15}$$
$$\dot{c} = j(c,c_D)$$

where u_D and c_D are the desired velocity and curvature relations established by Eq. (14). We then perform a change of coordinates,

$$Y = y$$
$$U = u - u_D \tag{16}$$
$$C = c - c_D.$$

where U and C are error states for our desired velocity and curvature. The transformed system becomes,

$$\dot{Y} = f(e,\alpha,\theta,U + u_D, C + c_D)$$
$$= F(e,\alpha,\theta,U,C)$$
$$\dot{U} = \dot{u} - \dot{u}_D \tag{17}$$
$$\dot{C} = \dot{c} - \dot{c}_D$$

After applying the feedback,

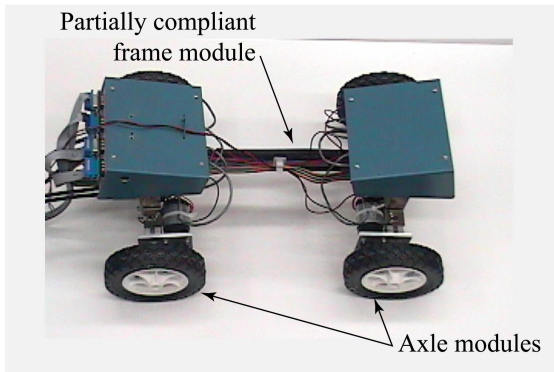Figure 4 Partially Compliant test-bed. Length $l = 0.366$ m, half wheel base, $a = 0.168$ m, wheel radius = 0.073 m

$$\dot{u} = k_u(u_D - u) + \dot{u}_D$$
$$\dot{c} = k_c(c_D - c) + \dot{c}_D \tag{18}$$

where k_u and k_c are positive constants, the closed-loop system becomes,

$$\dot{Y} = f(e,\alpha,\theta,U,C)$$
$$\dot{U} = -k_u U \tag{19}$$
$$\dot{C} = -k_c C$$

Thus, U and C are exponentially stabilized. As time goes to infinity, U=0 and C=0, and the system formulation becomes,

$$Y = y = F(e,\alpha,\theta,0,0)$$
$$= f(e,\alpha,\theta,u_D,c_D) \tag{20}$$

which is asymptotically stabilized about the origin as Bacciotti [26] Theorem 19.2 shows. In summary, substituting u_D and c_D into (18) and combining with (7) the polar state equations of the compliant framed mobile robot now become,

$$\dot{u} = k_u(\gamma e - u) + \dot{u}_D$$
$$\dot{c} = k_c(\frac{\sin\alpha}{e} + h\frac{\theta}{e}\frac{\sin\alpha}{\alpha} + \beta\frac{\alpha}{e} - c) + \dot{c}_D$$
$$\dot{e} = -u\cos\alpha \tag{21}$$
$$\dot{\alpha} = -u\left(c - \frac{\sin\alpha}{e}\right)$$
$$\dot{\theta} = u\frac{\sin\alpha}{e}.$$

5.   SIMULATION AND EXPERIMENTAL RESULTS

The controller was tested on the Partially Compliant Test Bed at the University of Utah, Figure 4.   Simulation results of the above controller, shown in Figure 5 and Figure 6, compare experimental results to those predicted by computer simulation.   As the results indicate, the algorithm performed well. The robot appears to conform to conditions 1, 2, and 3 from section 3 and stabilizes to the origin well considering the dynamics of the robot are not currently accounted for in the control of the robot. Some instability of the robot was noted very near the origin, as any error near the origin causes the controller to output unreasonable values of c.   Future work is

concentrating on reducing this instability near the origin, implementing path tracking capability based upon this curvature based technique, compensating for the dynamics of the robot in control, and implementing a sensor fusion algorithm that improves relative posture measurements using the beam itself as an internal configurations sensor.

6. CONCLUSIONS

This paper has presented a new breed of wheeled mobile robot, the compliant framed modular mobile robot. The robot has several advantages over existing compliant robots including a simple modular design that provides suspension and steering capability. The complicated robot kinematics are simplified by developing an equivalent curvature based model employing the pure bending condition, which is shown to be the most effective steering method. A time invariant control law is developed utilizing discontinuous polar mapping based upon the work of Indiveri [11] and is extended to compensate for non-ideal initial conditions and drift. Simulation and experimental results are presented that show the proposed control law performs as expected.

Figure 5 Robot paths with $e_0=5l$ and $\gamma=.3$, $\beta=2.1$, $h=2$, $k_u=2$, $k_c=2$, $\bar{u}=.3$, for $\theta_0=0$, $\pi/4$, $\pi/2$, $3\pi/4$, π, $5\pi/4$, $3\pi/2$, $7\pi/4$, $\phi_0=0$. Robot equivalent center, O, is represented by solid lines, wheel paths by dashed.
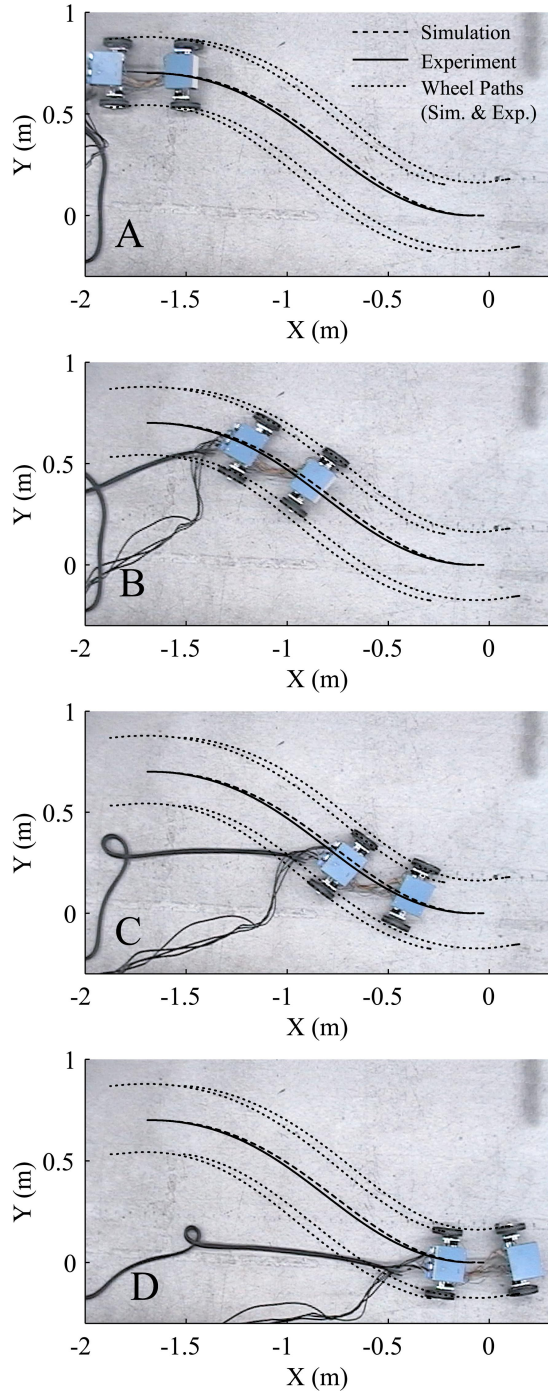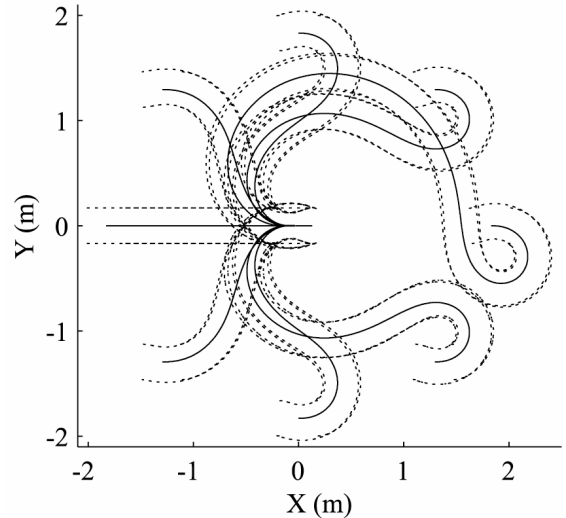
Figure 6 Robot snapshots at A: $t=0$, B: $t=3$, C: $t=5$, D: $t=12$ sec., for $\gamma=.3$, $\beta=2.1$, $h=2$, $k_u=2$, $k_c=2$, $\bar{u}=.3$, $e_0=5l$, $\theta_0=7\pi/8$, $\phi_0=0$. Simulation and experimental wheel paths are essentially coincident.

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