A spherical, iron cannon ball of diameter 15.0 cm rests on the surface of the Earth at elevation $z = 0.00$ m.

a) Calculate the mass (kg) of the ball.
b) Calculate the force (N) exerted by the cannon ball on the surface.
c) Calculate the work (J and kJ) required to raise the cannon ball 1000 m above the surface of the earth ($z = 1000$ m).

From Table A-3, the density of iron, $\rho$, is 7840 kg/m$^3$. The diameter, $D$, is 0.15 m. The mass, $m$, is the product of density and volume.

$$m = \rho (Vol) = \rho \frac{4}{3} \pi \left( \frac{D}{2} \right)^3 = 7840 \frac{kg}{m^3} \frac{4}{3} \pi \left( \frac{0.15 \frac{m}{2}}{2} \right)^3 = 13.85 \text{ kg}$$
I. Concepts and Definitions

H. Review of basic physics
1. Mass, force, and work
   b. Force exerted by ball on surface

   The standard acceleration of gravity, \(g\), from the inside front cover of the text, is 9.81 m/s\(^2\). Newton’s second law, Equation 1-1, p. 6 of the text, defines the force.

   \[
   \text{force} = \text{mass} \times \text{acceleration}
   \]

   \[
   F = mg = 13.85 \text{ kg} \left(9.81 \frac{m}{s^2}\right) = 153.9 \text{ N}
   \]

   \[
   N = \frac{9.153}{7.81} \cdot \frac{8}{5.13} = \frac{135.9}{10} \cdot 10^{-6} \text{ kg m/s}\]

   Note that 1 J = 1 kg m\(^2\)/s\(^2\).

I. Concepts and Definitions

H. Review of basic physics
1. Mass, force, and work
c. Work performed on ball in raising it 1000 m

   The work, \(W\), performed on a body that is displaced a distance \(\Delta z\) in the direction of the constant force \(F\) is

   \[
   W = F \Delta z
   \]

   \[
   W = mg \Delta z = 13.85 \text{ kg} \left(9.81 \frac{m}{s^2}\right) 1000 \text{ m}
   \]

   \[
   = 1.359 \times 10^5 \text{ J} = 135.9 \text{ kJ}
   \]

   \[
   z = 1000 \text{ m}
   \]

   \[
   z = 0 \text{ m}
   \]

   \[
   \text{Note that 1 J = 1 kg m}^2/\text{s}^2.
   \]

   \[
   \text{Note that 1 J = 1 kg m}^2/\text{s}^2.
   \]
I. Concepts and Definitions

H. Review of basic physics

2. Potential energy

The potential energy of the ball at elevation \( z = 1000 \text{ m} \) is

\[
\Delta P_E = mg \Delta z = 13.85 \text{ kg} \left( 9.81 \frac{m}{s^2} \right) 1000 \text{ m} = 1.359 \times 10^5 \text{ J}
\]

The total (system + surroundings) change in energy for this process is

\[
\Delta E_{\text{total}} = \Delta E_{\text{system}} + \Delta E_{\text{surroundings}} = 0
\]

The change in energy of the system (the ball) is

\[
\Delta E_{\text{system}} = \Delta P_E = mg \Delta z = 1.359 \times 10^5 \text{ J}
\]

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I. Concepts and Definitions

H. Review of basic physics

3. Kinetic energy

If the cannon ball is released from elevation \( z = 1000 \text{ m} \) and allowed to fall freely to the ground, calculate its velocity just before it strikes the surface of the earth. Neglect drag between the ball and the surrounding air.

\[
\Delta E_{\text{total}} = \Delta E_{\text{system}} + \Delta E_{\text{surroundings}} = 0
\]

\[
\Delta E_{\text{system}} = \Delta U + \Delta P_E + \Delta K_E = 0
\]

\[
m g (z_2 - z_1) + \frac{1}{2} m (V_2^2 - V_1^2) = 0
\]

\[
V_2 = \sqrt{2g z_1} = \left[ 2 \left( 9.81 \frac{m}{s^2} \right) 1000 \text{ m} \right]^{1/2} = 140 \text{ m/s}
\]
I. Concepts and Definitions

H. Review of basic physics

4. Internal energy

a. Ideal, monatomic gas (helium, neon, argon, xenon)

From statistical thermodynamics, the average kinetic energy of a He atom is

\[ \frac{1}{2} m \langle V^2 \rangle = \frac{3}{2} k T \]

Boltzmann constant = 1.381x10^{-23} J/K

The gas constant is related to the Boltzmann constant and the Avogadro constant \( N_A \) by

\[ R_u = N_A k \text{ where } N_A = 6.022x10^{23} \text{ mol}^{-1} \]

The internal energy of a kilomole of an ideal, monatomic gas is (kJ/kmol)

\[ \bar{u} = \frac{3}{2} R_u T \]

The internal energy of a kilogram of an ideal, monatomic gas is (kJ/kg)

\[ u = \frac{3}{2} \frac{R_u}{M W} T = \frac{3}{2} R T \]
I. Concepts and Definitions

H. Review of basic physics
4. Internal energy
   b. Ideal diatomic gases at room temperature (oxygen and nitrogen are both diatomic)

\[ \bar{u} = \frac{5}{2} R_u T \]
\[ u = \frac{5}{2} R T \]

There are 5 degrees of freedom: three translational (x, y, z directions) and two rotational (about x- and z-axes).

These equations for diatomic molecules are limited to room temperature because at higher temperatures, vibration of the O-O bond further increases the number of degrees of freedom.

Example. The internal energy of a kilogram of air (a mixture of N\textsubscript{2} and O\textsubscript{2}) at 300 K is

\[ u = \frac{5}{2} \frac{R_u T}{MW} = \frac{5}{2} \frac{RT}{2} \left( \frac{8.314 \text{kJ/kmol K}}{29 \text{kg/kmol}} \right) 300 \text{ K} = 215 \text{kJ/kg} \]

Table A-17 in text, for air at 300 K, gives \( u = 214 \text{kJ/kg} \).