CS 6640: IMAGE PROCESSING
Fall 2011
Take Home Midterm Exam — Due Nov 23

Name: ________________________________
Student ID Number: ________________________________

Rules:

- You may use online or written resources, but you *may not* work with anyone else or talk to anyone else except for the professor and the TA.
- Answers must fit on this test (front and back). No additional pages.

Hints:

- The term "describe" does not mean complete sentences and paragraphs or essays. If it's easier you may use simple bullets and meaningful phrases to answer such questions.
- If you split answers across pages (or on the backs of pages) make a clear note on the page where the question is posed to indicate you have done so. Clearly note the question number (and part) on the separate page.
- There are **six** questions for a total of 100 points. Point values are roughly correlated with the amount of time you should devote to each question.

1. **[20 pts.]** This question deals with the topic of *median filtering*.

   Write a pseudocode implementation of the median filter that runs over an $N \times N$ image for a specified $M \times M$ window size. You may assume the existence of an "Array" datastructure/object and that a "Sort" command exists for this object.

   For partial (most) credit, start with a simple version of the algorithm that ignores efficiency and image boundaries. However, to get full credit, you should take care of image boundaries and build an efficient algoirthm for larger windows (e.g. better than $M^2 \log(M) N^2$ run time).

2. **[20 pts.]** Consider the image alignment or warping problem based on correspondences $(\bar{c}_1, \bar{c}_1'), (\bar{c}_2, \bar{c}_2'), \ldots (\bar{c}_N, \bar{c}_N')$, where $\bar{c}_i = (x_i, y_i)$ and $\bar{c}_i' = (x_i', y_i')$. (Hint: Don't generate a lot of text for this question. Give short answers and equations.)

   (a) Suppose we wish to find a simple translation (x_0, y_0), and we wish to over constrain the system. What penalty function would we minimize (to get a good answer) and what linear system would we use to solve the problem? Give the equations/matrices.

   (b) Suppose we wish to find an affine transformation (linear in x and y) from one image to the other. What linear system would we use? Give equations/matrices.

3. **[15 pts.]** Give a continuous Fourier transforms for the following.

$$f(x) = \begin{cases} \frac{1}{2}\left(\cos(\pi x) + 1.0\right) & -1 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

4. **[15 pts]** Consider the filter which is, in the spatial domain a *differenece of Gaussians*:

$$d(x, y) = g_{\sigma_1}(x, y) - g_{\sigma_2}(x, y)$$

where g_σ is a Gaussian with standard deviation σ, and $\sigma_1 < \sigma_2$

(a) Give the equations for the Fourier tranform of $d(x, y)$

(b) In the Fourier domain, we often chacterize filters as low pass, band pass, or high pass. What type of filter is d? Use graphs to explain.

(c) Explain how the choices of $\sigma_{1,2}$ affect d's behavior as a (LP or BP or HP) filter.

5. **[15 pts.]** The derivative of an image is often computed using finite differences. We can write the continuous form of this finite difference operator as (for the x derivative):

$$d(x, y) = \frac{1}{2}[f(x+1, y) - f(x-1, y)]$$

Give the continuous Fourier transform, $D(u, v)$ of this finite difference operator. Use Euler's identity to reduce redundant terms. If we were to characterize this as a "filter" in the Fourier domain, what type of filter would it be?

6. **[15 pts.]**

(a) Use the convolution theorem and the Fourier transform of sinusoids to prove that any function convolved with a sinusoid is another sinusoid of the same frequency.

(b) Show that the multiplication (in the space domain) of a cosine of frequency ω with a low-pass kernel (kernel associated with a low-pass transform) results in a band-pass kernel. (Use diagrams if necessary) What restrictions are there on the low-pass transform for this to work? What role does ω play?