CS6640, Spring 2009, Homework Assignment #1

- Please show your intermediate results as well as final solutions.
- You may hand in your assignments on paper (under my office door).

1. Prove that the convolution of an even function and an odd function is and odd function.
2. The total energy content of a function (with bounded domain) is the same in both the frequency domain and the time domain. I.e.
$$\int_{-\infty}^{\infty} |f(t)|^2 dt = \int_{-\infty}^{\infty} F(s)F^*(s)ds.$$
This is called Rayleighs Theorem. Prove it.
3. Problem 4.12 in the book.
4. Problem 4.20 in the book.
5. A very useful function for image processing is the "rect" or "box" function, defined below.
$$\text{rect(x)} = \begin{cases} 1, & |x| < \frac{1}{2} \\ 0, & |x| > \frac{1}{2} \end{cases} \tag{1}$$
Compute the Fourier transform of $rect(x)$, and make a rough graph of this function (Use L'Hopital's rule to figure out how it behaves at the origin). Another useful function is the triangle function, which is defined as the box convolved with itself. Give a description (like that of rect above) of the triangle filter and give an equation of its Fourier transform.
6. Based on your results from the previous problem given an expression for the Fourier transform of
$$f(x) = \begin{cases} 0, & x < 0, or\ x > a \\ 1, & 0 \leq x \leq a \end{cases} \tag{2}$$
7. Calculate the Fourier transform of $f = \{1, 1, 1, 1\}$.
8. For a linear system, the input is
$$f(t) = \begin{cases} -\frac{1}{2}, & t < 0 \\ 0, & t = 0 \\ +\frac{1}{2}, & t > 0 \end{cases} \tag{3}$$
and the output is
$$h(t) = \begin{cases} -\frac{1}{2}, & t < -1 \\ t, & -1 \leq t \leq 1 \\ +\frac{1}{2}, & t > 1 \end{cases} \tag{4}$$
Calculate the characteristic function $g(t)$ of this system.