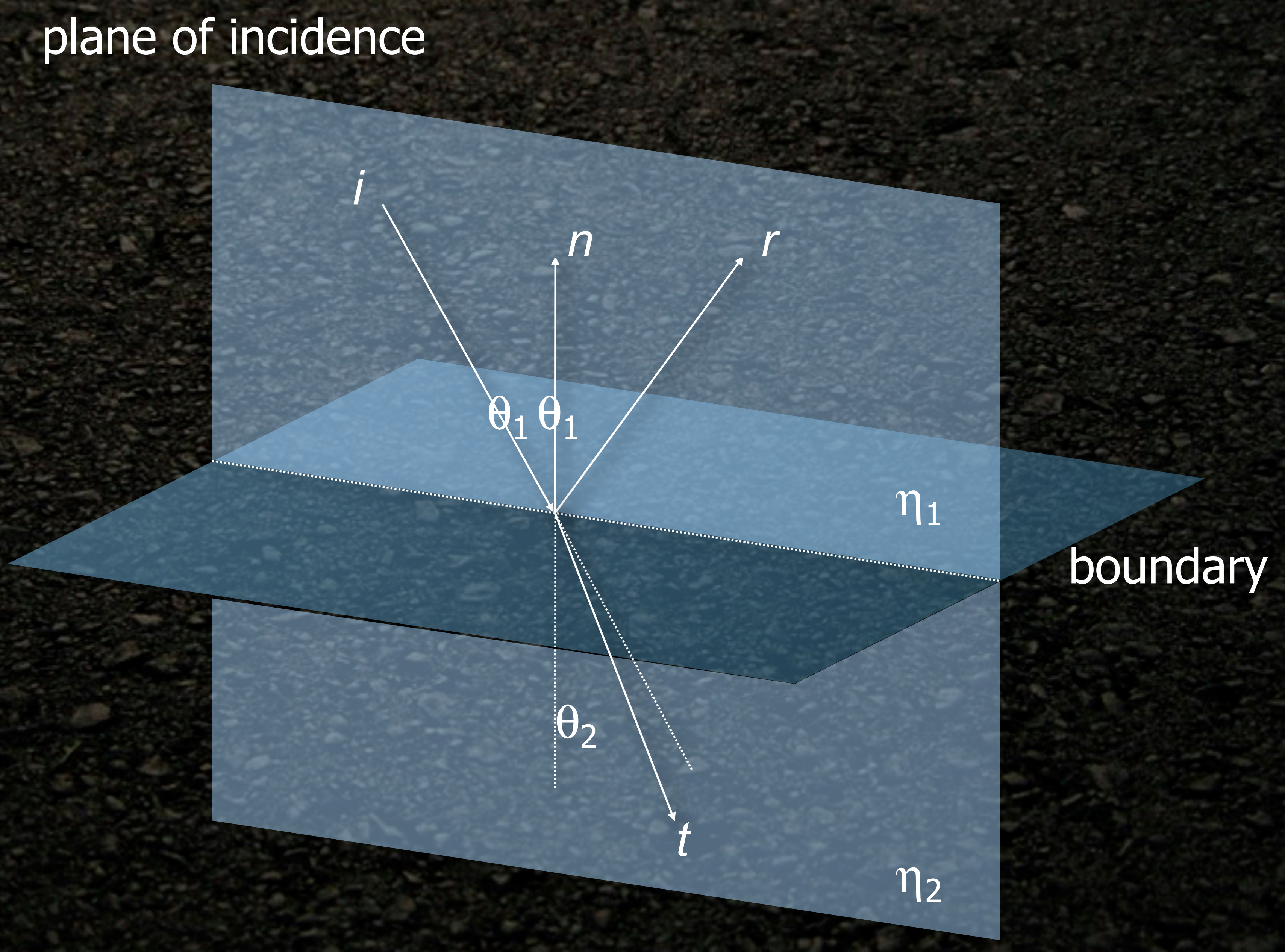
Reflected and transmitted rays



Index of refraction

Absolute index of refraction:

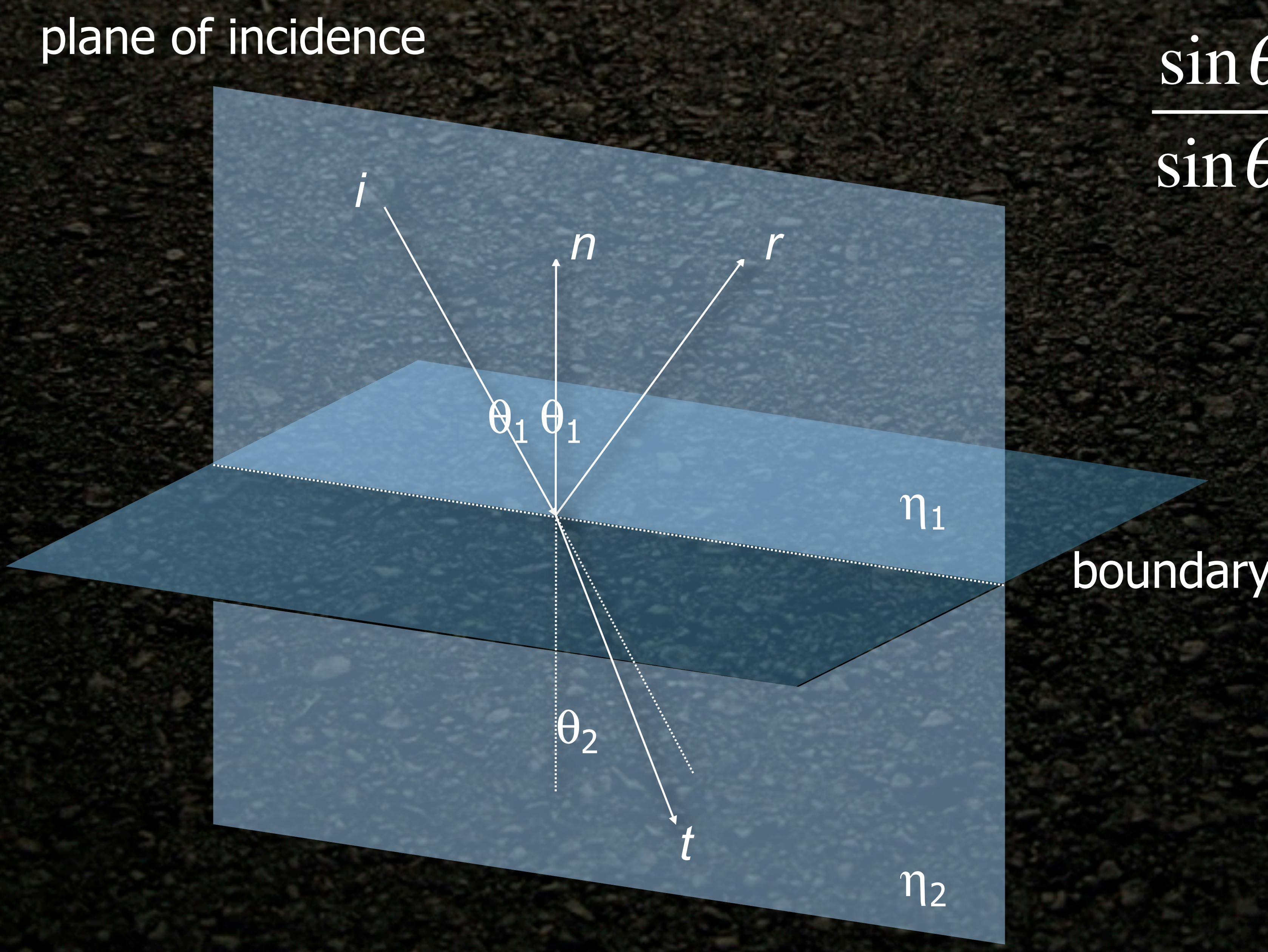
$$\eta_{abs} = \frac{c}{v}$$

where c = 2.99 x 10^8 m/s. Note: $v \leq c$ for all transparent materials, so $\eta \geq 1$.

Relative index of refraction:

$$\eta = \frac{\eta_2}{\eta_1}$$

Snell's Law



$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{\eta_2}{\eta_1} = \eta = \eta_{12}$$

Transmission

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\eta_2}{\eta_1} = \eta = \frac{S_1}{S_2}, S_2\eta = S_1$$

$$\cos\theta_1 = C_1 = -\vec{N}\cdot\vec{V}$$

$$\cos\theta_2 = C_2 = -\vec{N}\cdot\vec{T}$$

Square both sides of $S_1\eta = S_2$:

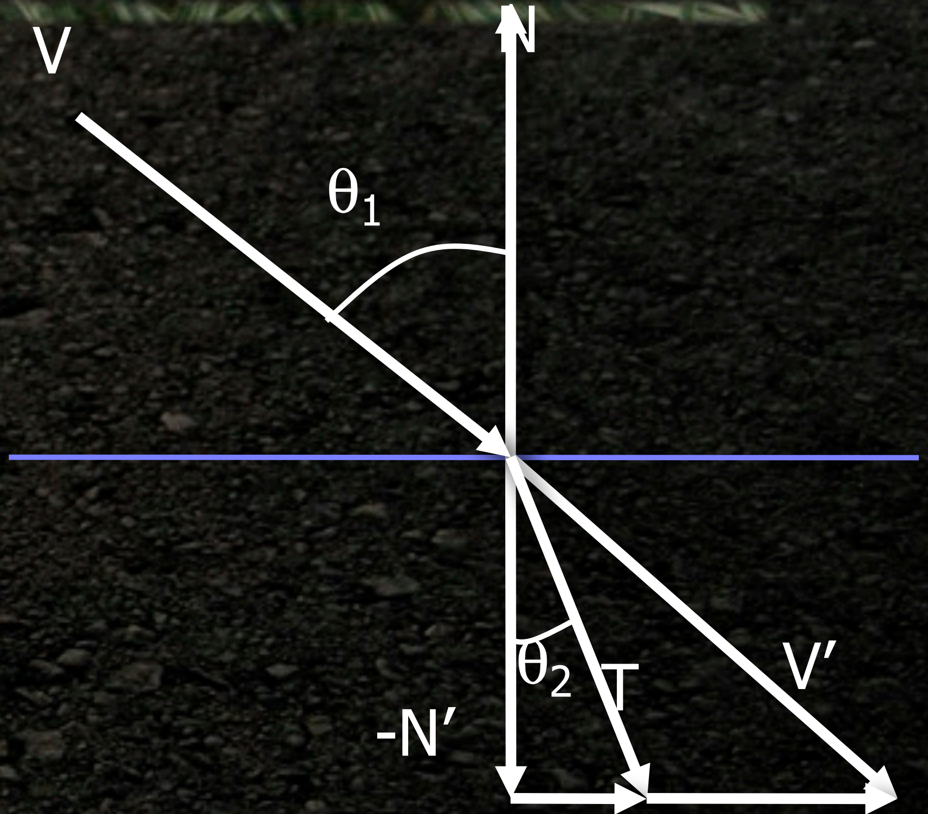
$$S_2^2\eta^2 = S_1^2$$

And use $S^2 + C^2 = 1$ to get cosine forms :

$$\left(1 - C_2^2\right)\eta^2 = 1 - C_1^2$$

$$C_2^2 = 1 + \frac{\left(C_1^2 - 1\right)}{\eta^2}$$

$$C_2 = \sqrt{1 + \frac{\left(C_1^2 - 1\right)}{\eta^2}}$$



Transmission

$$\vec{T} = -\overrightarrow{N'} + k\left(\overrightarrow{V'} + \overrightarrow{N'}\right) = k\overrightarrow{V'} + (k-1)\overrightarrow{N'}$$

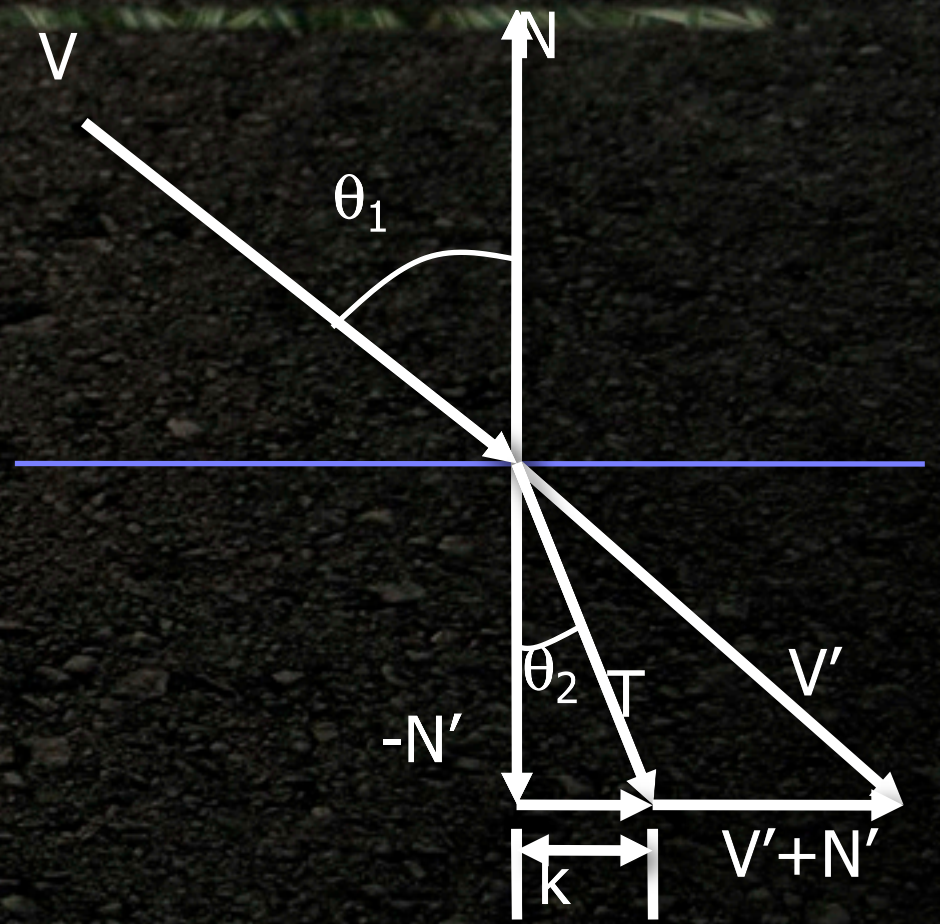
$$\left|\vec{T}\right| = 1$$

$$\overrightarrow{N'} = C_2\vec{N}$$

$$\left|\overrightarrow{V'}\right|C_1 = \left|-\overrightarrow{N'}\right|, \left|\overrightarrow{V'}\right| = \frac{\left|-\overrightarrow{N'}\right|}{C_1}, \left|\overrightarrow{V'}\right| = \frac{C_2}{C_1}$$

$$\overrightarrow{V'} = \frac{C_2}{C_1}\vec{V}$$

$$k = \frac{S_2}{S_1}\frac{\left|\vec{T}\right|}{\left|\overrightarrow{V'}\right|} = \frac{S_2}{S_1}\frac{1}{\frac{C_2}{C_1}} = \frac{S_2C_1}{C_2S_1}$$



Transmission

$$\vec{T} = -\vec{N'} + k\left(\vec{V'} + \vec{N'}\right) = k\vec{V'} + (k-1)\vec{N'}$$

$$k = \frac{S_2}{S_1}\frac{\left|\vec{T}\right|}{\left|\vec{V'}\right|} = \frac{S_2}{S_1}\frac{1}{\frac{C_2}{C_1}} = \frac{S_2 C_1}{C_2 S_1}$$

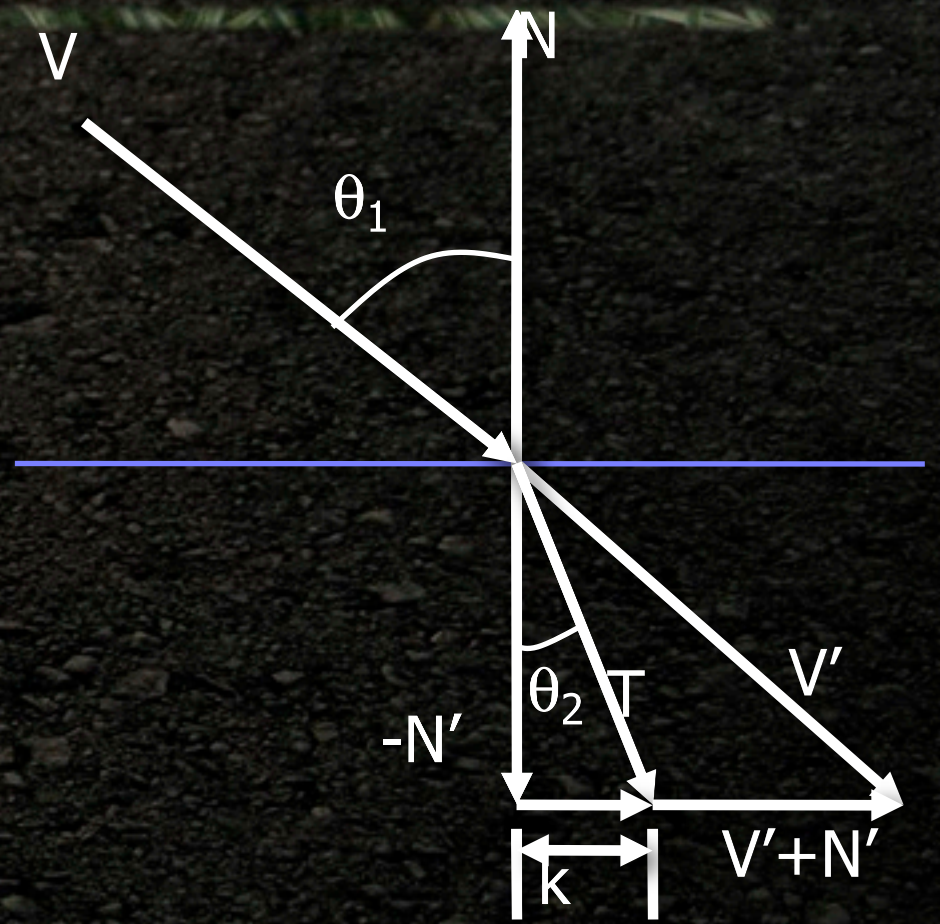
$$C_2 = \sqrt{1 + \frac{\left(C_1^2 - 1\right)}{\eta^2}}$$

$$\vec{T} = \frac{S_2 C_1}{C_2 S_1}\frac{C_2}{C_1}\vec{V} + \left(\frac{S_2 C_1}{C_2 S_1} - 1\right) C_2 \vec{N}$$

$$= \frac{S_2}{S_1}\vec{V} + \left(\frac{S_2}{S_1} C_1 - C_2\right)\vec{N}$$

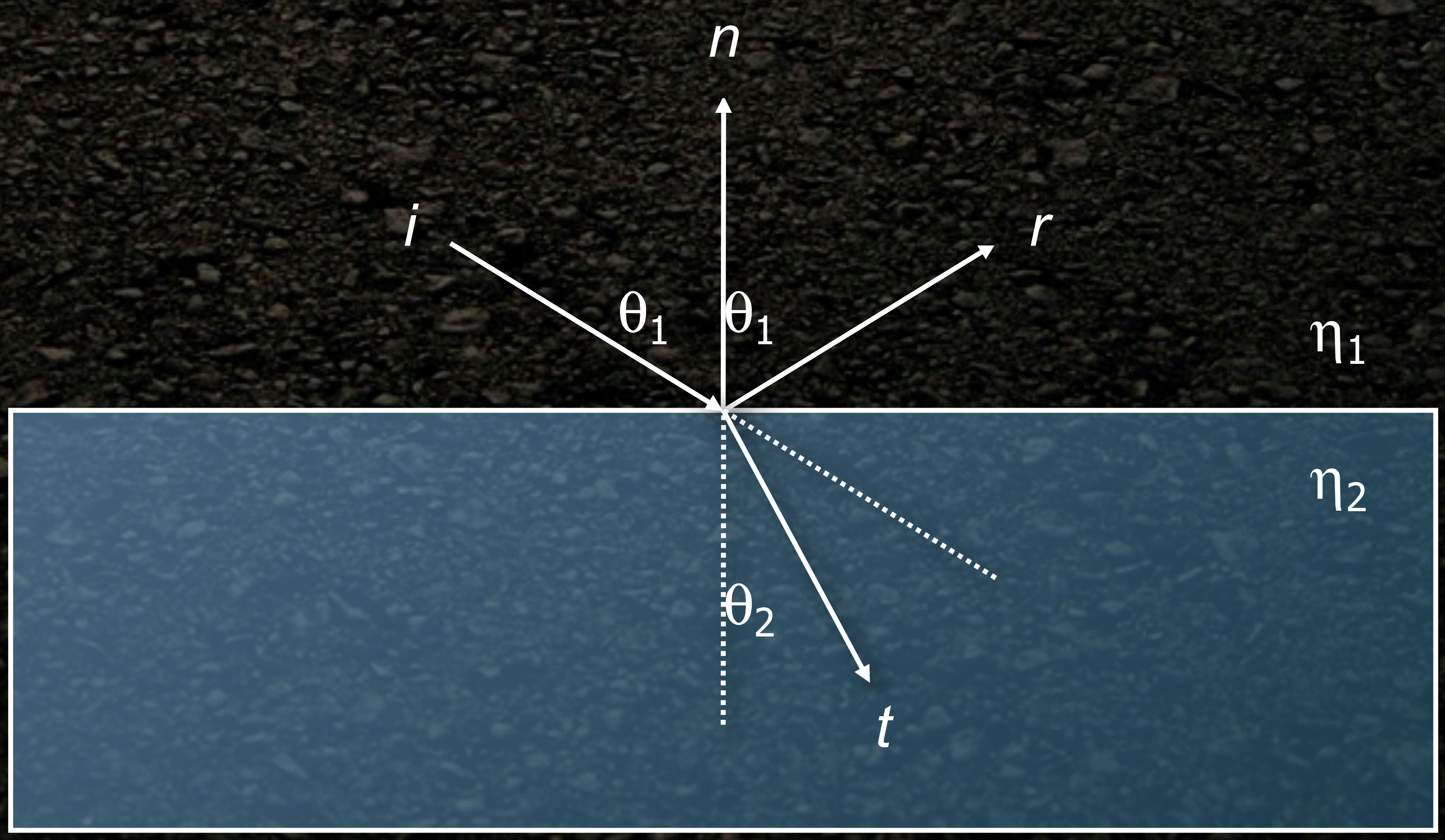
$$= \frac{1}{\eta}\vec{V} + \left(\frac{1}{\eta} C_1 - C_2\right)\vec{N}$$

$$= \frac{1}{\eta}\vec{V} + \left(\frac{C_1}{\eta} - \sqrt{1 + \frac{\left(C_1^2 - 1\right)}{\eta^2}}\right)\vec{N}$$



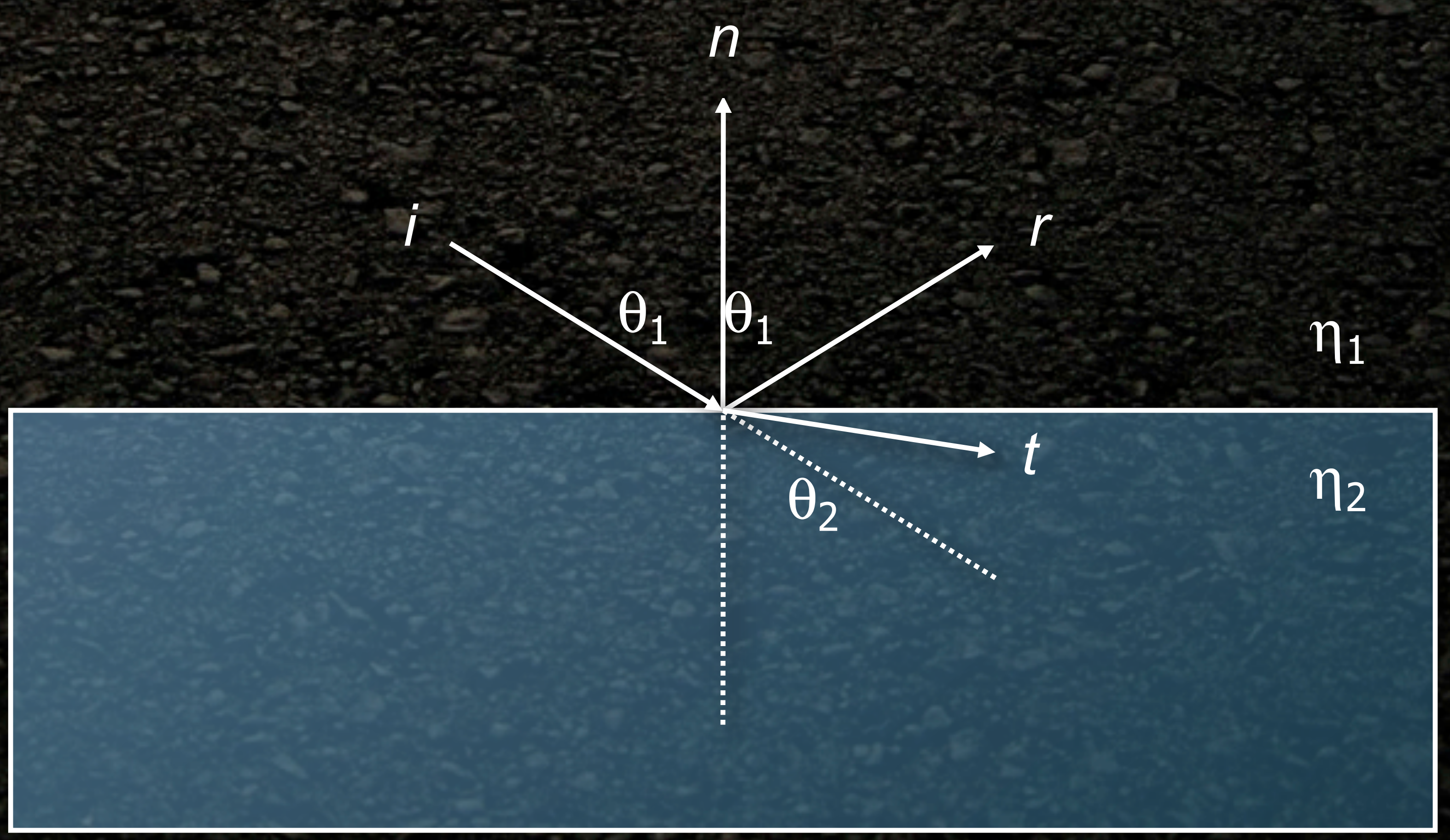
Transmitted ray direction, t (cont'd.)

When $\eta_1 < \eta_2$, t bends toward the normal direction at the hit point



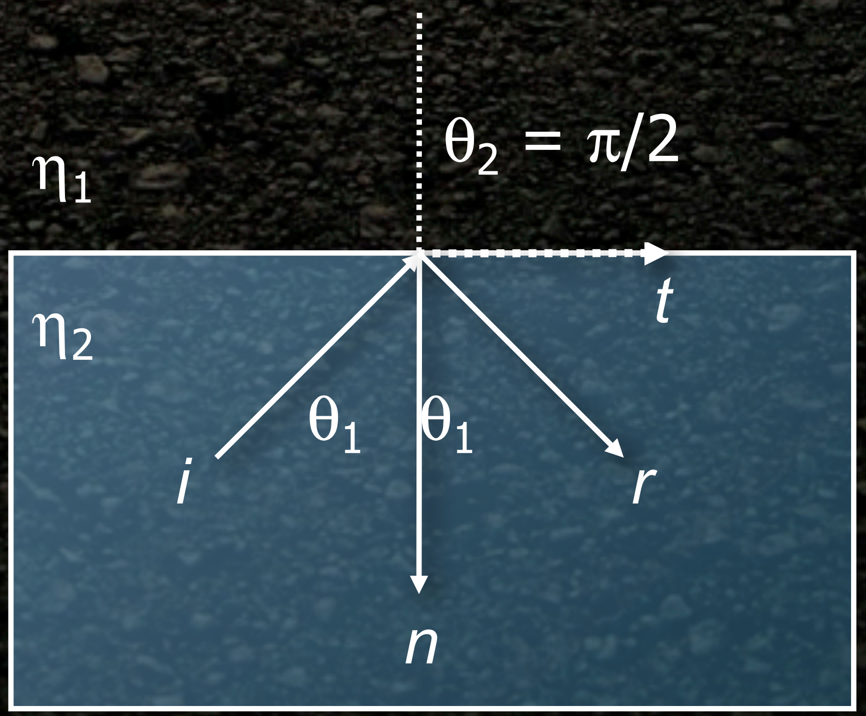
Transmitted ray direction, *t* (cont'd.)

When $\eta_1 > \eta_2$, *t* bends away from the normal direction at the hit point



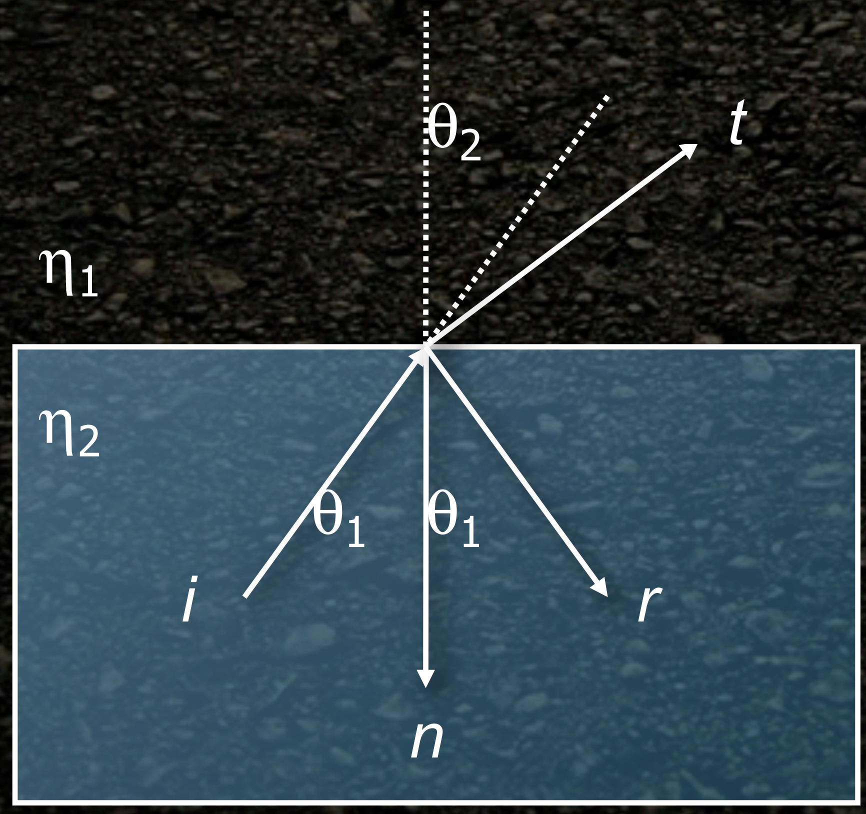
Total internal reflection

There exists a *critical angle*, θ_c, when the transmitted ray direction *t* is parallel to the boundary and $\theta_2 = \pi/2$



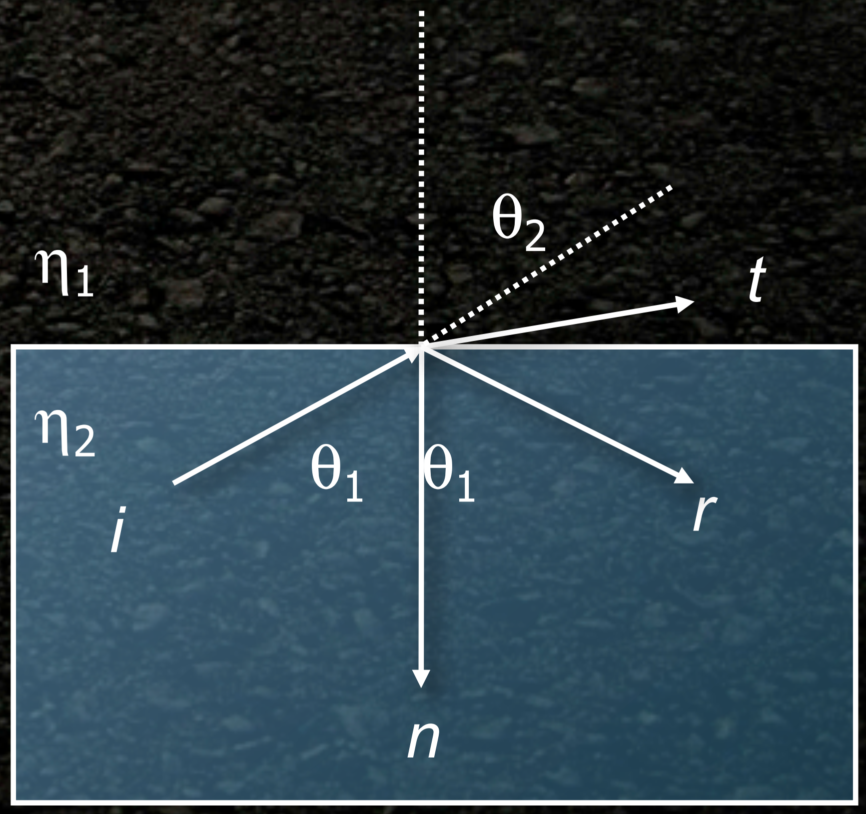
θ_1 approaches θ_c

As θ_1 increases, *t* bends toward the boundary



$\theta_1 << \theta_c$

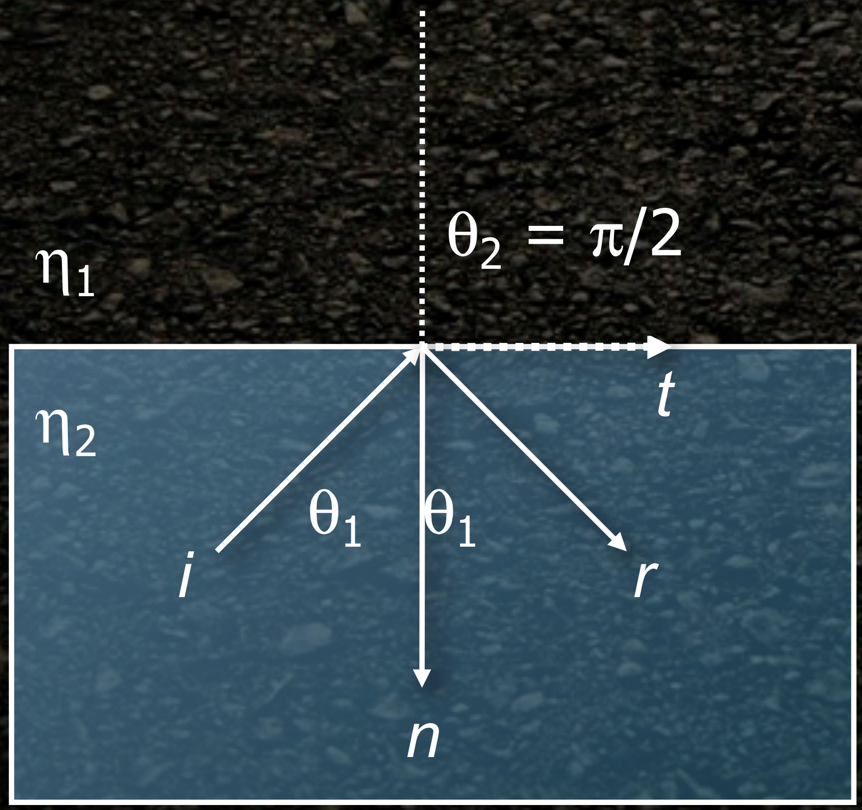
…



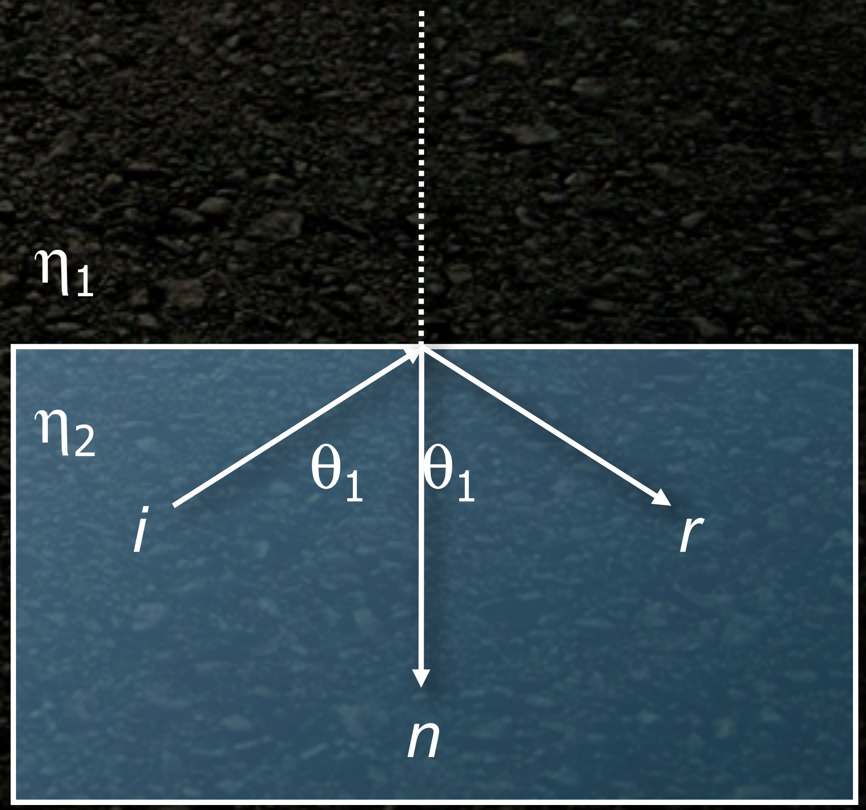
$\theta_1 < \theta_c$

θ_1 is greater than or equal to θ_c

When $\theta_1 \geq \theta_c$, *t* is parallel to the boundary and it carries no energy (total internal reflection occurs)



$$\theta_1 = \theta_c$$



$$\theta_1 > \theta_c$$

Checking for TIR

Recall that the *angle of refraction* θ_2 is given by:

$$\cos\theta_2 = [1 - \frac{1}{\eta^2}(1 - \cos^2\theta_1)]^{1/2}$$

TIR occurs when $\theta_1 = \theta_c$

When $\theta_1 = \theta_c$, the expression:

$$1 - \frac{1}{\eta^2}(1 - \cos^2 \theta_1)$$

becomes zero.

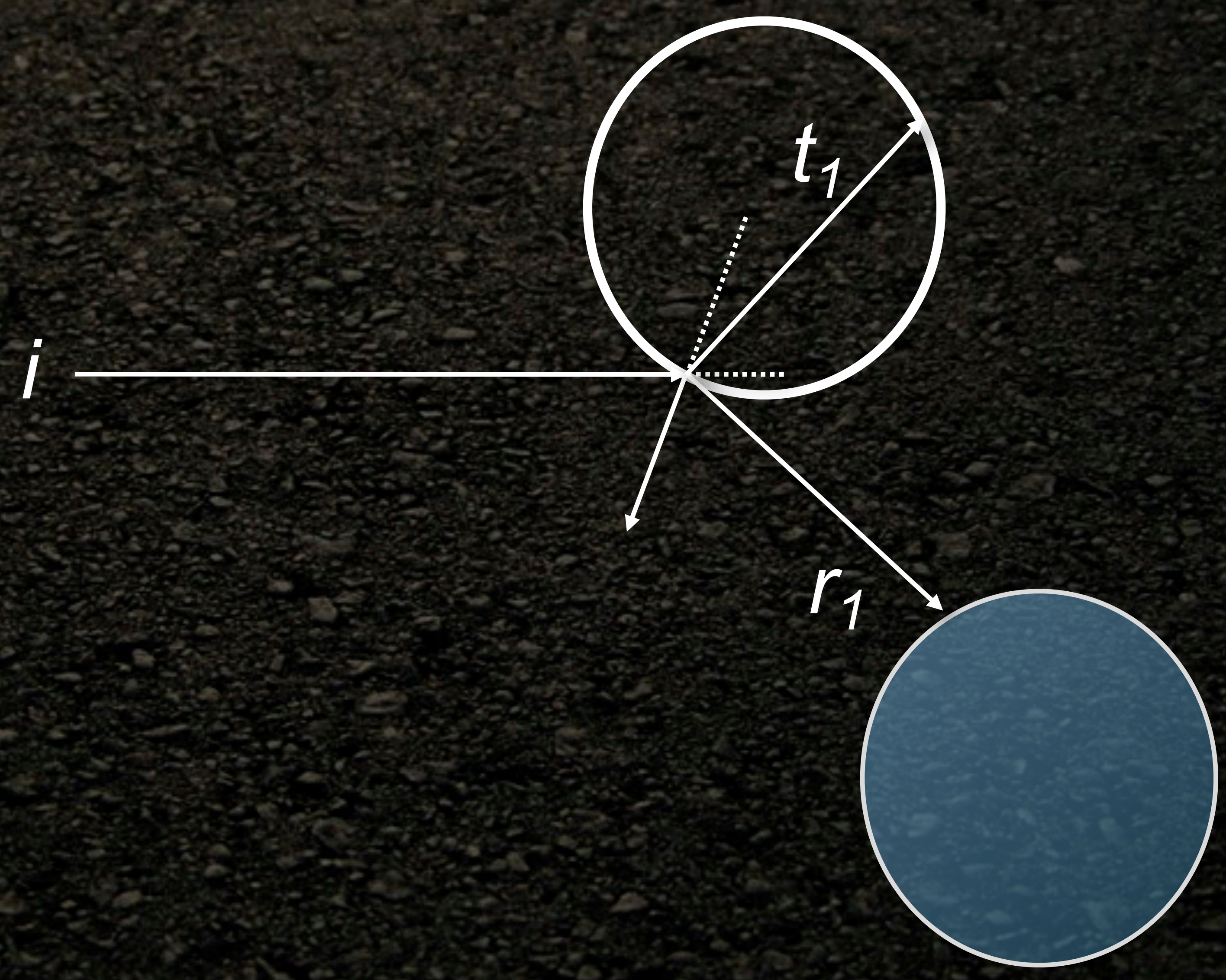
When $\theta_1 > \theta_c$, this expression is negative, so θ_2 is imaginary and *t* is a complex number.

TIR condition

Thus, TIR occurs when:

$$1 - \frac{1}{\eta^2}(1 - \cos^2 \theta_1) < 0$$

Spawning secondary rays



$$c(p) = c_{local}(p) + k_r(p) * c_r(p_r) + k_t(p) * c_t(p_t)$$