CS 6320 Computer Vision Practice Questions

Spring 2018
Name:
UID:

Each question carries 20 points. Show the necessary steps to compute the final solutions. You are free to use extra papers.

1. **Pose Estimation:** Let us consider a calibrated camera. Let the origin of the camera be given by $O(0, 0, 0)$. The image resolution is $640 \times 480$ and the principal point is given by $(320, 240)$. We assume the following parameters for the camera:

\[
K = \begin{pmatrix}
1 & 0 & 320 \\
0 & 1 & 240 \\
0 & 0 & 1
\end{pmatrix}
\]

where $(u, v)$ correspond to pixel coordinates and $I$ denotes the $3 \times 3$ identity matrix.

The projections of two 3D points $A$ and $B$ on the image are given by $a(120, 240)$ and $b(320, 240)$, respectively. Find the coordinates of the two 3D points $A(X_1, Y_1, Z_1)$ and $B(X_2, Y_2, Z_2)$ that satisfy the 2 conditions: (1) the length of the line segment $OA$ is given by $|OA| = 200$, and (2) the line segments $OA$ and $AB$ are perpendicular to each other. [20 points]

\[
A = \lambda_1 K^{-1} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \lambda_1 \begin{pmatrix} 1 \\ 200 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & -320 \\ 0 & 1 & -240 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 120 \\ 240 \\ 1 \end{pmatrix} = \lambda_1 \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}
\]

\[
B = \lambda_2 K^{-1} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \lambda_2 \begin{pmatrix} 1 \\ 200 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & -320 \\ 0 & 1 & -240 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 320 \\ 240 \\ 1 \end{pmatrix} = \lambda_2 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}
\]

$|OA| = 200 \Rightarrow \lambda_1^2 + \lambda_2^2 = 40000 \Rightarrow \lambda_1 = \sqrt{10000}$

$OA \perp AB \Rightarrow (-100\sqrt{2})(+100\sqrt{2}) + (+100\sqrt{2})(\lambda_2 - 100\sqrt{2}) = 0$

$\Rightarrow 2 \times 100\sqrt{2} \lambda_2 = 40000$

$\lambda_2 = 200\sqrt{2}$
2. **Model fitting:** In Figure 1, you are given a set of 6 2D points \((A, B, C, D, E, F)\). We can obtain a line equation by choosing 2 points at a time. Given two 2D points \((x_1, y_1)\) and \((x_2, y_2)\), we can obtain the line equation using the expression \((y - y_1)(x_1 - x_2) = (x - x_1)(y_1 - y_2)\). The distance of a point \((x_0, y_0)\) from a line \(ax + by + c = 0\) is given by \(\frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}\). Let the threshold in selecting the inliers for a given line equation is given by 0.6.

- (a) Find a pair of points that gives the line equation with the maximum number of inliers. Show the line equations and inliers.
- (b) Find a point pair that gives the line equation with the minimum number of inliers. Show the line equations and inliers.

[20 points]
3. **Vocabulary tree**: We are given three images $I_1$, $I_2$ and $I_3$. Each of these images have two 2D descriptors as given below.

\[ I_1 : \{(1.5, 0.5), (-1.5, -1.5)\}, \quad I_2 : \{(1.5, 1.5), (1.5, -1.5)\}, \quad I_3 : \{(1.5, 0.5), (1.5, -0.5)\} \]

In Figure 2, we show a vocabulary tree with branch factor 4. Using this vocabulary tree find the best image match among the 3 possible pairs \{(I_1, I_2), (I_1, I_3), (I_2, I_3)\}. Assume that the nodes in the tree has the same weight $w_i = 1$. Show the normalized difference in each of the three pairs. [20 points]
4. Belief Propagation: You have 2 Boolean variables \((x_1, x_2 \in \{0,1\})\) and 3 equations as shown below:

\[
\begin{align*}
x_1 + 4x_2 &= 5 \quad \text{a} \\
-x_1 + 2x_2 &= 1 \quad \text{b} \\
-3x_1 + 2x_2 &= -1.2 \quad \text{c}
\end{align*}
\]

Show the factor graph and use Belief propagation to solve the equations. Please show the messages in each iteration till the algorithm terminates. [20 points]

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**Step 1:** All zero msg from var to factor nodes.

**Step 2:** \(m_{a \rightarrow 1} = (1, 0)\) \(m_{a \rightarrow 2} = (4, 0)\) \(m_{b \rightarrow 1} = (1, 0)\) \(m_{b \rightarrow 2} = (1, 0)\) \(m_{c \rightarrow 1} = [1.2, 0.2]\) \(m_{c \rightarrow 2} = [1.2, 0.2]\)

**Step 3:** \(b_1 = \left(\begin{array}{c} 3.2 \\ 0.2 \end{array}\right)\) \(b_2 = \left(\begin{array}{c} 6.2 \\ 0.2 \end{array}\right)\)

**Step 4:** \(x_1 \leftarrow 1\), \(x_2 \leftarrow 1\)

**Step 5:** \(m_{1 \rightarrow a} = \left(\begin{array}{c} 2.2 \\ 0.2 \end{array}\right)\) \(m_{2 \rightarrow a} = \left(\begin{array}{c} 2.2 \\ 0.2 \end{array}\right)\) \(m_{1 \rightarrow b} = \left(\begin{array}{c} 2.2 \\ 0.2 \end{array}\right)\) \(m_{2 \rightarrow b} = \left(\begin{array}{c} 5.2 \\ 0.2 \end{array}\right)\) \(m_{1 \rightarrow c} = \left(\begin{array}{c} 2.2 \\ 0.2 \end{array}\right)\) \(m_{2 \rightarrow c} = \left(\begin{array}{c} 5.2 \\ 0.2 \end{array}\right)\)

**Step 6:** \(m_{a \rightarrow 1} = \left(\begin{array}{c} 1.2 \\ 0.2 \end{array}\right)\) \(m_{a \rightarrow 2} = \left(\begin{array}{c} 4.2 \\ 0.2 \end{array}\right)\) \(m_{b \rightarrow 1} = \left(\begin{array}{c} 1.2 \\ 0.2 \end{array}\right)\) \(m_{b \rightarrow 2} = \left(\begin{array}{c} 2.2 \\ 0.2 \end{array}\right)\) \(m_{c \rightarrow 1} = \left(\begin{array}{c} 3.2 \\ 0.2 \end{array}\right)\) \(m_{c \rightarrow 2} = \left(\begin{array}{c} 1.8 \\ 0.2 \end{array}\right)\)

**Step 7:** \(l_1 = \left(\begin{array}{c} 5.6 \\ 0.6 \end{array}\right)\) \(b_2 = \left(\begin{array}{c} 8.2 \\ 0.4 \end{array}\right)\)

**Step 8:** \(x_1 \leftarrow 1\), \(x_2 \leftarrow 1\) **Terminate**