Consider the following linear advection-diffusion equation:

\[ u_t + au_x = \nu u_{xx} \]

where \( a \) is a real-valued wave speed and \( \nu \geq 0 \) denotes the diffusion coefficient. For this problem, we will take \( a = 1 \) and \( \nu = 1 \). Assume a domain of \([0, 1)\) with periodic boundary conditions. Take as your initial condition:

\[ u(x, 0) = \sin(2\pi x). \]

Consider the difference approximation given by:

\[ u_j^{n+1} = u_j^n + k \left( \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{h^2} - \frac{a}{2h} \left( u_{j+1}^n - u_{j-1}^n \right) \right) \]

for \( N = 10, 20, 40, 80, 160 \). Choose the time step \( k \) such that if \( h \) denotes your grid spacing and if \( \alpha = \frac{2\nu k}{h^2} \) and \( \lambda = \frac{ak}{h} \) then you satisfy either:

- \( \lambda^2 \leq \alpha \leq 1 \) or
- \( a^2 k \leq 2\nu \leq h^2 / k \).

Compare the solutions and explain the differences in their behavior. What is the impact of which relation you choose to satisfy? Is it possible to satisfy both simultaneously?