Lecture 15
Scientific and Data Computing I
Intro to Topological Optimization

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Lecture

- Wednesday – Introduction to Topological Optimization
- Monday – Polynomial Interpolation and Projection (Chapter 5.2 and 5.5)
- Wednesday – Splines (Chapter 5.3 and 5.4)
Topological Optimization

• Examples
Where do you find it used?

- Stiffing Ribs
Topological Optimization

- Distribution of the material resources for a given set of loads and boundary conditions that minimizes a cost function subject to performance criteria

\[ \min_{\rho} \quad C(\rho) \]

subject to \[ V(\rho) \leq \bar{V} \]
\[ Ku = F \]
\[ \rho_{\text{min}} \leq \rho \leq 1 \]

- \( \rho \) the volume fraction of each element; void \( \rho = 0 \) solid \( \rho = 1 \)
- Objective function: Compliance \( C \), Constraint: Volume \( V \)
Topo Opt (continued)

- Compliance:

\[ C = U^T K U = \sum_{e=1}^{N} E_e(\rho_e) u_e^T k_0 u_e \]
\[ k_e = E_e(\rho_e) k_0 \]

- Compliance derivative: Adjoint Analysis (suitable for large number of design variables)

\[ (F + K\Lambda) \frac{\partial U}{\partial \rho} = 0 \rightarrow (F + K\Lambda) = 0 \]

Adjoint solution: \( \Lambda = -U \)

\[ \frac{\partial C}{\partial \rho} = \Lambda^T \frac{\partial K}{\partial \rho} U = -\sum_{e=1}^{N} \frac{\partial E_e(\rho_e)}{\partial \rho_e} u_e^T k_0 u_e \]
Compliance:

\[ C = U^T K U = \sum_{e=1}^{N} E_e(\rho_e) u_e^T k_0 u_e \]
\[ k_e = E_e(\rho_e) k_0 \]

- Basis functions (Chap 5)
- Gaussian integration (Chap 6)
- Linear Solver Methods (Chap 3 and extra sources)
Topo Opt (continued)

- Compliance derivative: Adjoint Analysis (suitable for large number of design variables)

\[(F + K\Lambda)\frac{\partial U}{\partial \rho} = 0 \rightarrow (F + K\Lambda) = 0\]

Adjoint solution: \(\Lambda = -U\)

\[\frac{\partial C}{\partial \rho} = \Lambda^T \frac{\partial K}{\partial \rho} U = -\sum_{e=1}^{N} \frac{\partial E_e(\rho_e)}{\partial \rho_e} u_e^T k_0 u_e\]

- Linear and non-linear optimization (Chap 8)
Topo Opt (continued)

- Filtering: impose length scale via kernel $K$ in the ball $B_r$ with radius $r$

$$\hat{\rho}(x) = \int_{B_r(y)} K(x - y) \rho(y) \, dv$$

- Thresholding: generating sharp boundaries i.e. removing the gray elements produced due to filtering

$$\tilde{\rho}(x) = \frac{\tanh(\beta \eta) + \tanh(\beta (\hat{\rho}(x) - \eta))}{\tanh(\beta \eta) + \tanh(\beta (1 - \eta))}$$

- Penalization: Solid Isotropic Material with Penalization (SIMP) to invoke black and white design $\rho \to \rho^3$

$$\tilde{\rho}(x) \to \tilde{\rho}^3(x)$$
Topo Opt (continued)

An example of processed design variables

An example of Heaviside Thresholding for different thresholds $\eta$ and $\beta = 5$
Topo Opt (continued)

We need the function (compliance) and the function’s sensitivity to use gradient based optimizer. Common approaches in topology optimization:

- Interior Point Method: finding the Hessian via BFGS (a secant method)
For Next Time

• Read Chapter 5 and 6
• Start working on Practicums
• Start working on Homework 4