1 History of Calculus

1.1 Integral calculus

Greek geometers are credited with a significant use ofi nfinitesimals. Democritus is the first person recorded to consider seriously the division of objects into an infinite number of cross-sections, but his inability to rationalize discrete cross-sections with a cone's smooth slope prevented him from accepting the idea. At approximately the same time, Zeno of Elea discredited infinitesimals further by his articulation of the paradoxes which they create.

Antiphon and later Eudoxus are generally credited with implementing the method of exhaustion, which made it possible to compute the area and volume of regions and solids by breaking them up into an infinite number of recognizable shapes. Archimedes developed this method further, while also inventing heuristic methods which resemble modern day concepts somewhat. It was not until the time of Newton that these methods were made obsolete. It should not be thought that infinitesimals were put on rigorous footing during this time, however. Only when it was supplemented by a proper geometric proof would Greek mathematicians accept a proposition as true.

In the third century Liu Hui wrote his Nine Chapters and also Haidao suanjing, which dealt with using the Pythagorean theorem (already stated in the Nine Chapters), known in China as the Gougu theorem, to measure the size of things. He discovered the usage of Cavalieri's principle to find an accurate formula for the volume of a cylinder, showing a grasp of elementary concepts associated with the differential and integral calculus.

Indian mathematicians produced a number of works with some ideas of calculus. The formula for the sum of the cubes was first written by Aryabhata circa 500 AD, which was an important step in the development ofi ntegral calculus.

Around 1000 AD, Ibn al-Haytham (known as Alhazen in the West), an Iraqi mathematician working in Egypt, was the first mathematician to derive the formula for the sum of the fourth powers. In turn, he developed a method for determining the general formula for the sum of any integral powers, which was fundamental to the development ofi ntegral calculus.

In the 17th century, Pierre de Fermat, among other things, is credited with an ingenious trick for evaluating the integral of any power function directly, thus providing a valuable clue to Newton and Leibniz in their development of the fundamental theorems of calculus.

At around the same time, there was also a great deal of work being done by Japanese mathematicians, particularly Kowa Seki. He made a number of contributions, namely in methods of determining areas of figures using integrals, extending the method of exhaustion. While these methods of finding areas were made largely obsolete by the development of the fundamental theorems by Newton and Leibniz, they still show that a sophisticated knowledge of mathematics existed in 17th century Japan.

1.2 Differential calculus

The Greek mathematician Archimedes was the first to find the tangent to a curve, other than a circle, in a method akin to differential calculus. While studying the spiral, he separated a point's motion into two components, one radial motion component and one circular motion component, and then continued to add the two component motions together thereby finding the tangent to the curve.

The Indian mathematician-astronomer Aryabhata in 499 used a notion of infinitesimals and expressed an astronomical problem in the form of a basic differential equation.Manjula, in the 10th century, elaborated on this different ial equation in a commentary. This equation eventually led Bhskara II in the 12t h century to develop the concept of a derivative representing infinitesimal change, and he described an early form of "Rolle's theorem".

In the late 12th century, the Persian mathematician Sharaf al-Din al-Tusi was the first to discover the derivative of cubic polynomials, an important result in differential calculus.

In the 15th century, an early version of the mean value theorem was first described by Parameshvara (13701460) from the Kerala school of astrono my and mathematics in his commentaries on Govindasvmi and Bhaskara II.

In 17th century Europe, Isaac Barrow, Pierre de Fermat, Blaise Pascal, John Wallis and others discussed the idea of a derivative. The first proof of Rolle's theorem was given by Michel Rolle in 1691 after the invention of m odern calculus. The mean value theorem in its modern form was stated by Augustin Louis Cauchy (1789-1857) also after the invention of modern calculus.

2 Fundamental Theorem of Calculus

We are all used to evaluating definite integrals without giving the reason for the procedure much thought. The definite integral is defined, however, not by our regular procedure but rather as a limit of Riemann sums. We often view the definite integral of a function as the area under the graph of the function between two limits. It is not intuitively clear, then, why we proceed as we do in computing definite integrals. The Fundamental Theorem of Calculus justifies our procedure of evaluating an antiderivative at the upper and lower limits of integration and taking the difference.

Let f be continuous on $[a,b]$. If F is any antiderivative for f on $[a,b]$, then

$\int_a^b f(t)\,dt = F(b) - F(a)$.

Again, Let f be a continuous real-valued function defined on a closed interval $[a, b]$.

$F(x) = \int_a^x f(t)\,dt$

Then, F is differentiable on $[a, b]$, and for every x in $[a, b]$,

$F'(x) = f(x)$.

The operation

$\int_a^x f(t)\,dt$

is a definite integral with variable upper limit, and its result F(x) is one of the infinitely many antiderivatives off .

3 Corollary

Let f be a real-valued function defined on a closed interval [a, b]. Let F be a function such that, for all x in [a, b],

$f(x) = F'(x)$

Then, for all x in [a, b],

$F(x) = \int_a^x f(t)\,dt + F(a)$

and

$f(x) = \frac{d}{dx}\int_a^x f(t)\,dt.$