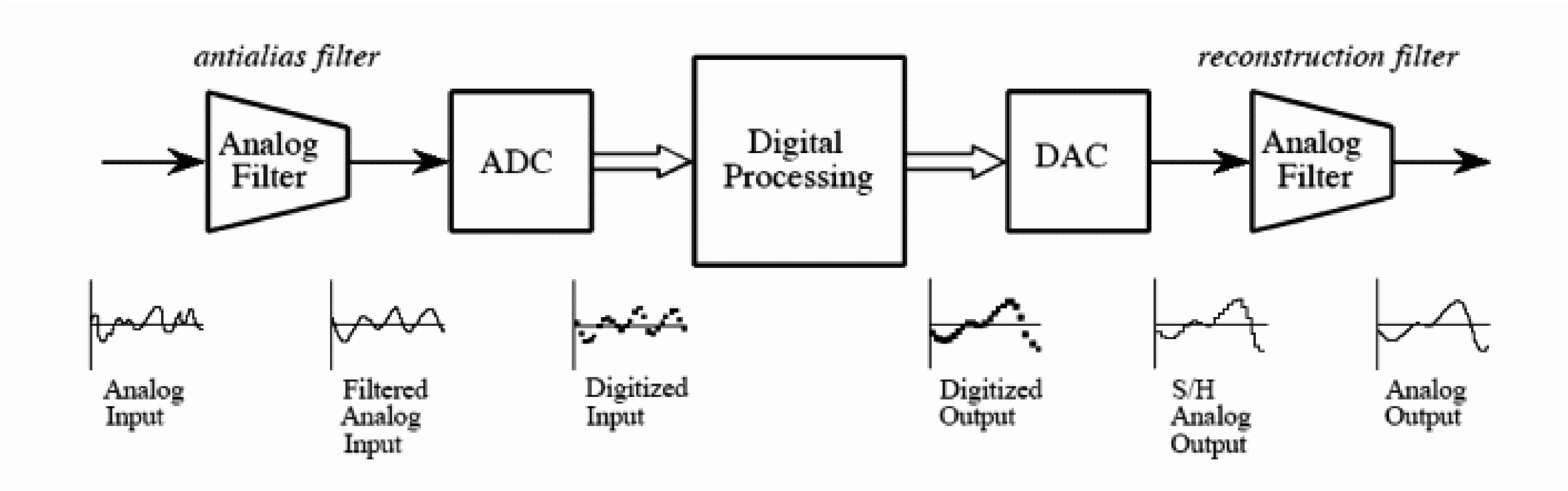
Today

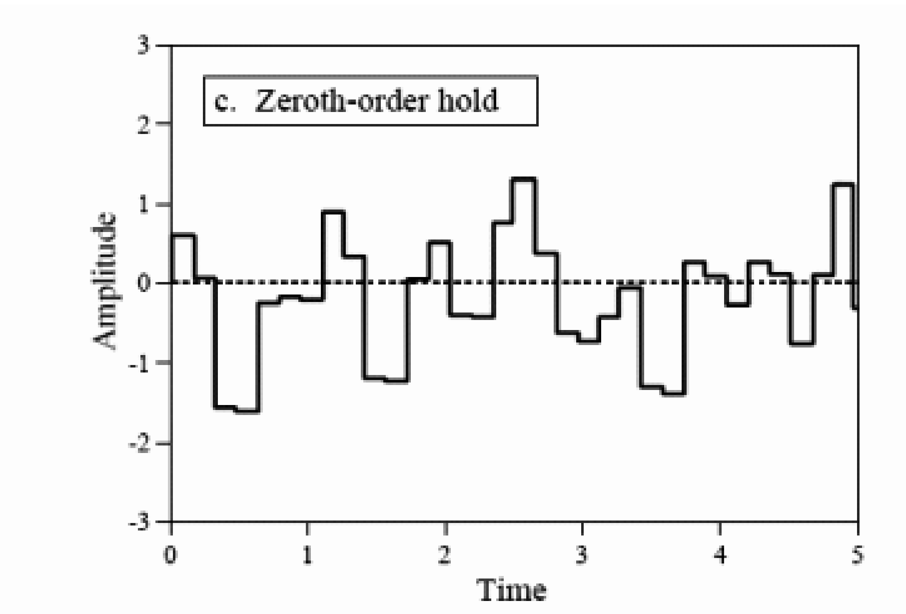
- Digital filters and signal processing
  - Filter examples and properties
  - FIR filters
  - Filter design
  - Implementation issues
  - DACs
  - PWM

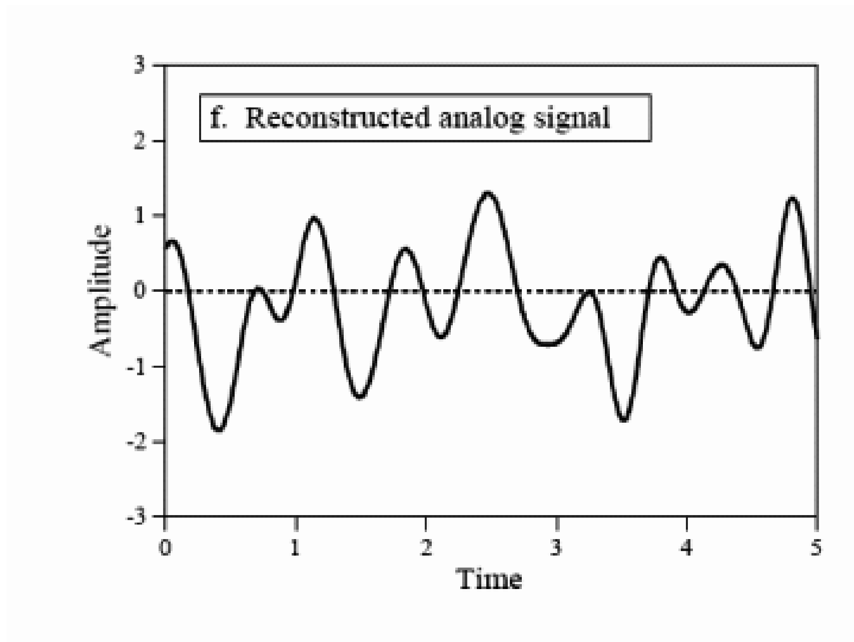
DSP Big Picture



Signal Reconstruction

- Analog filter gets rid of unwanted high-frequency components





Data Acquisition

- Signal: Time-varying measurable quantity whose variation normally conveys information
  - Quantity often a voltage obtained from some transducer
  - E.g. a microphone
- Analog signals have infinitely variable values at all times
- Digital signals are discrete in time and in value
  - Often obtained by sampling analog signals
  - Sampling produces sequence of numbers
    - E.g. { ... , x[-2], x[-1], x[0], x[1], x[2], ... }
  - These are time domain signals

Sampling

- **Transducers**
  - **Transducer turns a physical quantity into a voltage**
  - **ADC turns voltage into an *n*-bit integer**
  - **Sampling is typically performed periodically**
  - **Sampling permits us to reconstruct signals from the world**
    - **E.g. sounds, seismic vibrations**
- **Key issue: *aliasing***
  - ***Nyquist rate*: 0.5 * sampling rate**
  - **Frequencies higher than the Nyquist rate get mapped to frequencies below the Nyquist rate**
  - **Aliasing cannot be undone by subsequent digital processing**

Sampling Theorem

- Discovered by Claude Shannon in 1949:

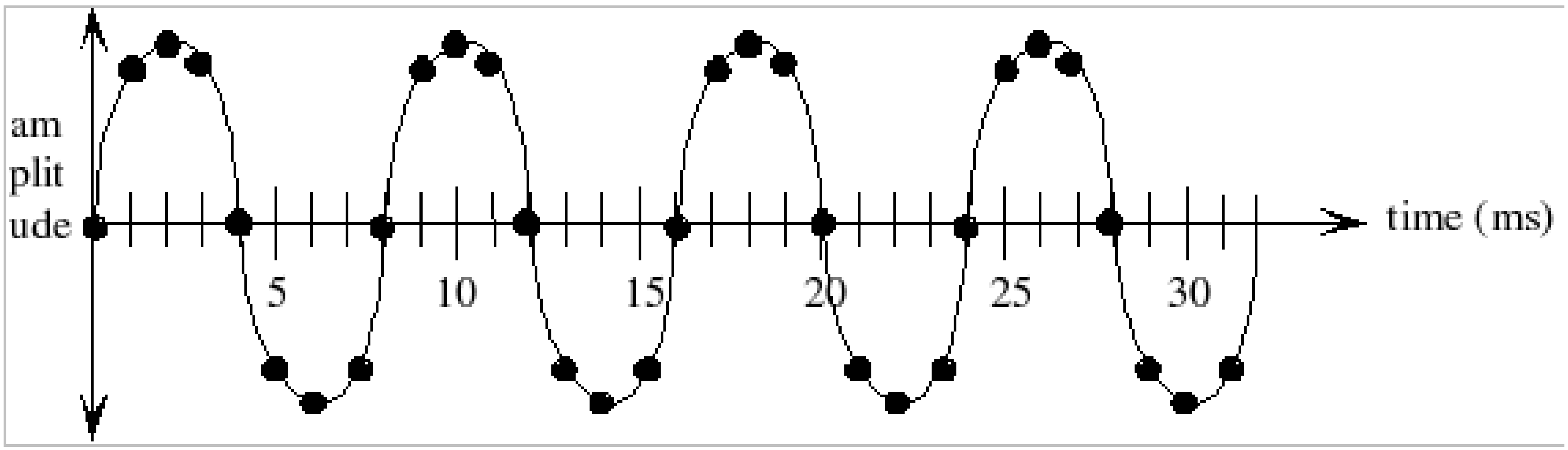
  A signal can be reconstructed from its samples without loss of information, if the original signal has no frequencies above 1/2 the sampling frequency

- This is a pretty amazing result
  - But note that it applies only to discrete time, not discrete values

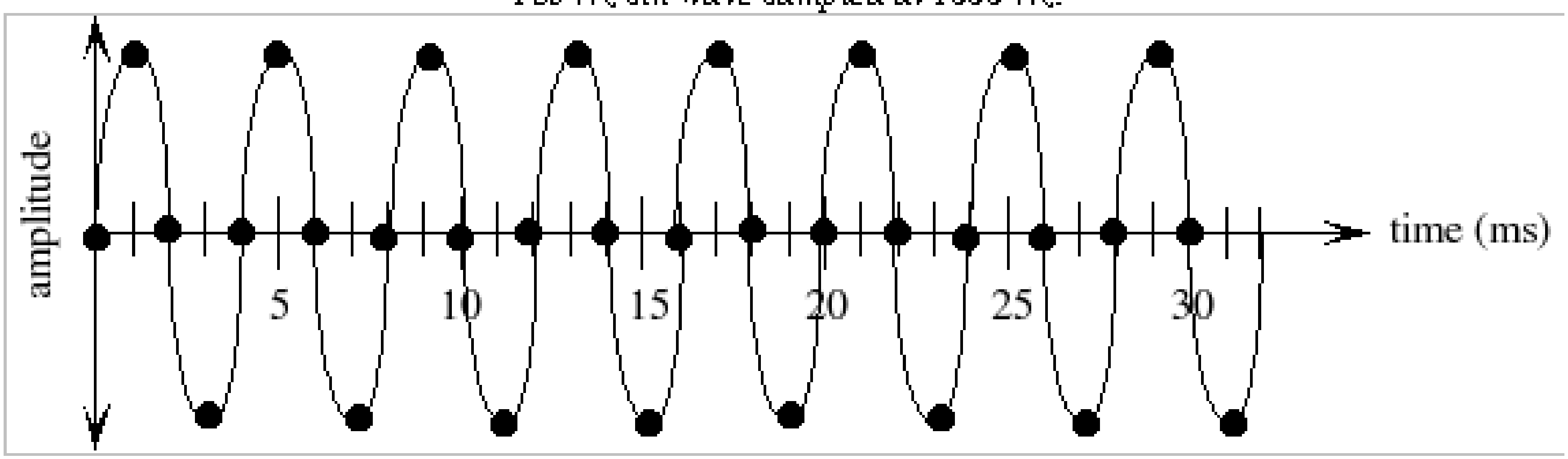
Aliasing Details

- Let N be the sampling rate and F be a frequency found in the signal
  - Frequencies between 0 and 0.5*N are sampled properly
  - Frequencies >0.5*N are aliased
    - Frequencies between 0.5*N and N are mapped to (0.5*N)-F and have phase shifted 180°
    - Frequencies between N and 1.5*N are mapped to f-N with no phase shift
    - Pattern repeats indefinitely
- Aliasing may or may not occur when N == F*2*X where X is a positive integer

No Aliasing

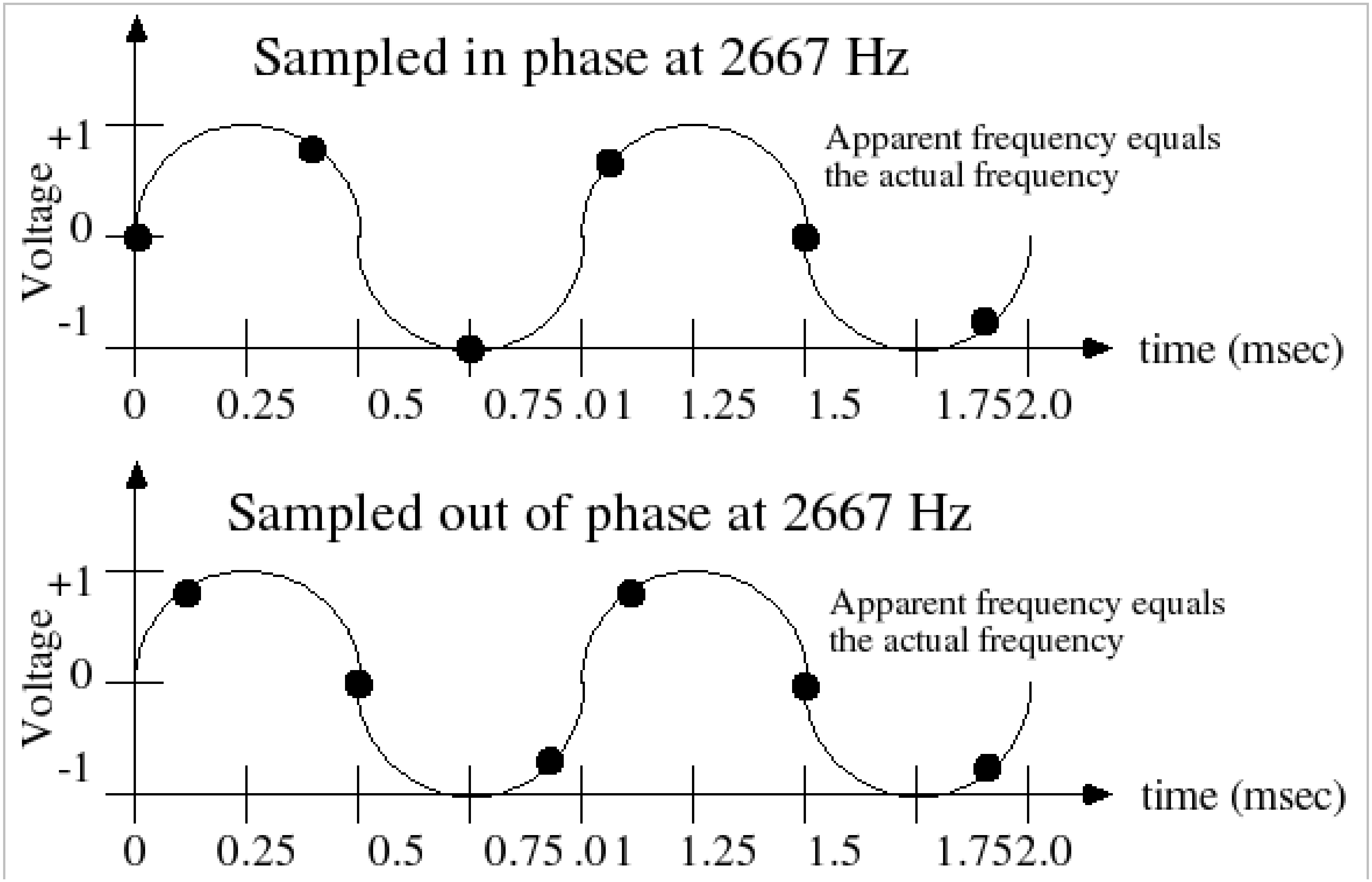


125 Hz sin wave sampled at 1000 Hz.

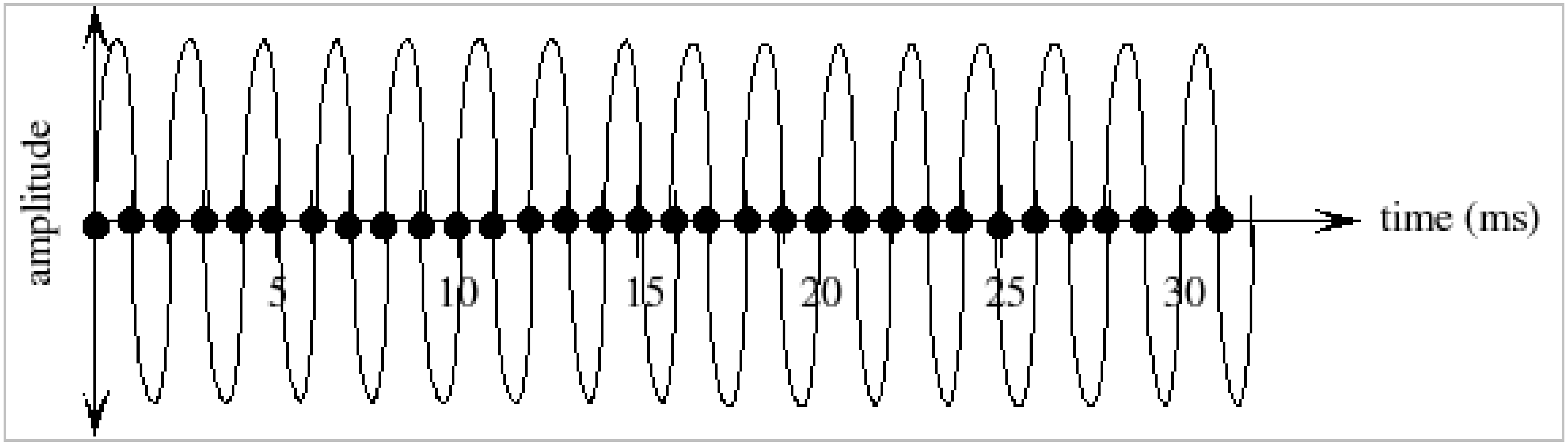


250 Hz sin wave sampled at 1000 Hz.

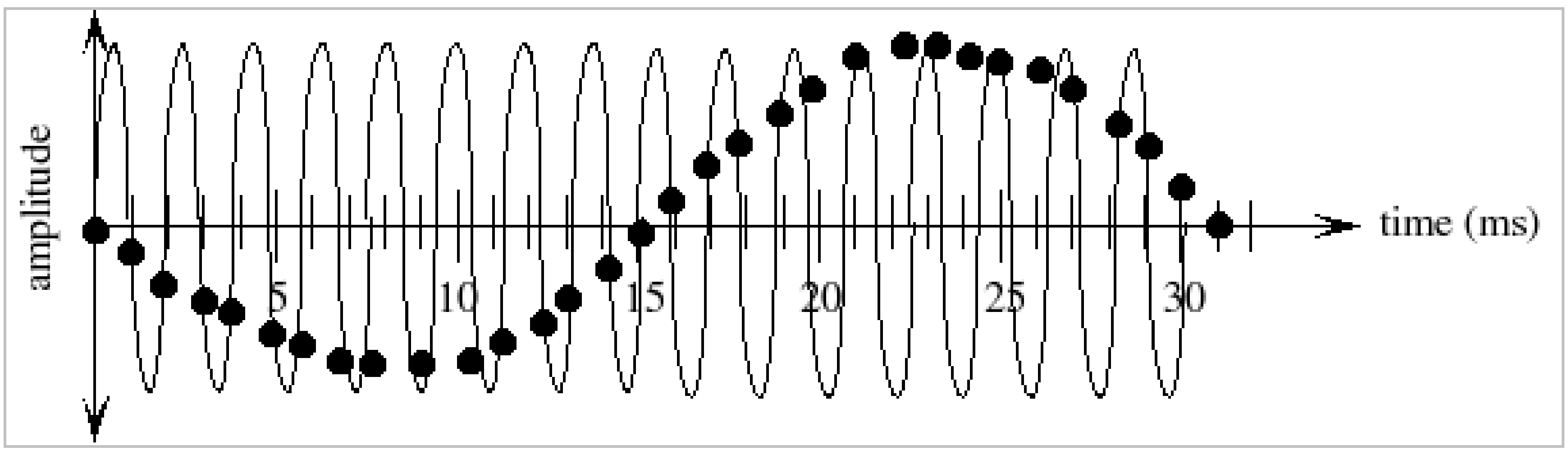
1 kHz Signal, No Aliasing



Aliasing

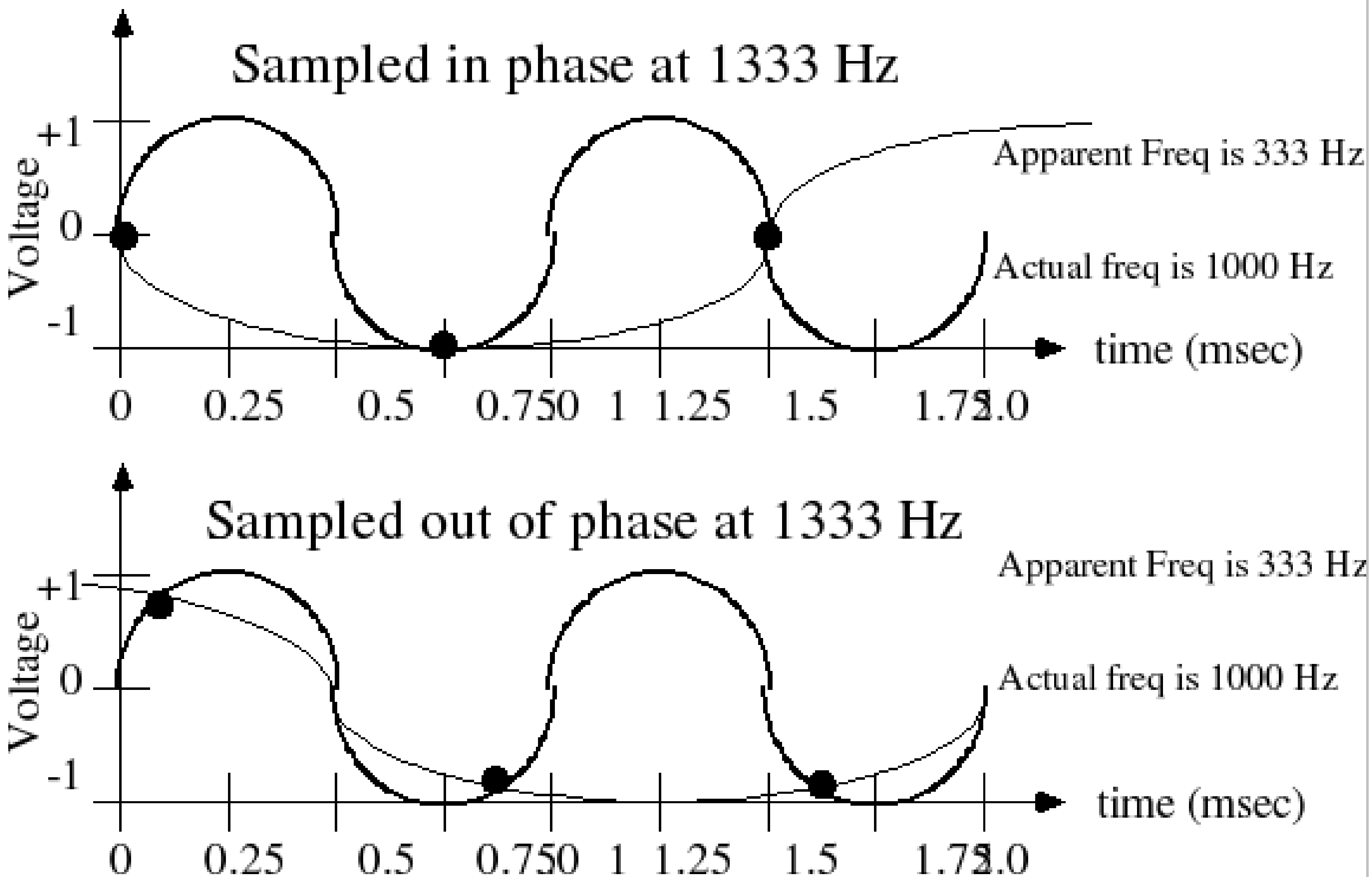


500 Hz sin wave sampled at 1000 Hz.

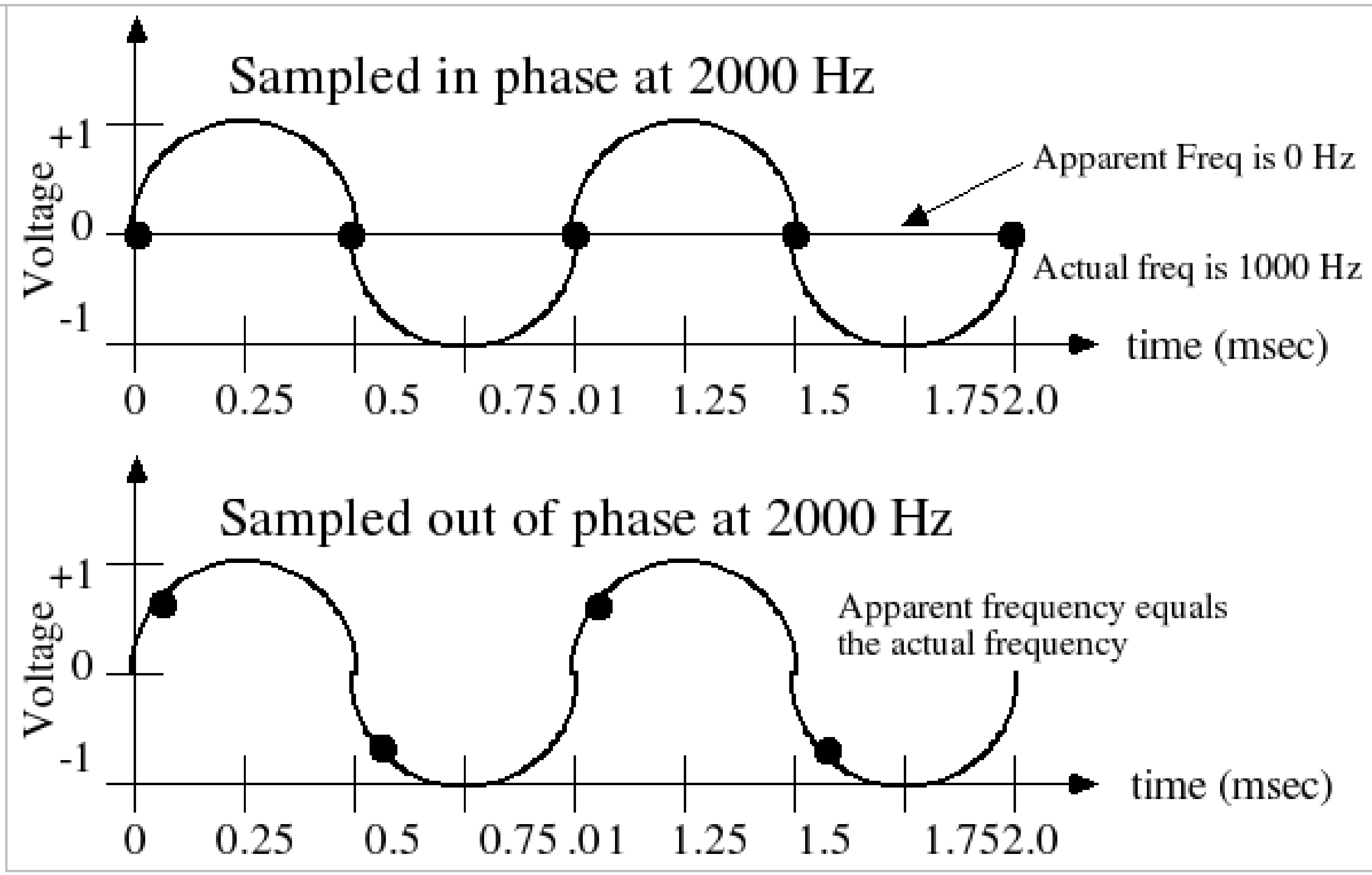


533 Hz sin wave sampled at 1000 Hz.

More Aliasing



N == 2*F Example

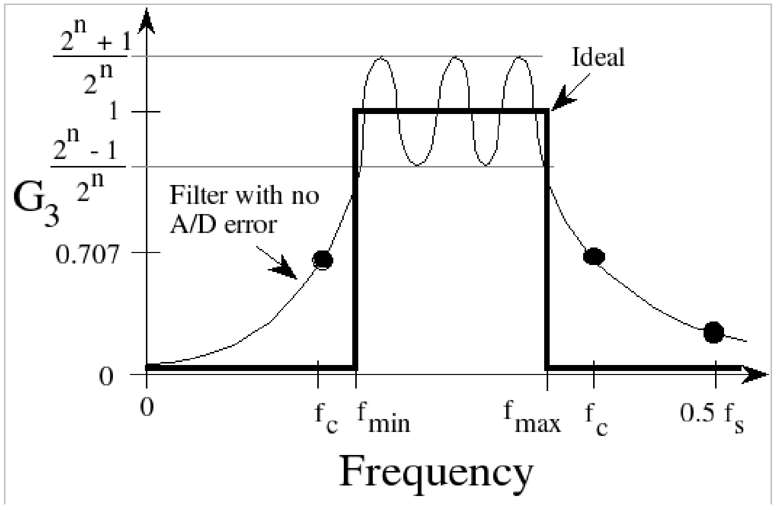


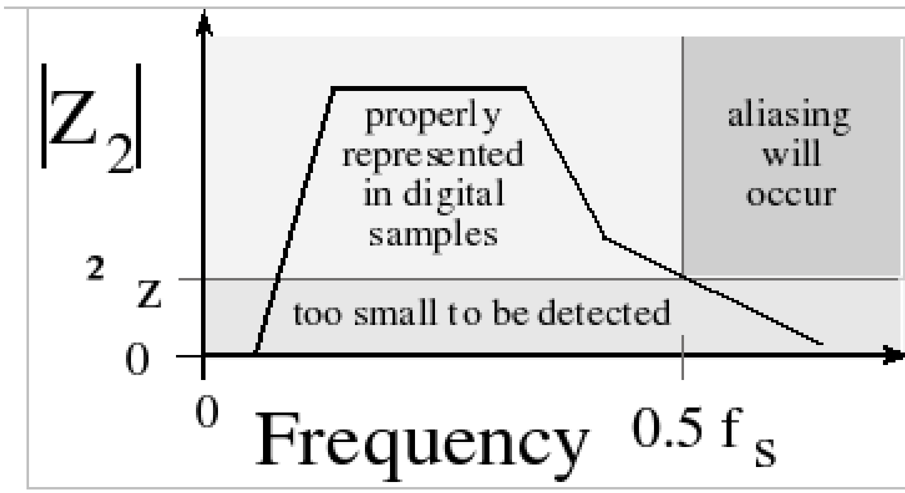
Sampled in phase at 2000 Hz
Voltage
+1
0
-1
Apparent Freq is 0 Hz
Actual freq is 1000 Hz
time (msec)
0
0.25
0.5
0.75
.01
1.25
1.5
1.75
2.0
Sampled out of phase at 2000 Hz
Voltage
+1
0
-1
Apparent frequency equals
the actual frequency
time (msec)
0
0.25
0.5
0.75
.01
1.25
1.5
1.75
2.0

Avoiding Aliasing

1. **Increase sampling rate**
   - ➢ **Not a general-purpose solution**
     - • **White noise is not band-limited**
     - • **Faster sampling requires:**
       - – **Faster ADC**
       - – **Faster CPU**
       - – **More power**
       - – **More RAM for buffering**
2. **Filter out undesirable frequencies before sampling using analog filter(s)**
   - ➢ **This is what is done in practice**
   - ➢ **Analog filters are imperfect and require tradeoffs**

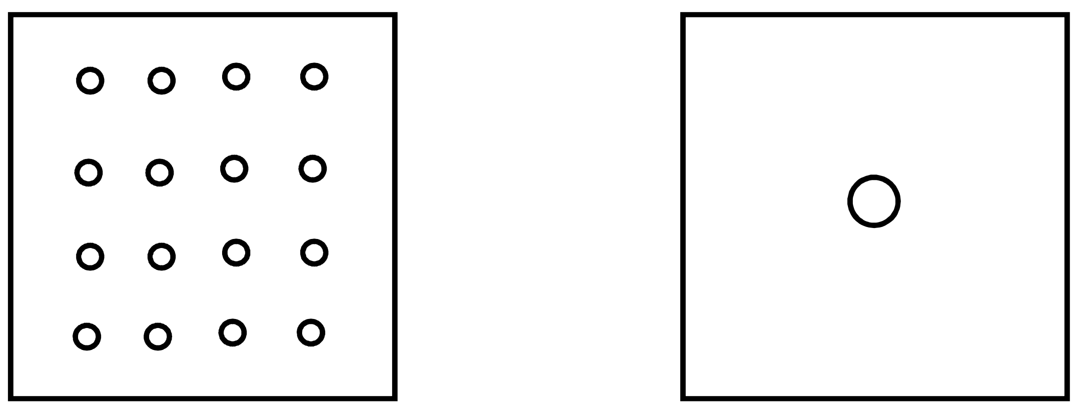
Signal Processing Pragmatics





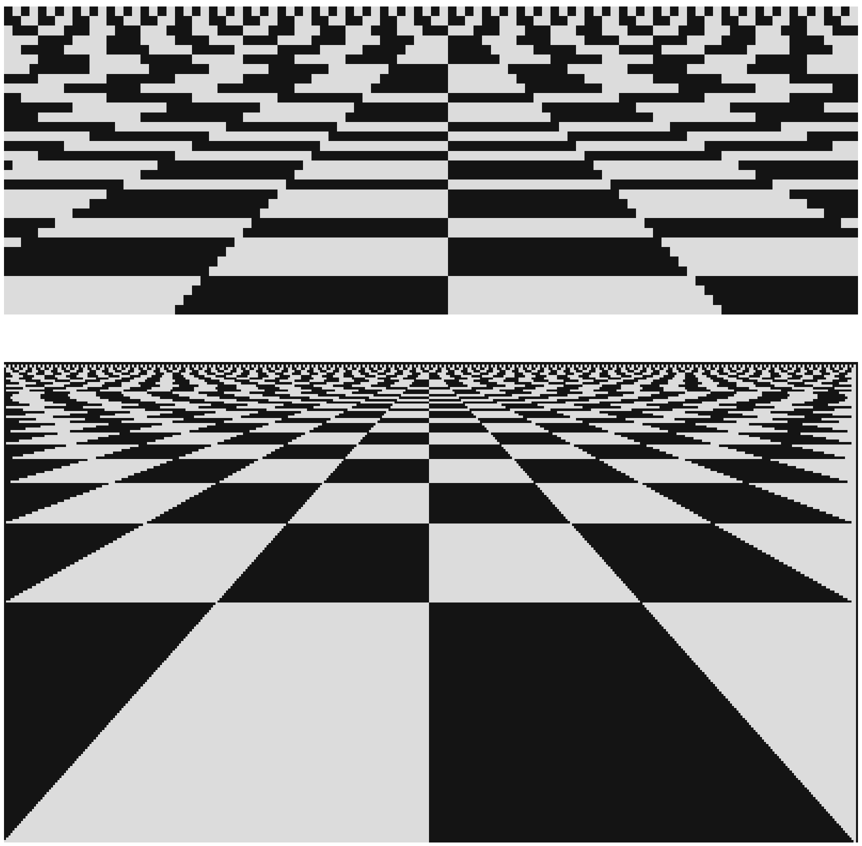
Aliasing in Space

- Spatial sampling incurs aliasing problems also
- Example: CCD in digital camera samples an image in a grid pattern
  - Real world is not band-limited
  - Can mitigate aliasing by increasing sampling rate

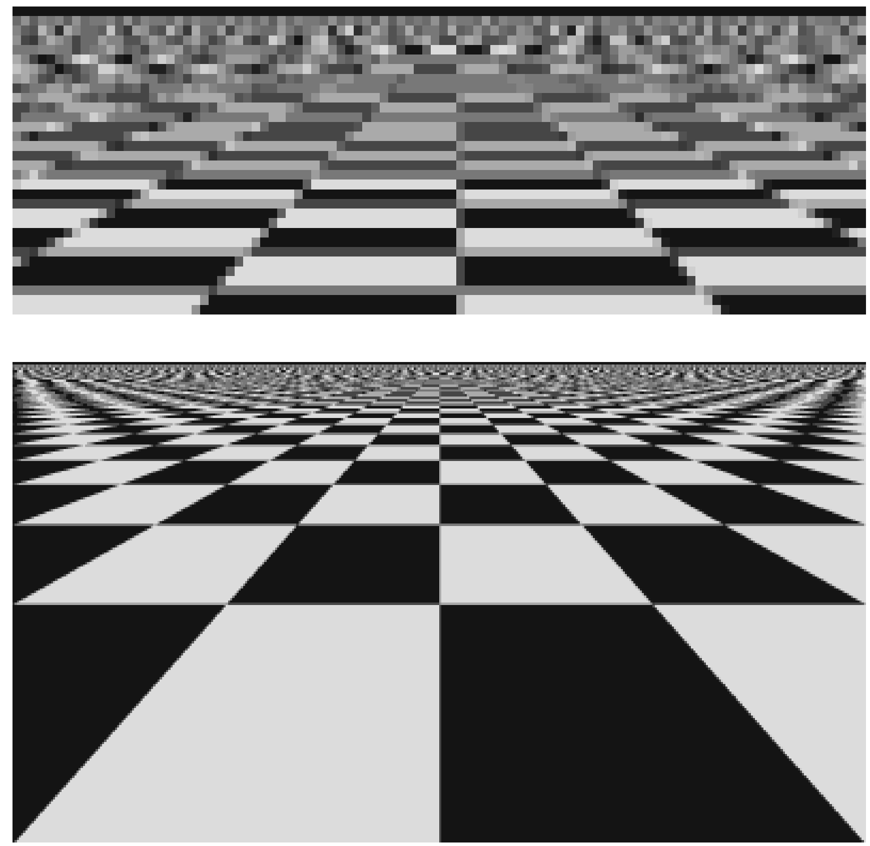


Samples   Pixel

Point vs. Supersampling



Point sampling



4x4 Supersampling

Digital Signal Processing

- Basic idea
  - Digital signals can be manipulated losslessly
  - SW control gives great flexibility
- DSP examples
  - Amplification or attenuation
  - Filtering – leaving out some unwanted part of the signal
  - Rectification – making waveform purely positive
  - Modulation – multiplying signal by another signal
    - E.g. a high-frequency sine wave

Assumptions

1. **Signal sampled at fixed and known rate f_s**
   - **I.e., ADC driven by timer interrupts**
2. **Aliasing has not occurred**
   - **I.e., signal has no significant frequency components greater than $0.5*f_s$**
   - **These have to be removed before ADC using an analog filter**
   - **Non-significant signals have amplitude smaller than the ADC resolution**

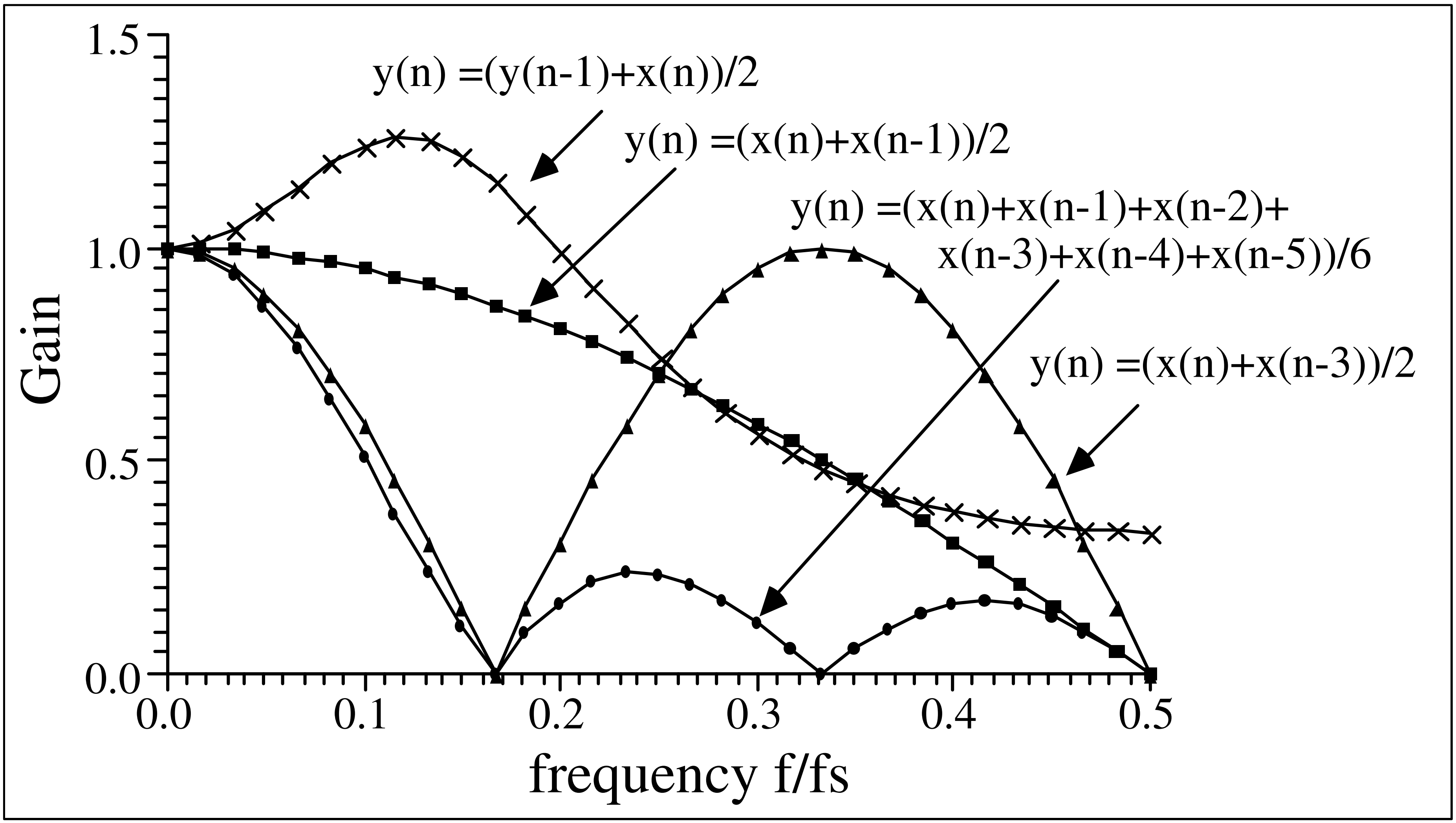
Filter Terms for CS People

- Low pass – lets low frequency signals through, suppresses high frequency
- High pass – lets high frequency signals through, suppresses low frequency
- Passband – range of frequencies passed by a filter
- Stopband – range of frequencies blocked
- Transition band – in between these

Simple Digital Filters

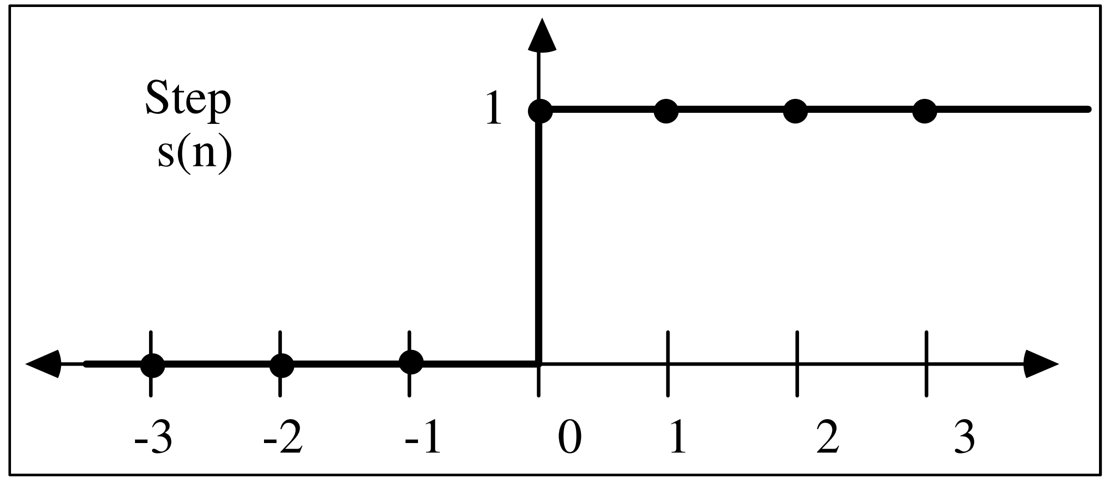
- $y(n) = 0.5 * (x(n) + x(n-1))$
  - Why not use $x(n+1)$?
- $y(n) = (1.0/6) * (x(n) + x(n-1) + x(n-2) + \ldots + y(n-5))$
- $y(n) = 0.5 * (x(n) + x(n-3))$
- $y(n) = 0.5 * (y(n-1) + x(n))$
  - What makes this one different?
- $y(n) = \text{median}\ [\ x(n) + x(n-1) + x(n-2)\ ]$

Gain vs. Frequency

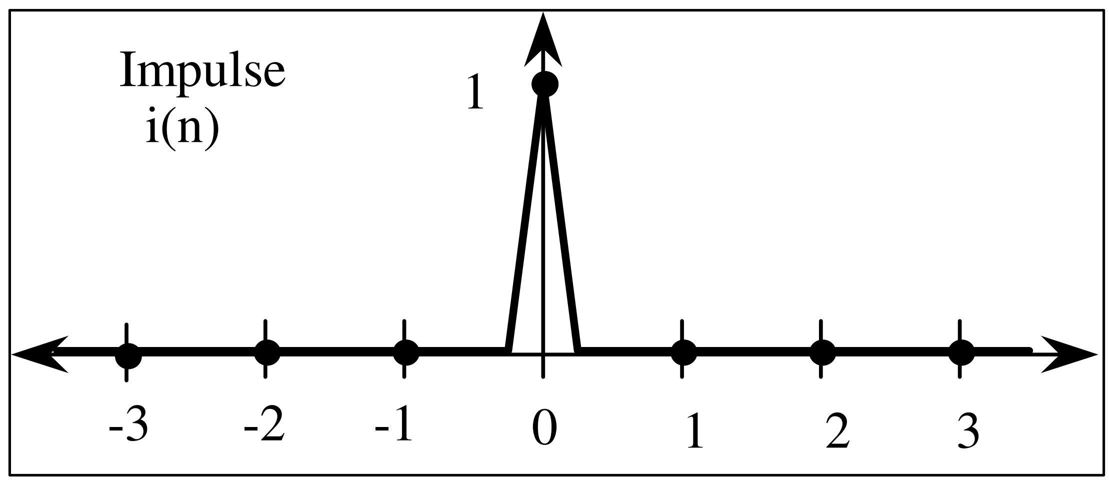


Useful Signals

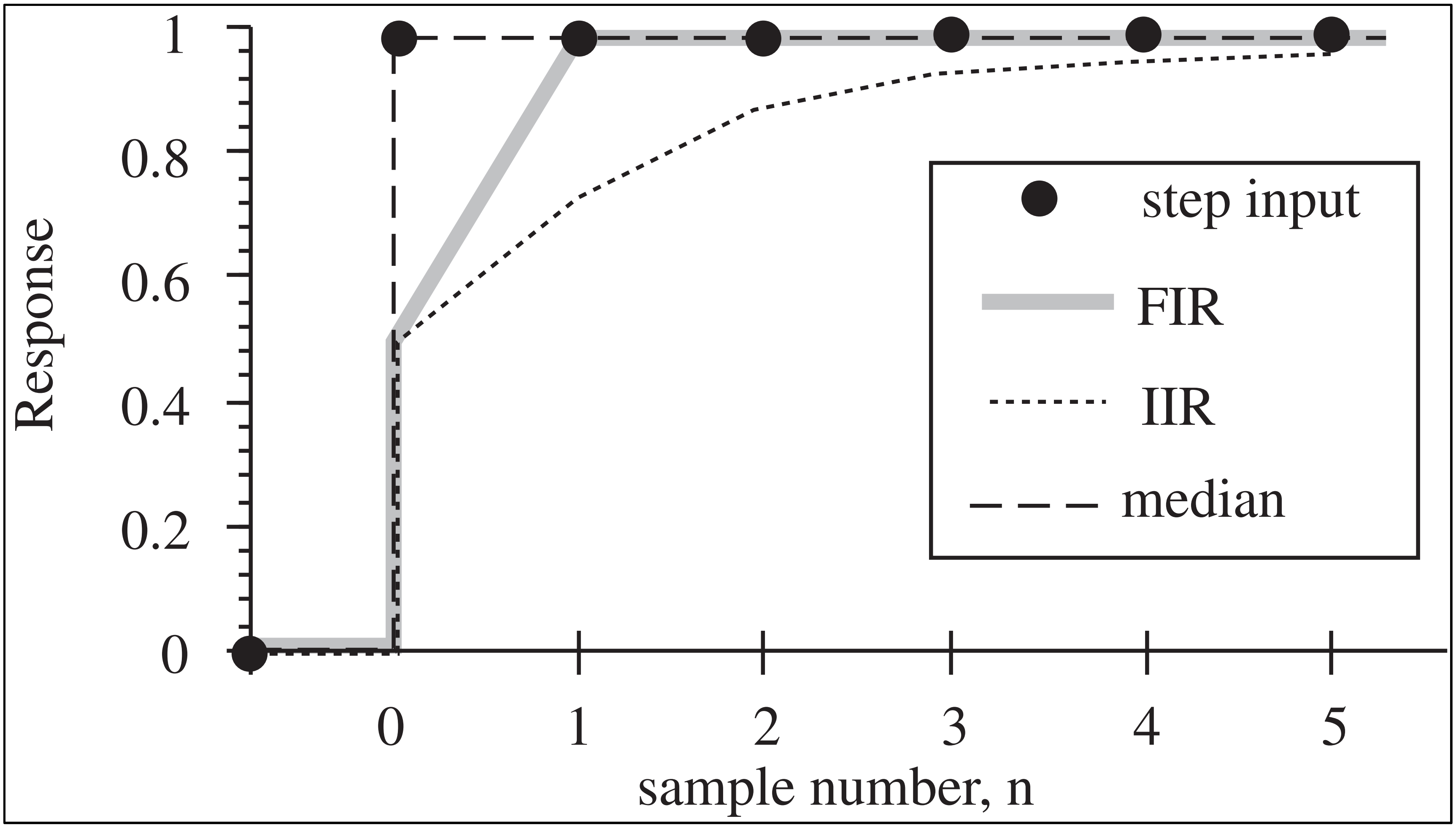
- **Step:**
  - **…, 0, 0, 0, 1, 1, 1, …**



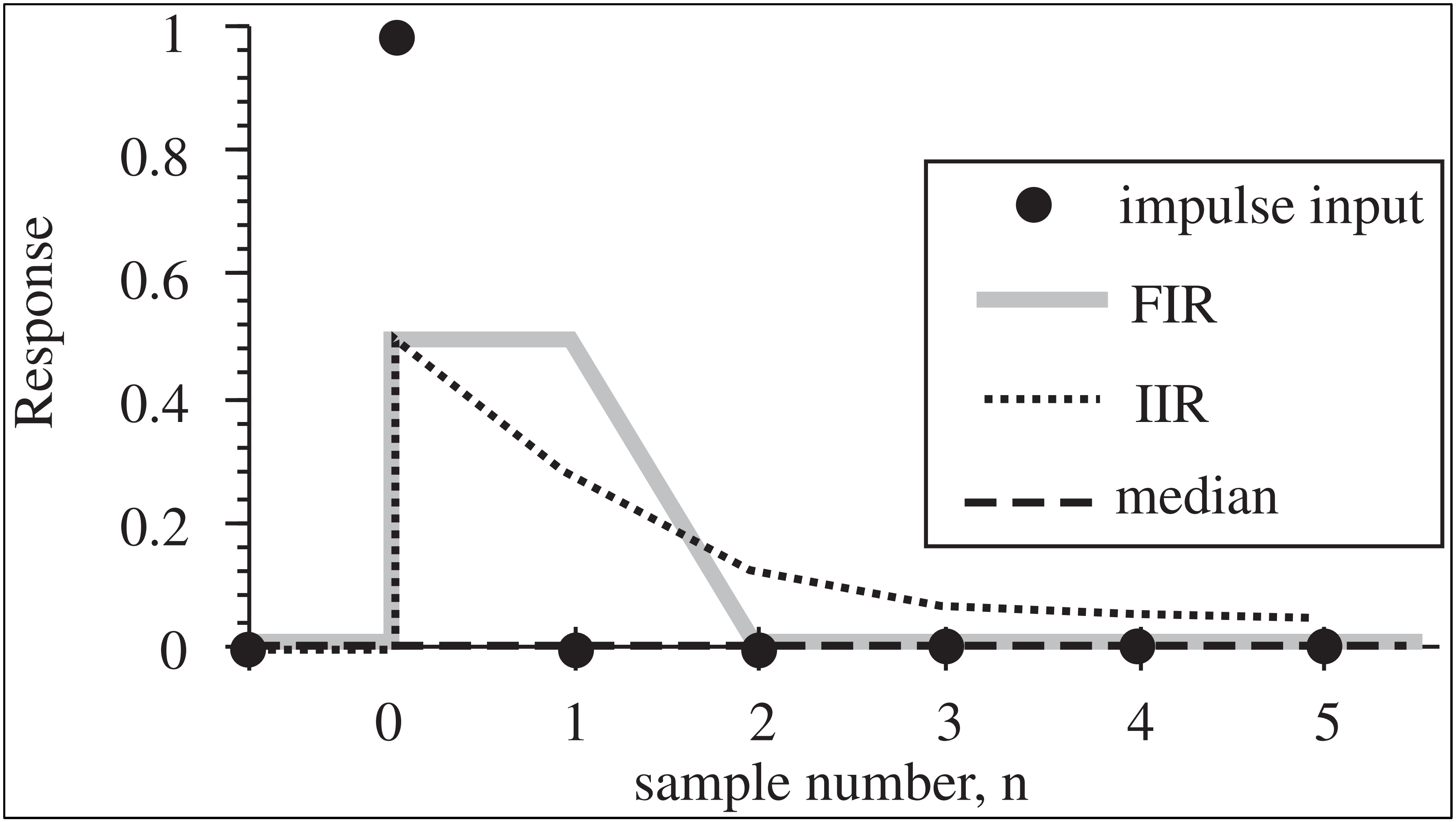
- **Impulse:**
  - **…, 0, 0, 0, 1, 0, 0, …**



Step Response



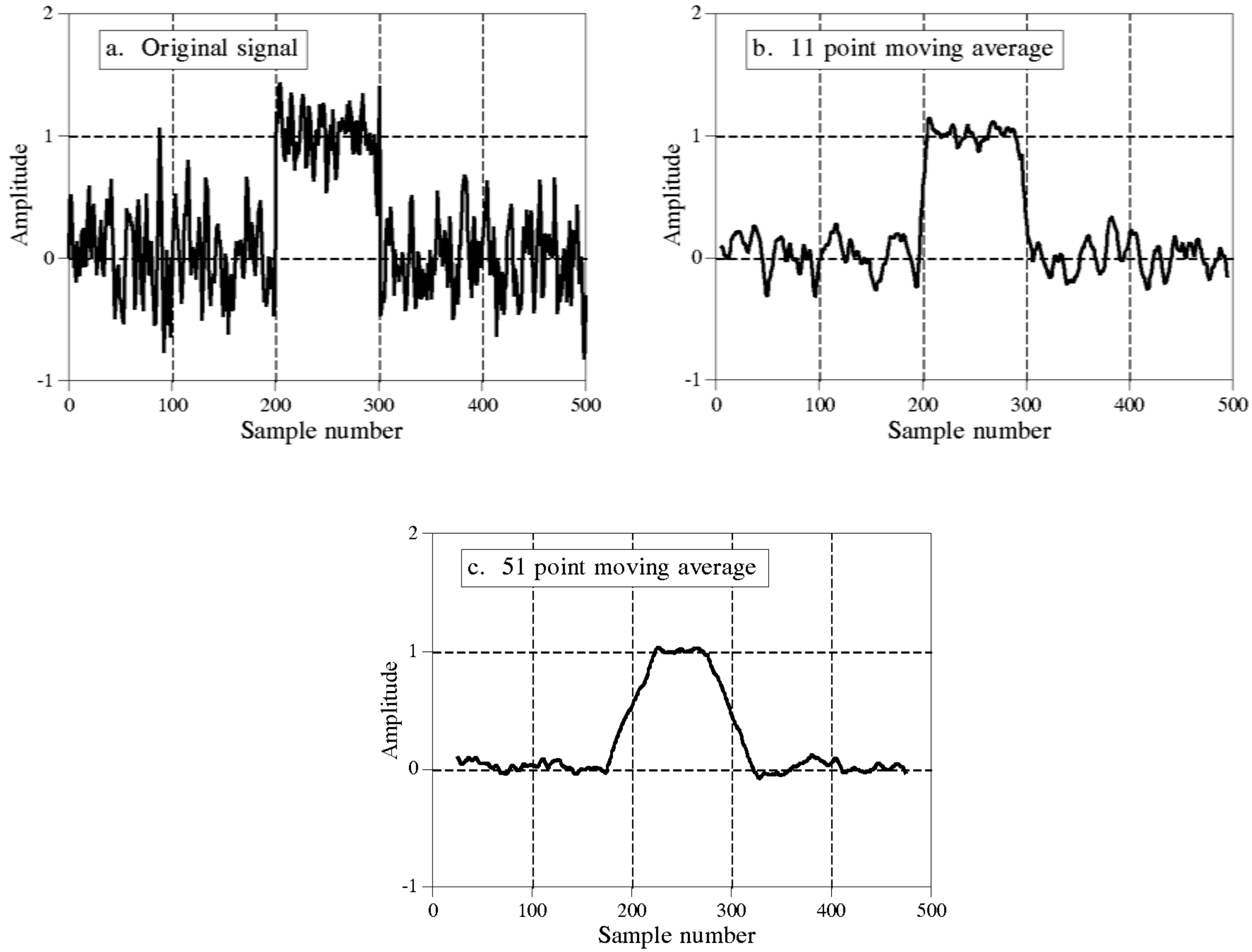
Impulse Response



FIR Filters

- Finite impulse response
  - Filter “remembers” the arrival of an impulse for a finite time
- Designing the coefficients can be hard
- Moving average filter is a simple example of FIR

Moving Average Example



FIR in C

```
SAMPLE fir_basic (SAMPLE input, int ntaps,
                  const SAMPLE coeff[],
                  SAMPLE z[])
{
  z[0] = input;
  SAMPLE accum = 0;
  for (int ii = 0; ii < ntaps; ii++) {
    accum += coeff[ii] * z[ii];
  }
  for (ii = ntaps - 2; ii >= 0; ii--) {
    z[ii + 1] = z[ii];
  }
  return accum;
}
```

Implementation Issues

- Usually done with fixed-point
- How to deal with overflow?
- A few optimizations
  - Put coefficients in registers
  - Put sample buffer in registers
  - Block filter
    - Put both samples and coefficients in registers
    - Unroll loops
  - Hardware-supported circular buffers

- Creating very fast FIR implementations is important

Filter Design

- Where do coefficients come from for the moving average filter?
- In general:
  1. Design filter by hand
  2. Use a filter design tool
- Few filters designed by hand in practice
- Filters design requires tradeoffs between
  1. Filter order
  2. Transition width
  3. Peak ripple amplitude
- Tradeoffs are inherent

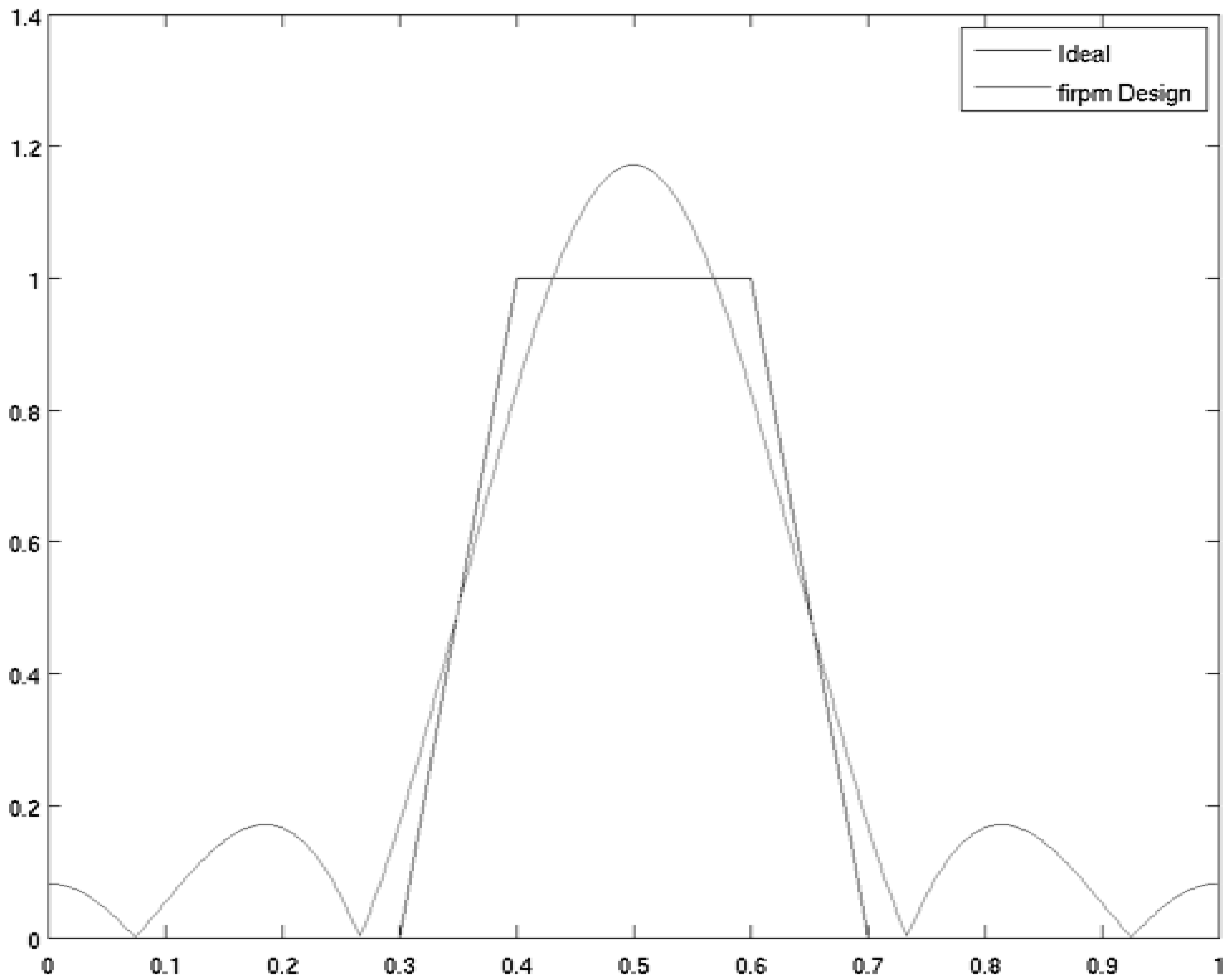
Filter Design in Matlab

- Matlab has excellent filter design support
  - `C = firpm (N, F, A)`
  - N = length of filter - 1
  - F = vector of frequency bands normalized to Nyquist
  - A = vector of desired amplitudes
- `firpm` uses minimax – it minimizes the maximum deviation from the desired amplitude

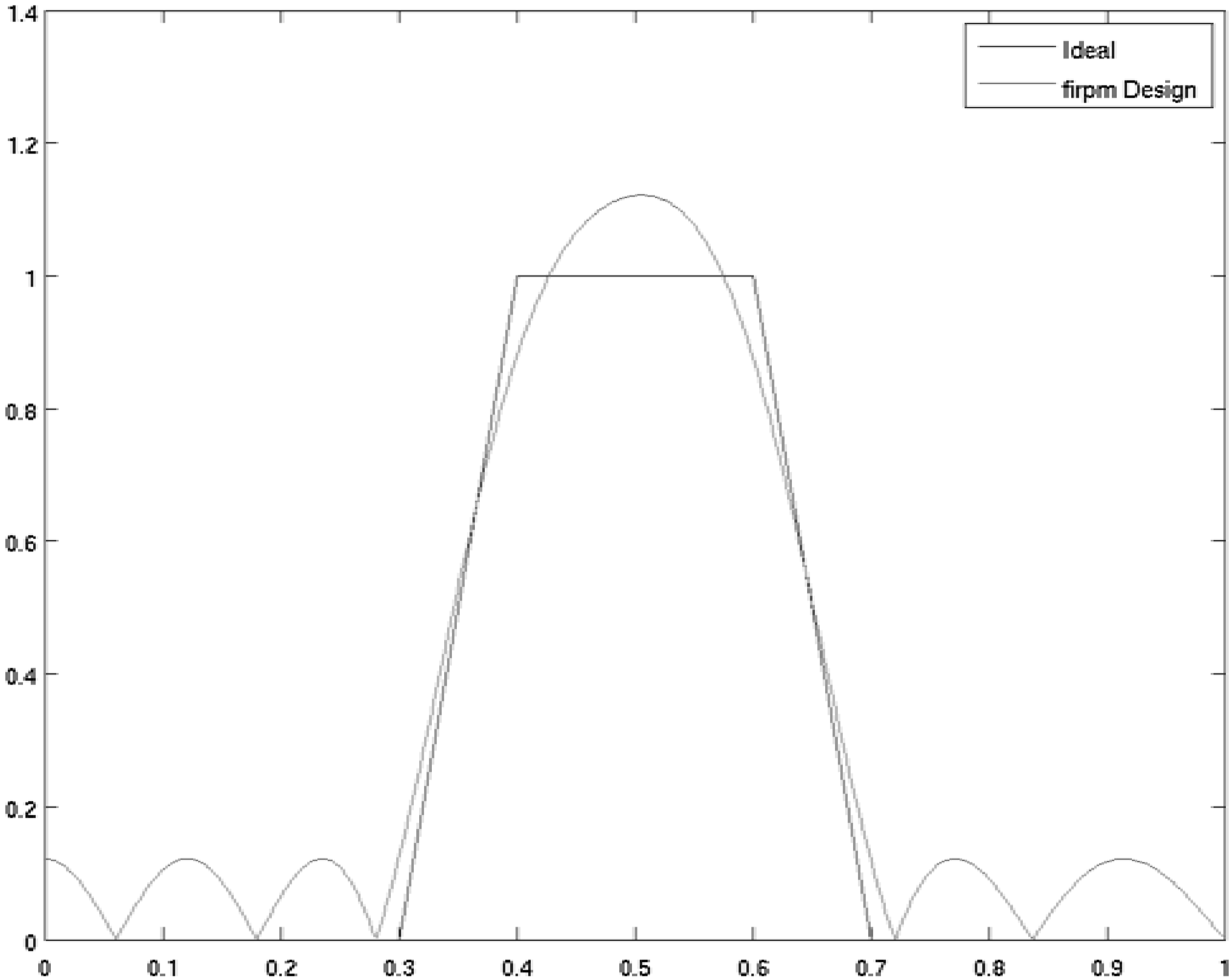
Filter Design Examples

```
f = [ 0.0 0.3 0.4 0.6 0.7 1.0];
a = [   0   0   1   1   0   0];
fil1 = firpm( 10, f, a);
fil2 = firpm( 17, f, a);
fil3 = firpm( 30, f, a);
fil4 = firpm(100, f, a);
```

```
fil2 =
  Columns 1 through 8
-0.0278 -0.0395 -0.0019 -0.0595  0.0928  0.1250 -0.1667 -0.1985
  Columns 9 through 16
 0.2154  0.2154 -0.1985 -0.1667  0.1250  0.0928 -0.0595 -0.001
  Columns 17 through 18
-0.0395 -0.0278
```



1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Ideal
firpm Design



Ideal
firpm Design
1.4
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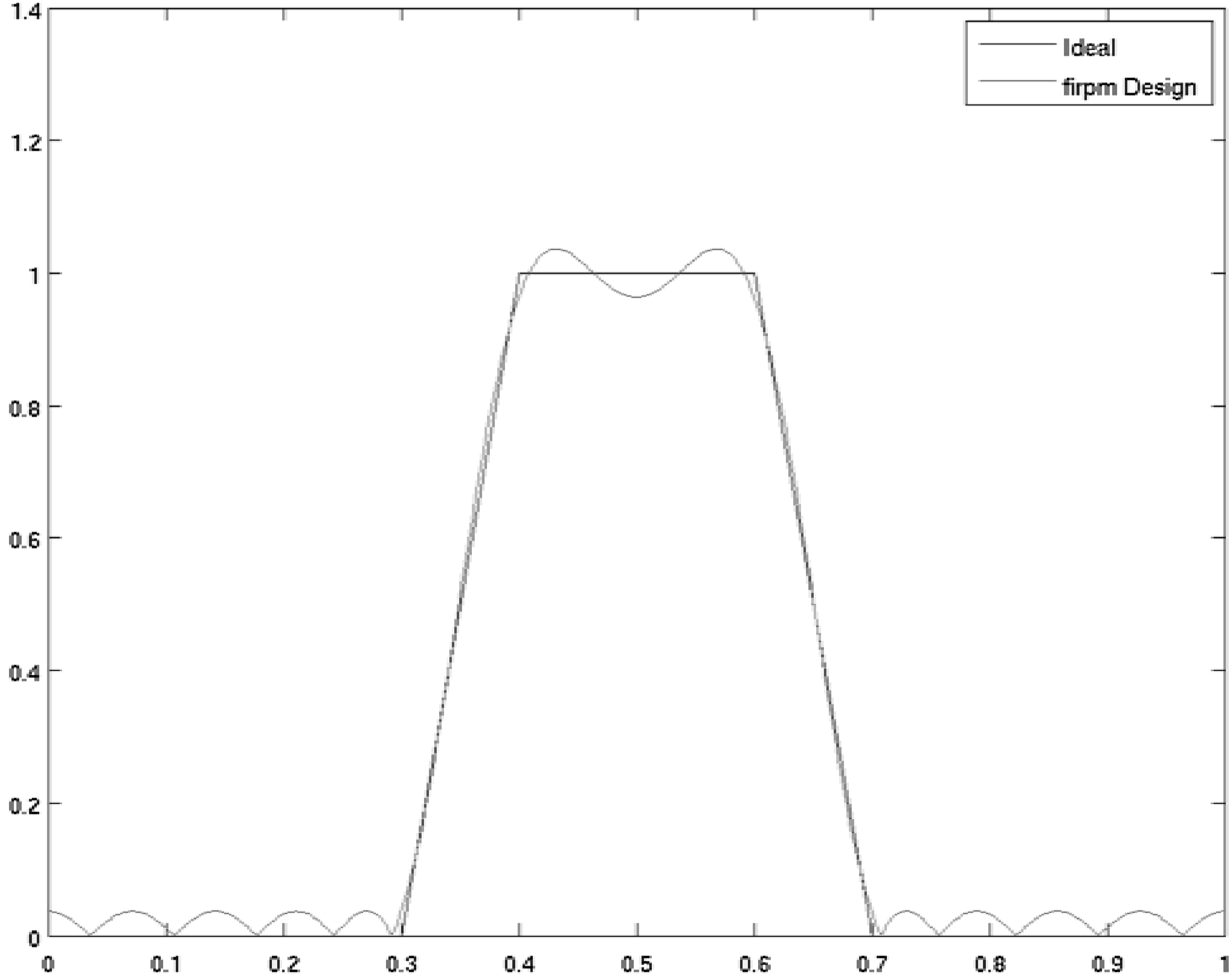
1



1.4
1.2
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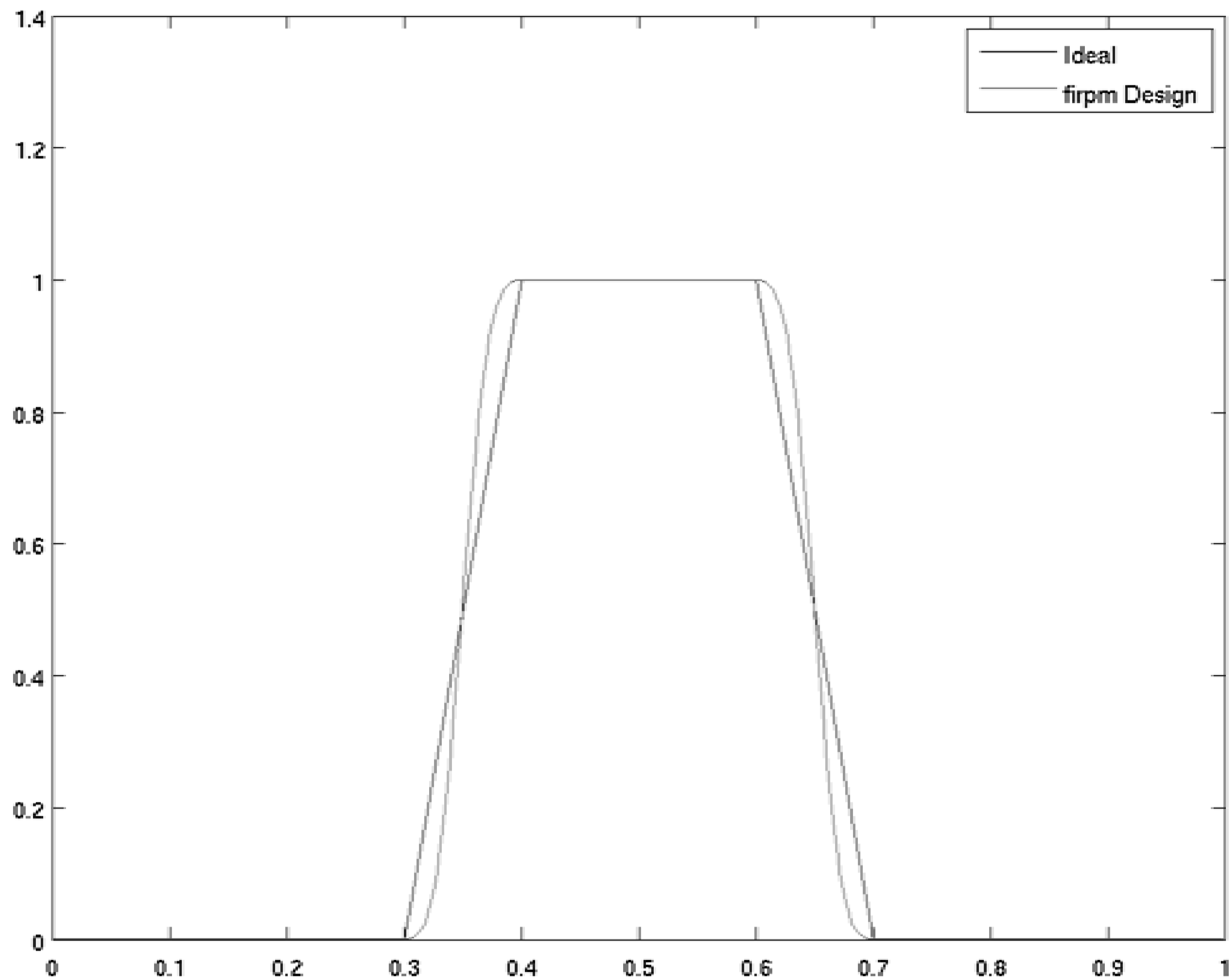
1
Ideal
firpm Design



Ideal
firpm Design
1.4
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0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Testing an FIR Filter

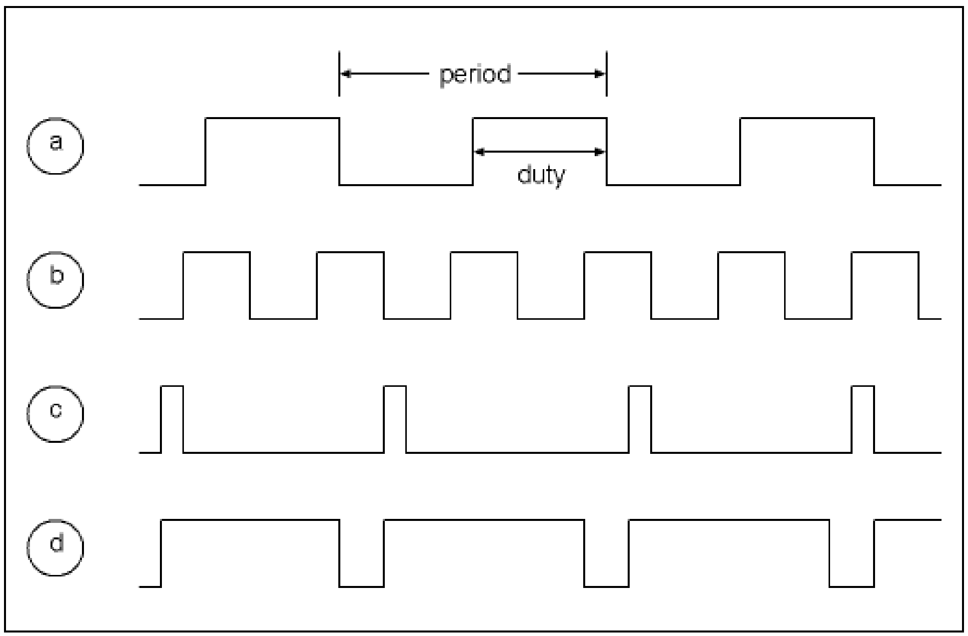
- Impulse test
  - Feed the filter an impulse
  - Output should be the coefficients
- Step test
  - Feed the filter a test
  - Output should stabilize to the sum of the coefficients
- Sine test
  - Feed the filter a sine wave
  - Output should have the expected amplitude

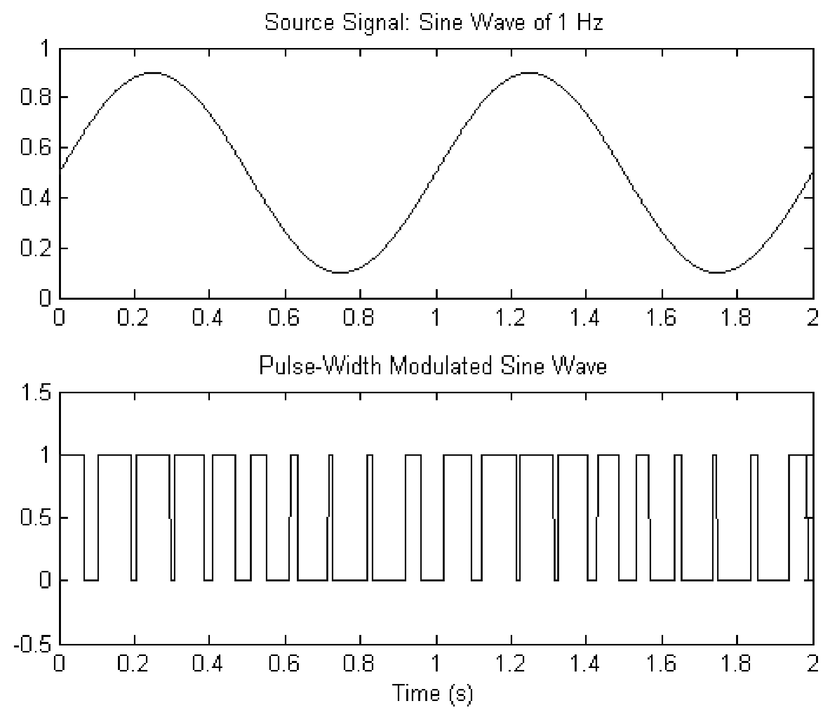
Digital to Analog Converters

- Opposite of an ADC
- Available on-chip and as separate modules
  - Also not too hard to build one yourself
- DAC properties:
  - Precision: Number of distinguishable alternatives
    - E.g. 4092 for a 12-bit DAC
  - Range: Difference between minimum and maximum output (voltage or current)
  - Speed: Settling time, maximum output rate
- LPC2129 has no built-in DACs

Pulse Width Modulation

- PWM answers the question: How can we generate analog waveforms using a single-bit output?
  - Can be more efficient than DAC





PWM

- Approximating a DAC:
  - Set PWM period to be much lower than DAC period
  - Adjust duty cycle every DAC period
- PWM is starting to be used in audio equipment
- Important application of PWM is in motor control
  - No explicit filter necessary – inertia makes the motor its own low-pass filter

Summary

- Filters and other DSP account for a sizable percentage of embedded system activity
- Filters involve unavoidable tradeoffs between
  - Filter order
  - Transition width
  - Peak ripple amplitude
- In practice filter design tools are used
- We skipped all the theory!
  - Lots of ECE classes on this