Radiosity

Not Radio City

The illumination at a given point in the environment is a combination of the light received directly from a light source and the light which is reflected one or more times from the surfaces of the environment.

Radiosity

- Equilibrium of energy balances within an enclosure.
- Based on radiative heat transfer (thermodynamics)

Light striking a surface is reflected in all directions, following the Lambertian reflection model. This diffuse reflection of light leads to color bleeding, as light striking a surface causes that surface's color into the environment.

What is Radiosity?

The radiosity of a surface is the rate at which energy leaves that surface (energy per unit time per unit area). It includes the energy emitted by a surface as well as the energy reflected from other surfaces.

Techniques of modeling the transfer of energy between surfaces based upon radiosity were first used in analyzing heat transfer between surfaces in an enclosed environment. The same techniques can be used to analyze the transfer of radiant energy between surfaces in computer graphics.

Radiosity methods allow the intensity of radiant energy arriving at a surface to be computed. These intensities can then be used to determine the shading of the surface.
Slide 25: Steel Mill.
This image of a steel rolling mill was created using progressive radiosity. The original model contains about 30,000 polygons, which were subdivided into about 55,000 elements during the solution. It was computed on a DEC VAX 8700 and displayed using a Hewlett-Packard SRX graphics device.

Radiosity Enclosures

- **Bi** – total rate of energy leaving surface I per unit area, sometimes called flux
- **Flux** contains 2 parts:
  - **Ei** – rate at which the surface I emits energy as a source
  - **piHi** – rate at which the incident light (energy) is reflected back into the environment
- **pi** – reflectivity of the surface
- **Hi** – radiant energy incident on surface I

Incident flux = \( \sum \text{flux from all surfaces} \)

\[ B_i = E_i + p_i H_i \]

Figure 5-81 A hypothetical environment

Diffuse surfaces have one radiance (color) for all viewing directions
Energy leaving patch $j$ that reaches patch $i$

\[ H_i = \sum_{j=1}^{N} B_j \frac{A_j F_{ji}}{A_i} \]

Per unit area of patch $i$

Radiosity reciprocity relation

\[ A_i F_{ij} = A_j F_{ji} \]

\[ H_i = \sum_{j=1}^{N} B_j \frac{A_j F_{ji}}{A_i} \]
Radiosity reciprocity relation

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Radiosity Equation

\[ B_i = E_i + p_i H_i \]

\[ B_i = E_i + \sum_{j=1}^{N} B_j F_{ij} \]

Classic Radiosity

Power balance

\[ B_i = E_i A_i + \rho_i \sum B_j A F_j \]

\[ B_i = E_i + \rho_i \sum B_j A_{ij} \]

Linear system of equations

\[
\begin{pmatrix}
1 - \rho F_{ii} & -\rho F_{ij} & \cdots & -\rho F_{in} \\
-\rho F_{ji} & 1 - \rho F_{jj} & \cdots & -\rho F_{jn} \\
\vdots & \vdots & \ddots & \vdots \\
-\rho F_{ni} & -\rho F_{nj} & \cdots & 1 - \rho F_{nn}
\end{pmatrix}
\begin{pmatrix}
B_i \\
B_j \\
B_k \\
B_n
\end{pmatrix}
= \begin{pmatrix}
E_i \\
E_j \\
E_k \\
E_n
\end{pmatrix}
\]
Radiosity Form Factors

\[ F_{i, j} = \frac{1}{A_i} \int_{A_j} \cos \phi_i \cos \phi_j \, dA \]

**The Form Factor**

The form factor is defined as the fraction of energy leaving one surface that reaches another surface. It is a purely geometric relationship, independent of viewpoint or surface attitudes.

Between differential areas, the form factor equals:

\[ F_{i, j} = \frac{\cos \phi_i \cos \phi_j}{\sigma_{i, j}} \]

\[ \sigma_{i, j} = \frac{1}{\pi} \int_{A_i} \frac{1}{r} dA \]

The overall form factor between \( i \) and \( j \) is found by integrating:

\[ F_{i, j} = \frac{1}{A_i} \int_{A_j} \frac{1}{r} dA \]

**Classic Radiosity Algorithm**

1. Mesh Surfaces into Elements
2. Compute Form Factors Between Elements
3. Solve Linear System for Radiosities
4. Reconstruct and Display Solution

**Simple Room Scene**

Table in room sequence from Cohen and Wallace

**patches visible to patch i**

Shooting methods: patch i can send its power to the patches based on percentage of area.
How Does OpenGL Fit In?

Figure 5-64  The equivalence of projected areas means A, B, C, D, and E all have the same form factors.

Figure 5-65  Projection of the environment onto the hemicube covering a patch.

Figure 5-66  A patch projected onto a subdivided hemicube.

Hemicube Delta Form Factors
Distributing the Energy

- Solve linear system
  - Diagonal dominant
  - Use Gauss-Seidel or other method
- Hierarchical methods
  - Do interaction between 2 far-away groups of patches all at once

Hierarchical Radiosity

Usually an adaptive hierarchy is used
**Progressive Solution**
The above images show increasing levels of global diffuse illumination: From left to right: 0 luxes, 1 luxes, 2 luxes.

**Accuracy**

Reference Solution  
Uniform Mesh

Table in room sequence from Cohen and Wallace

**Artifacts**

A. Blocky shadows  
B. Missing features  
C. Mesh bands  
D. Inappropriate shading discontinuities  
E. Unresolved discontinuities

**Increasing Resolution**

**Adaptive Meshing**

**Discontinuity Mesh**

From Campbell et al.
Summary

Remember assumptions
- Diffuse reflectance
- Polygons

Difficult to relax assumptions

Computation challenges
- Meshing
- Complex input geometry
- Complexity due to shadows
- Dense coupling
- $O(n^2)$ matrix elements
- HR leads to $O(n^2)$ algorithm (ignoring discontinuities)

OpenGL to do more?
- Relook at equations

Approximate one-bounce
- Item buffer rendered from light.
- Item buffer is scanned by multiple CPUs to determine polygons which reflect maximum energy.
- First bounce of brightest polygons approximated by point lights.
- Only one pass. If $p$ is number of pipes, then $8p$ virtual lights used.
- No visibility. Since intensity of virtual lights is low, incorrect illumination is not noticeable in most scenes.

Indirect lighting
- Location of 240 virtual lights
- Result
Slide Credits

- Pat Hanrahan, Stanford CS348B
- Pete Shirley, Siggraph 98 Radiosity Course