

What is “*Perspective*?”

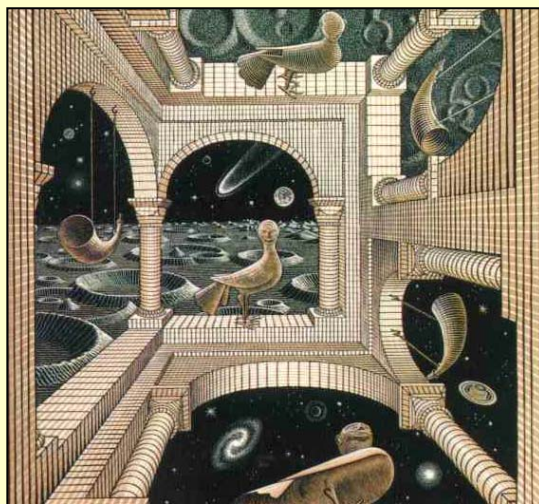
- A mechanism for portraying 3D in 2D
- “*True Perspective*” corresponds to projection onto a plane
- “*True Perspective*” corresponds to an ideal camera image

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Perspective (Mural) Games

M C Esher,
*Another
World II*
(1947)



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Perspective

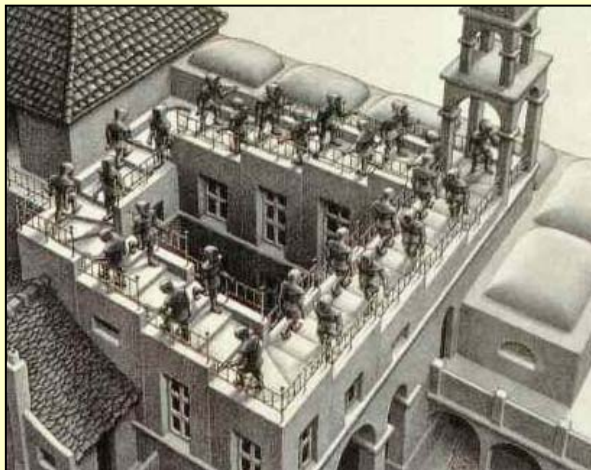


M.C. Escher,
Ascending
and
Descending
(1960)

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M. C. Escher



M.C. Escher,
Ascending
and
Descending
(1960)

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M. C. Esher

- Perspective is “local”
- Perspective consistency is not “transitive”
- Nonplanar (hyperbolic) projection

Nonplanar (Hyperbolic) Projection



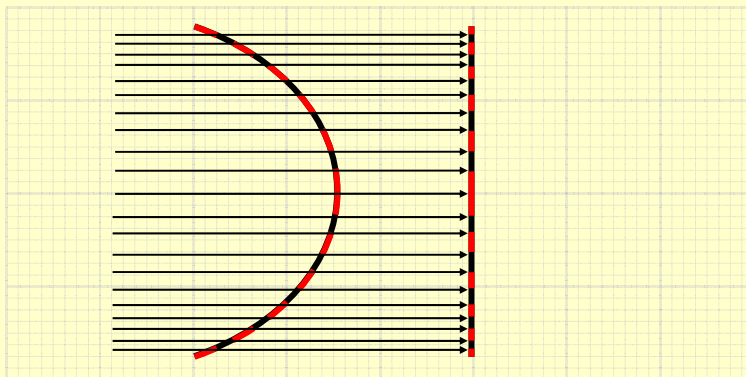
M C Esher,
Heaven and Hell

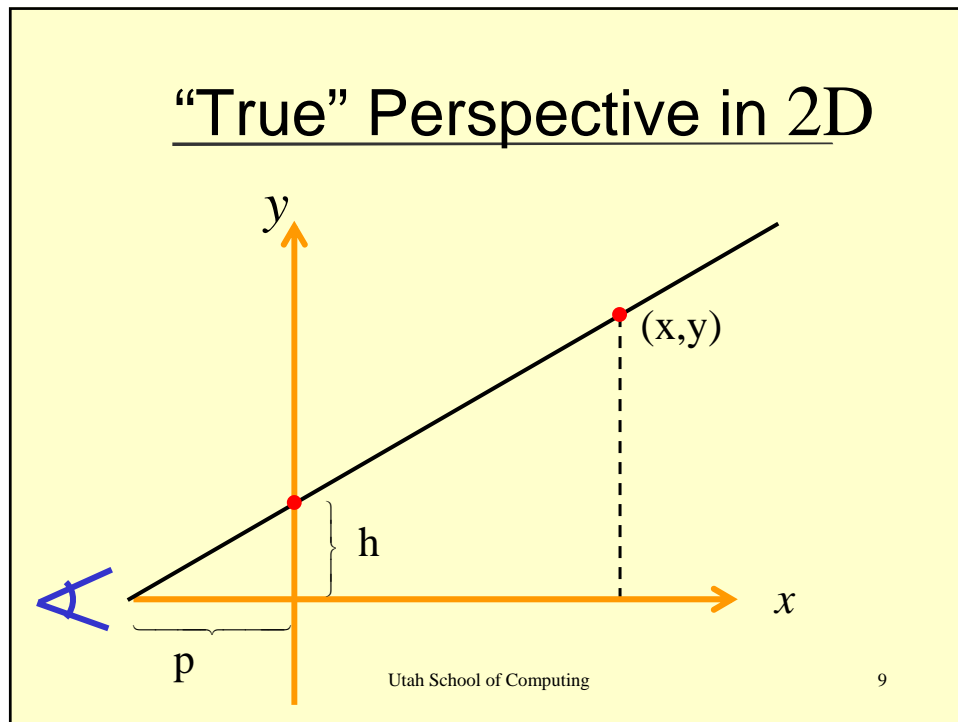
Nonplanar (*Hyperbolic*) Projection



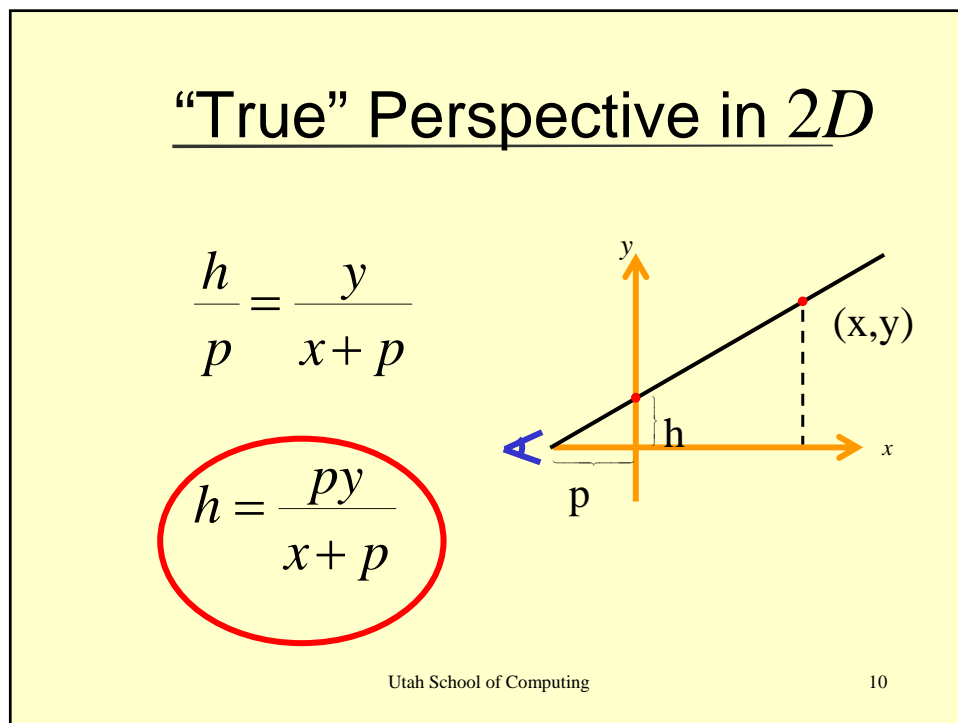
M C Escher,
Heaven and Hell

Curvilinear Projection





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“True” Perspective in 2D

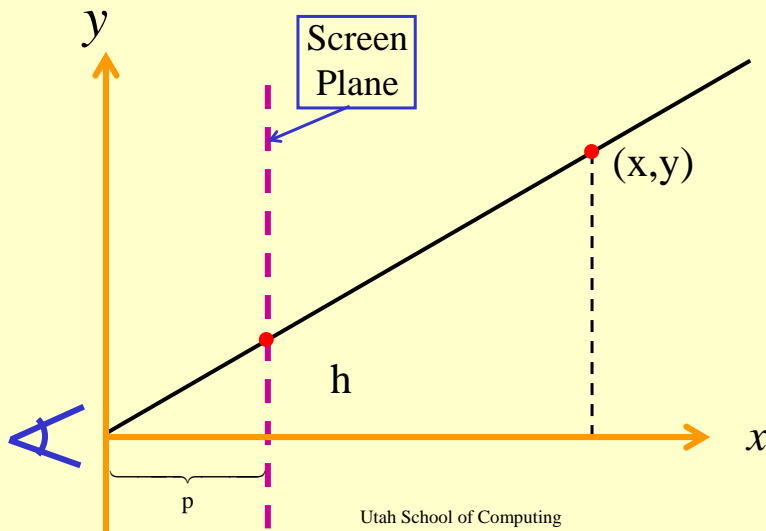
$$\begin{bmatrix} ? \\ \\ \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Leftrightarrow \begin{bmatrix} \frac{px}{x+p} \\ \frac{py}{x+p} \end{bmatrix}$$

“True” Perspective in 2D

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{p} & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ \frac{x}{p} + 1 \end{bmatrix}$$

$$= \begin{bmatrix} x \\ y \\ \frac{x+p}{p} \end{bmatrix} = \begin{bmatrix} \frac{px}{x+p} \\ \frac{py}{x+p} \\ 1 \end{bmatrix} \Leftrightarrow \begin{bmatrix} \frac{px}{x+p} \\ \frac{py}{x+p} \end{bmatrix}$$

Geometry Same for Eye at Origin



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What Happens to Special Points?

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{p} & 0 & 1 \end{bmatrix} \begin{bmatrix} -p \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -p \\ 0 \\ 0 \end{bmatrix}$$

What is this point??

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Let's Look at a Limit

Observe,

$$\lim_{n \rightarrow \infty} \begin{bmatrix} 1 \\ 0 \\ \left(\frac{1}{n}\right) \end{bmatrix} = \begin{bmatrix} n \\ 0 \\ 1 \end{bmatrix} \Leftrightarrow \begin{bmatrix} n \\ 0 \end{bmatrix}$$

We see that $\begin{bmatrix} n \\ 0 \end{bmatrix} \Leftrightarrow +\infty$ on x -axis

Where does Eye Point Go?

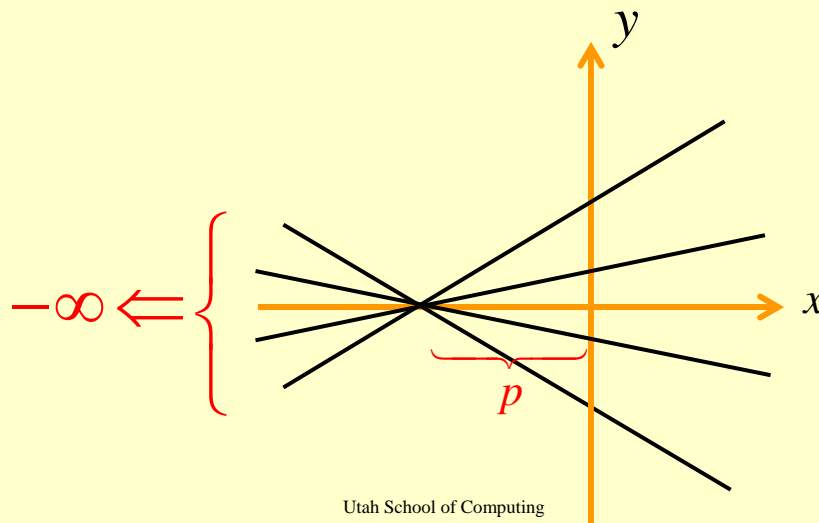
- It gets sent to $-\infty$ on x -axis
- Where does $+\infty$ on x -axis go?

What happens to $+\infty$?

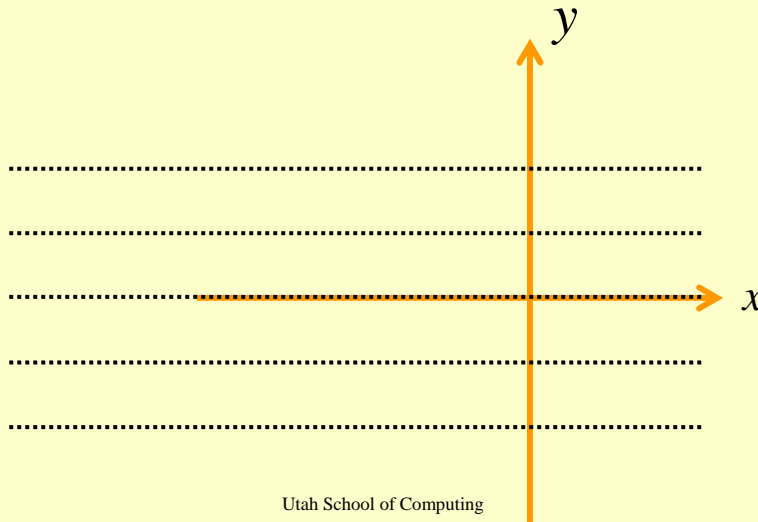
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{p} & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \frac{1}{p} \end{bmatrix} = \begin{bmatrix} p \\ 0 \\ 1 \end{bmatrix} \Leftrightarrow \begin{bmatrix} p \\ 0 \end{bmatrix}$$

It comes back to virtual eye point!

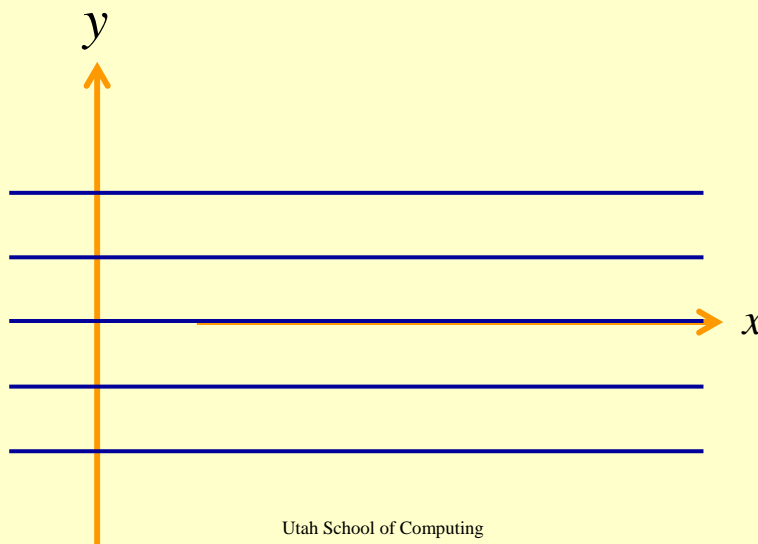
What Does This Mean?

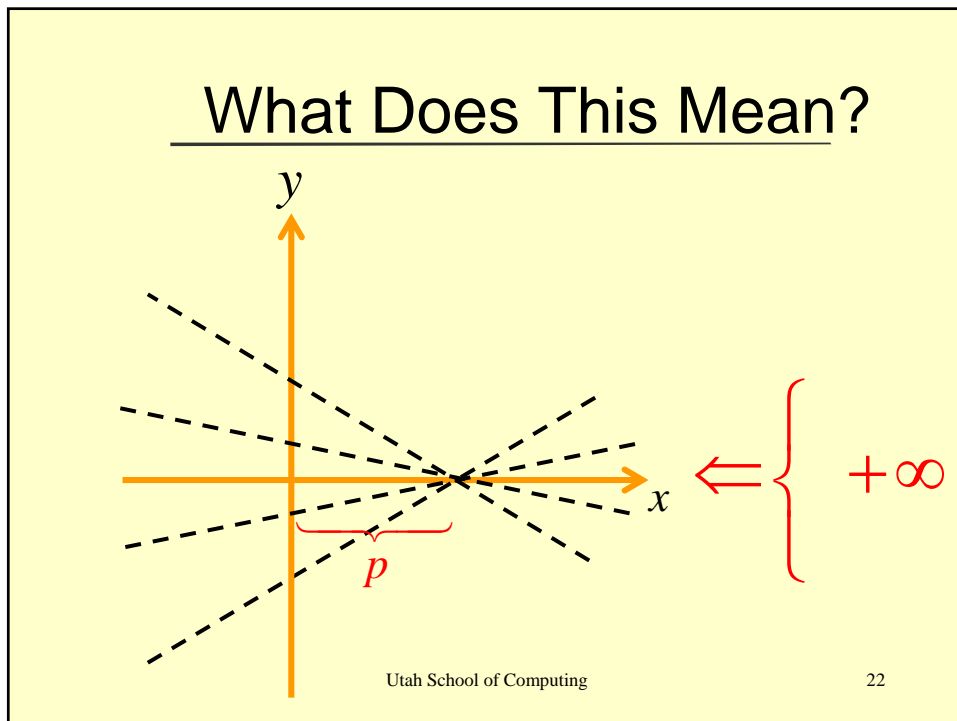
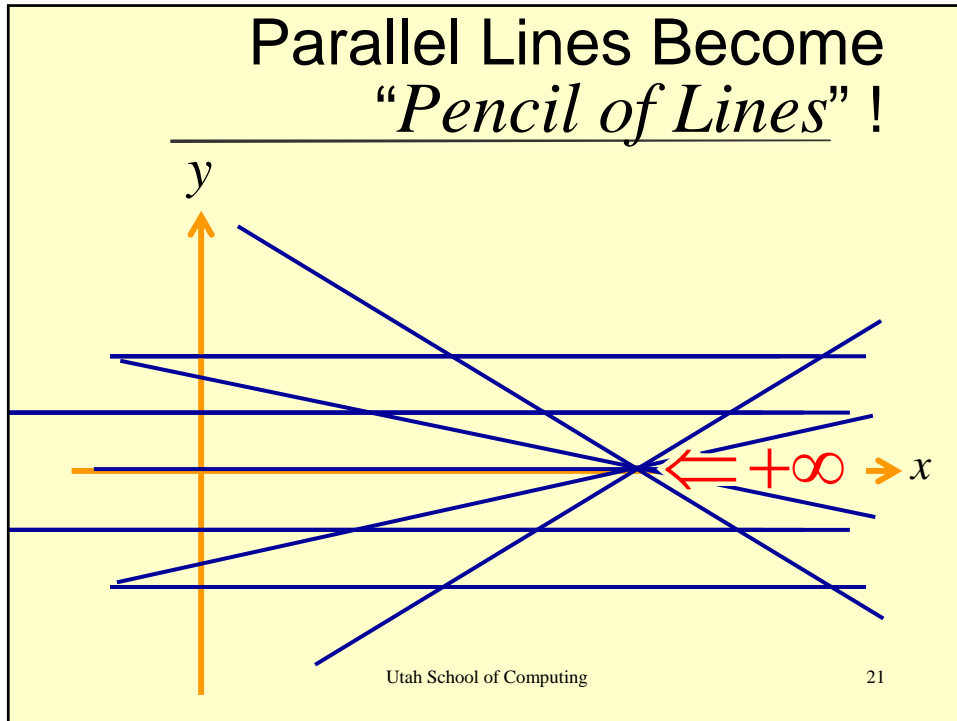


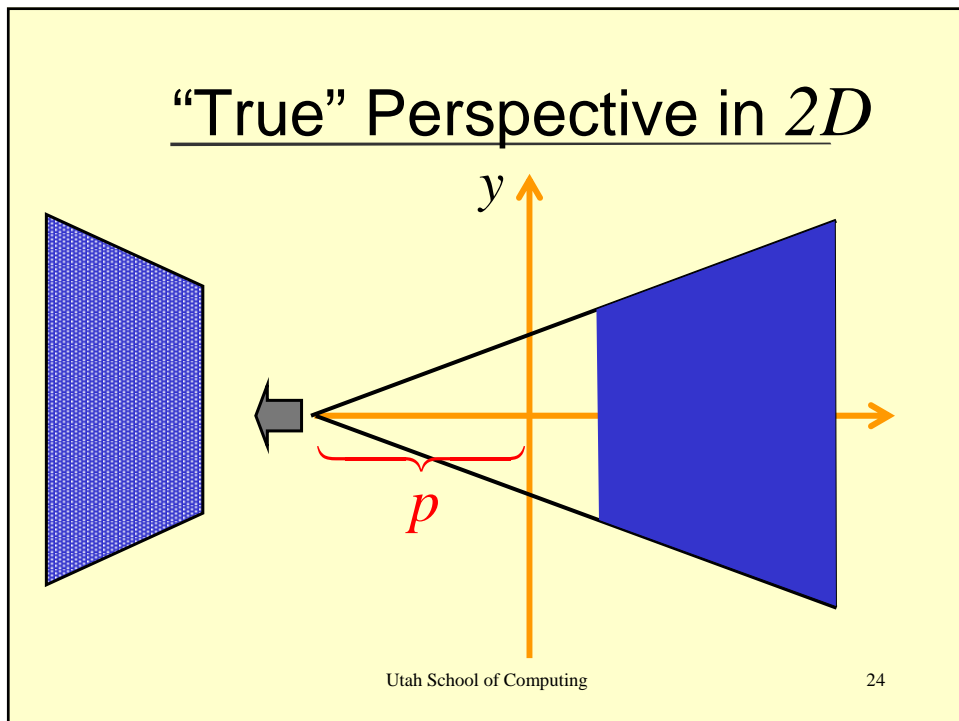
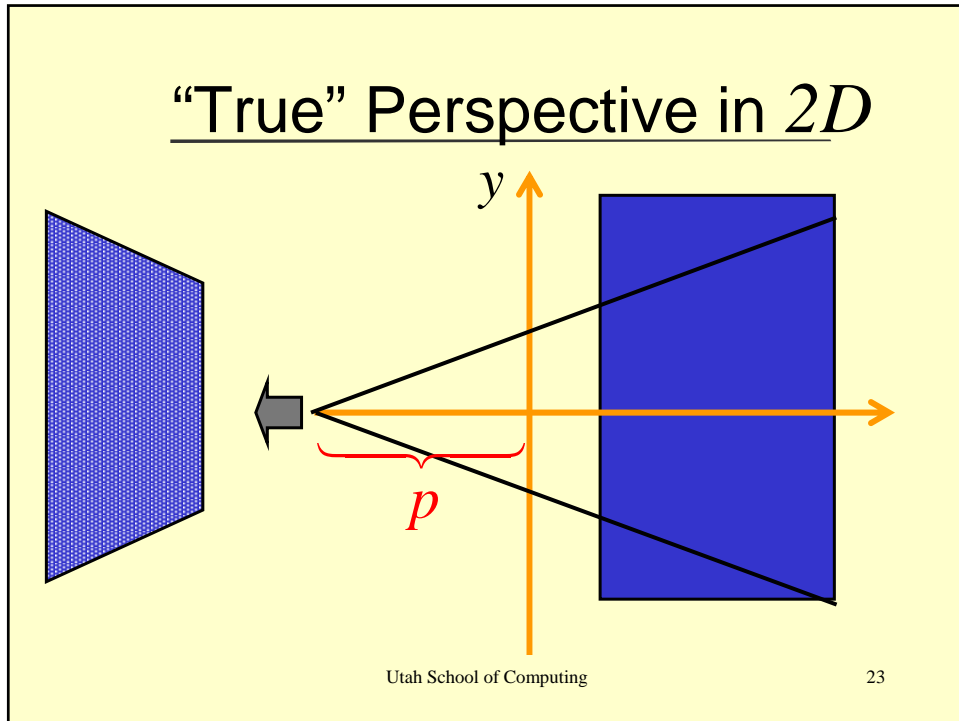
“Pencil of Lines” Becomes Parallel

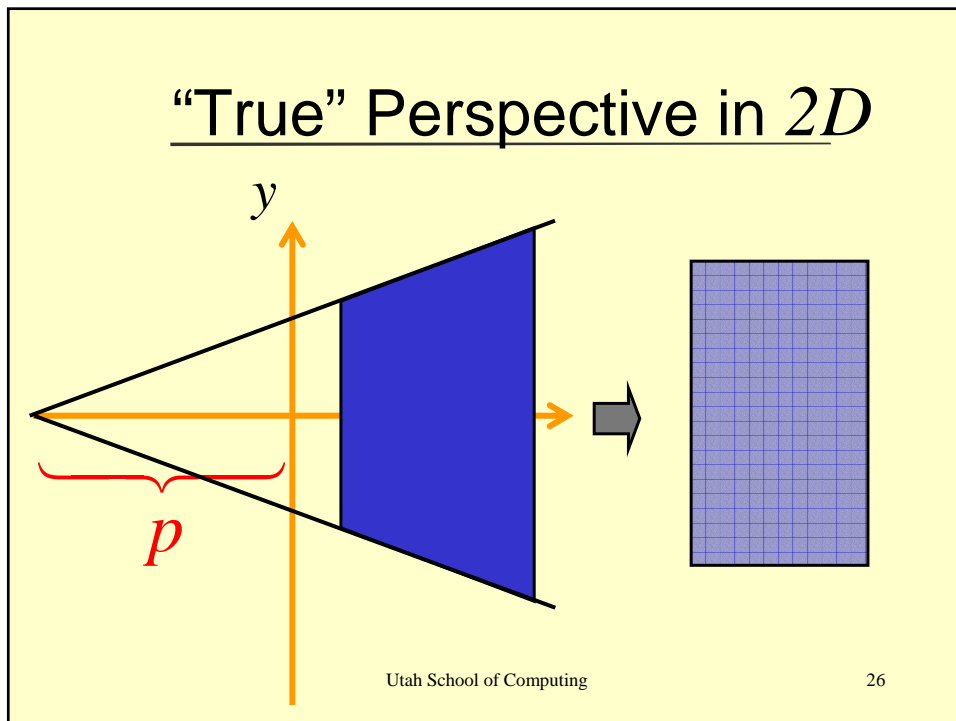
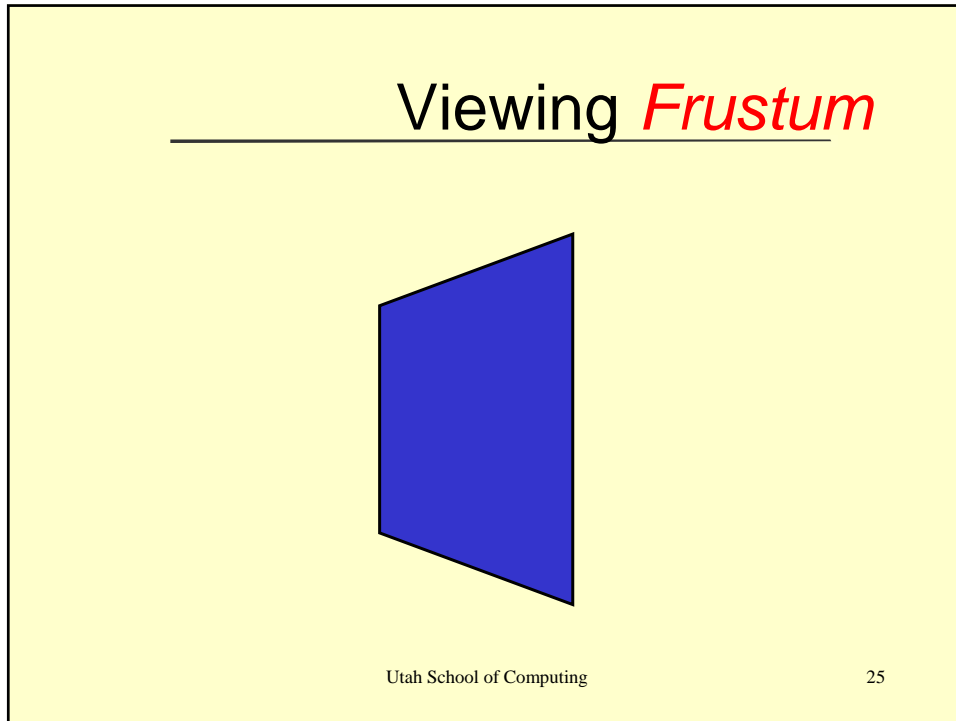


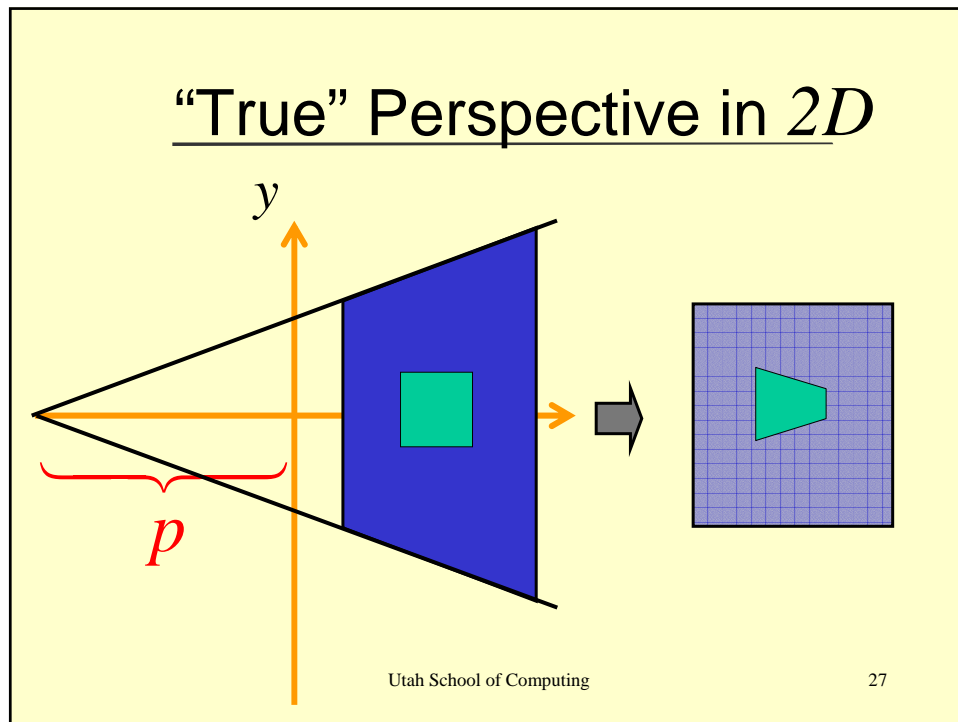
Parallel Lines Become a “Pencil of Lines” !











What happens for large p ?”

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{p} & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\lim_{p \rightarrow \infty} \frac{1}{p} = 0$$

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