Lecture Set 6

Transformations II

CS5600 *Computer Graphics*

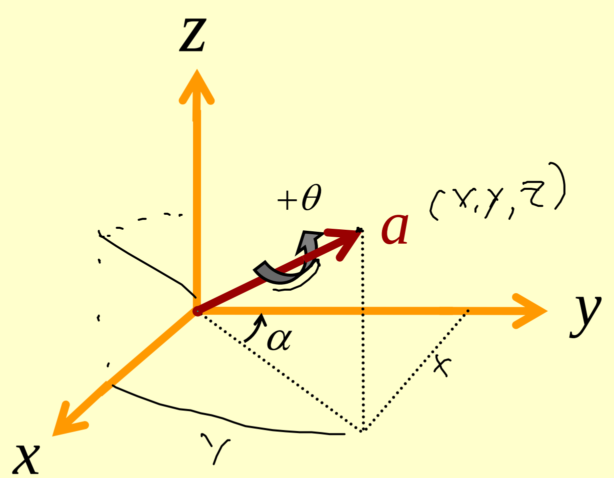
Rich Riesenfeld

Spring 2009

Arbitrary *3D Rotation*

- What is its inverse?
- What is its transpose?
- Can we constructively elucidate this relationship?

Rotate $+\theta$ *about axis* $a: R_a(+\theta)$



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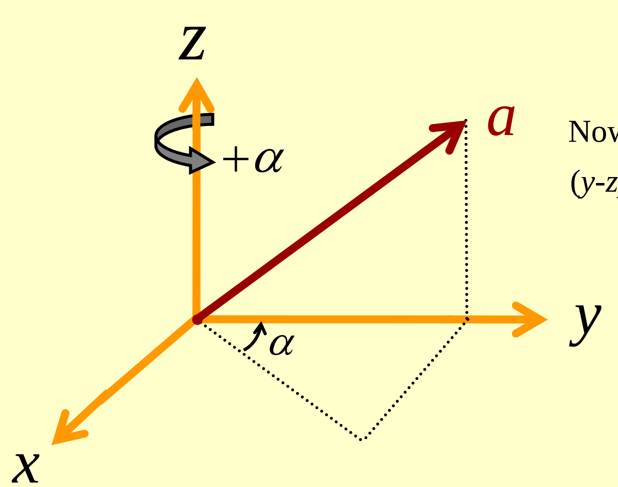
First, *Rotate about z by* $+\alpha$: $R_z(+\alpha)$



Now in the (*y-z*)-plane

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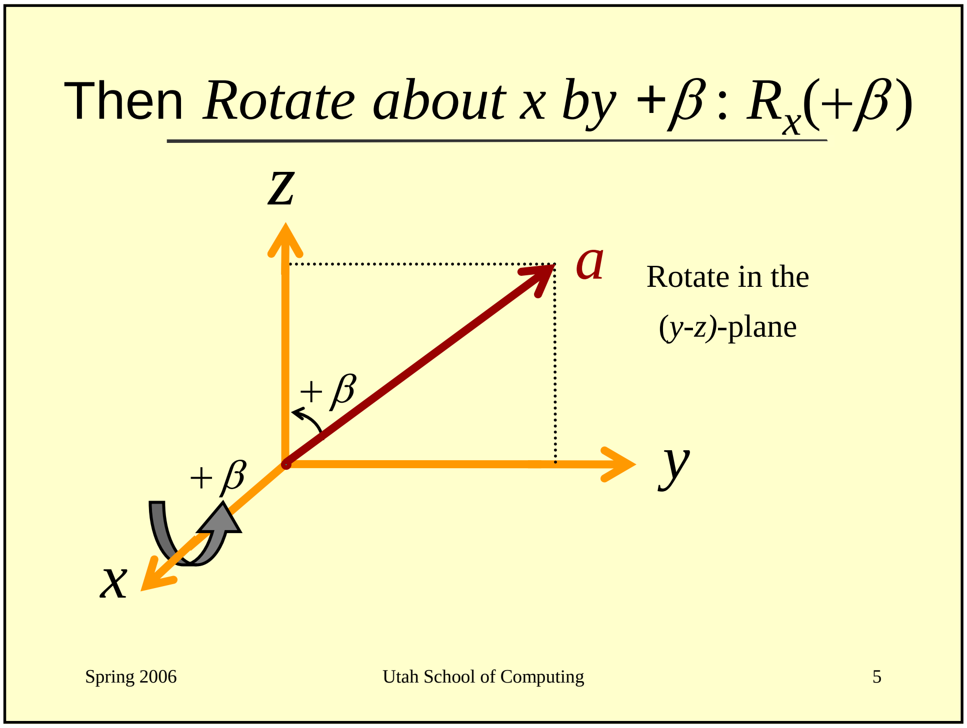
Then *Rotate about x by* $+\beta$: $R_x(+\beta)$
z
a
Rotate in the
(y-z)-plane
$+\beta$
y
$+\beta$
x
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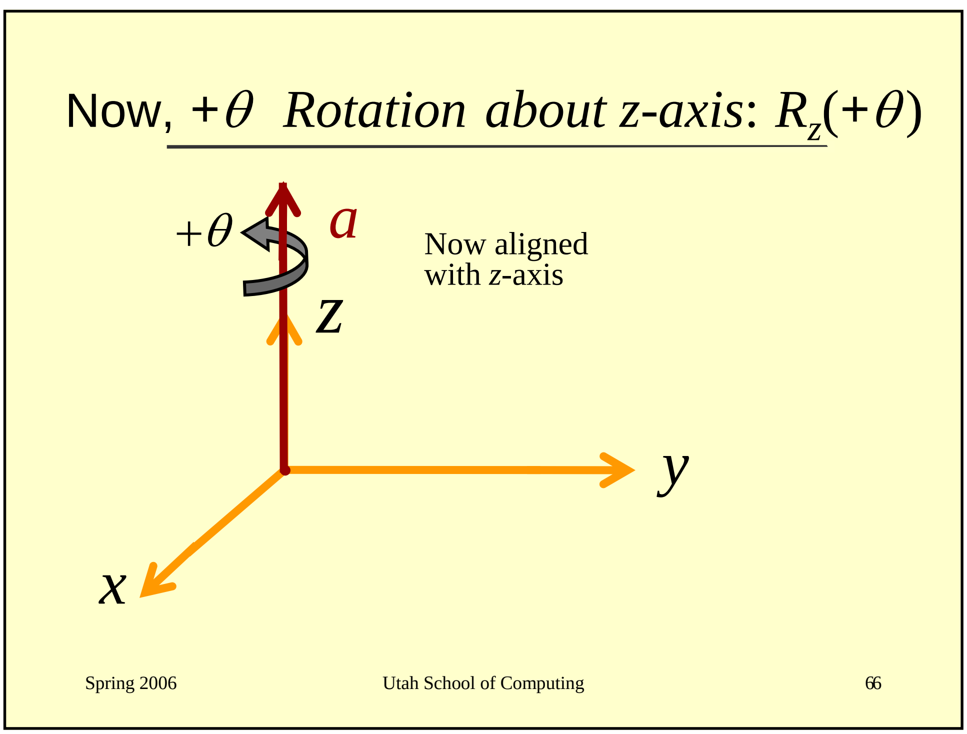
Now, $+\theta$ *Rotation about z-axis*: $R_z(+\theta)$
$+\theta$
a
Now aligned
with z-axis
z
y
x
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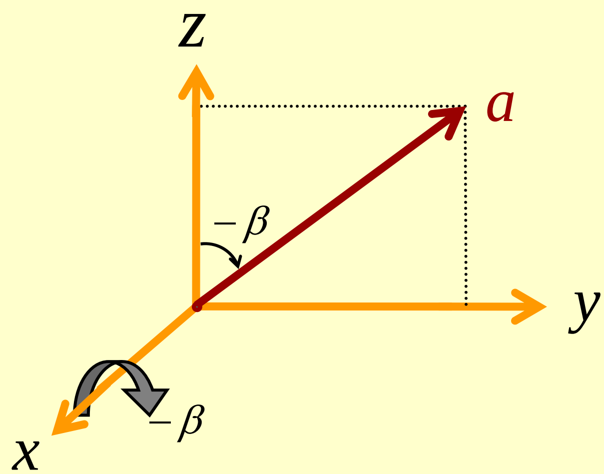
Then *rotate* about x by $-\beta$: $R_x(-\beta)$



Rotate again in the (y-z)-plane

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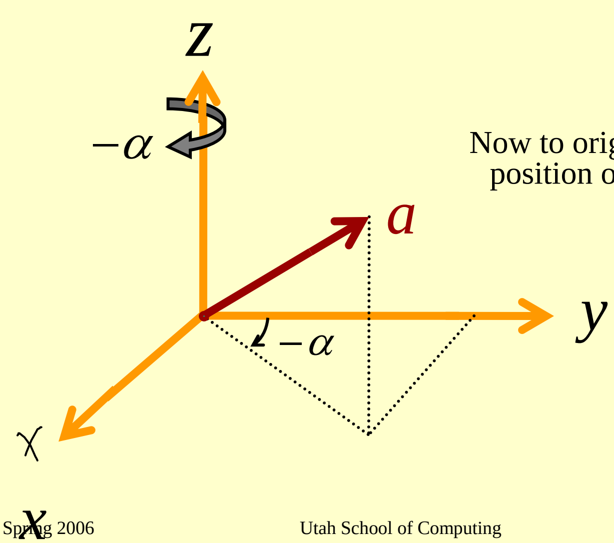
Now, $+\theta$ *Rotation* about z by $-\alpha$: $R_z(-\alpha)$



Now to original position of *a*

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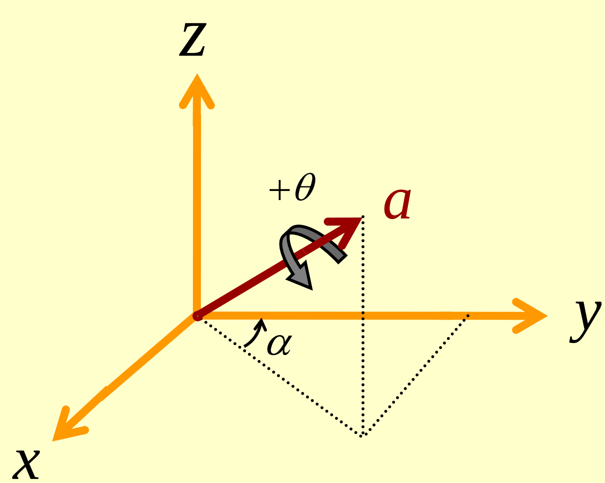
We Effected $+\theta$ *rotation* about Arbitrary *axis a*: $R_a(+\theta)$



We Effected $+\theta$ *rotation* about Arbitrary *axis a*: $R_a(+\theta)$

$$R_a(+\theta) = R_z(-\alpha)\ R_x(-\beta)\ R_z(+\theta)$$

$$\times\ R_x(+\beta)\ R_z(+\alpha)$$

Pts

Rotation about Arbitrary Axis

- Rotation about *a-axis* effected by (nonunique) composition of 5 elementary *rotations*
- We show arbitrary *rotation* as succession of 5 *rotations* about *principal axes*

In matrix terms, $R_a(+\theta) =$

$$R_a(+\theta) = \underbrace{\begin{bmatrix} \cos(-\alpha) & -\sin(-\alpha) & 0 & 0 \\ \sin(-\alpha) & \cos(-\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{R_Z(-\alpha)} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-\beta) & -\sin(-\beta) & 0 \\ 0 & \sin(-\beta) & \cos(-\beta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{R_X(-\beta)}$$

$$\times \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{R_Z(+\theta)} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta & 0 \\ 0 & \sin\beta & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{R_X(+\beta)} \underbrace{\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{R_Z(+\alpha)}$$

x y z 1

Similarly, $R_a^{-1}(+\theta) = R_a(-\theta)$, so

$$R_a(-\theta) = \begin{bmatrix} \cos(-\alpha) & -\sin(-\alpha) & 0 & 0 \\ \sin(-\alpha) & \cos(-\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-\beta) & -\sin(-\beta) & 0 \\ 0 & \sin(-\beta) & \cos(-\beta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\times \underbrace{\begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{R_z(-\theta)} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta & 0 \\ 0 & \sin\beta & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Recall, $[AB]^t = B^t A^t$

Consequently, for $A = R^t M R$,

$A^t = R^t M^t R$ because,

$$\left[R^t (M R)\right]^t = \left[M \cdot R\right]^t \left[R^t\right]^t$$

$$= R^t M^t R$$

It follows directly that,

$$\left[R^t S^t M S R\right]^t = R^t S^t M^t S R$$

$$\therefore \quad R_a^{-1}(+\theta) = R_a^t(\theta)$$

$$R_a(-\theta) = \begin{bmatrix} \cos(-\alpha) & -\sin(-\alpha) & 0 & 0 \\ \sin(-\alpha) & \cos(-\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-\beta) & -\sin(-\beta) & 0 \\ 0 & \sin(-\beta) & \cos(-\beta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\times \underbrace{\begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{R_z^t(\theta)} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta & 0 \\ 0 & \sin\beta & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Similarly, $R_a^{-1}(+\theta) = R_a(-\theta)$, so

$$R_a(-\theta) = \begin{bmatrix} \cos(-\alpha) & -\sin(-\alpha) & 0 & 0 \\ \sin(-\alpha) & \cos(-\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-\beta) & -\sin(-\beta) & 0 \\ 0 & \sin(-\beta) & \cos(-\beta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\times \underbrace{\begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{R_z(-\theta)} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta & 0 \\ 0 & \sin\beta & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In matrix terms, $R_a(+\theta) =$

$$R_a(+\theta) = \overbrace{\begin{bmatrix} \cos(-\alpha) & -\sin(-\alpha) & 0 & 0 \\ \sin(-\alpha) & \cos(-\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}^{R_z(-\alpha)} \overbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-\beta) & -\sin(-\beta) & 0 \\ 0 & \sin(-\beta) & \cos(-\beta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}^{R_x(-\beta)}$$

$$\times \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{R_z(+\theta)} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta & 0 \\ 0 & \sin\beta & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{R_x(+\beta)} \underbrace{\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{R_z(+\alpha)}$$

Constructively, we have shown,

$$R_a^{-1}(\theta) = R_a^{t}(\theta)$$

This will be useful later

3D *Translation in x*

$$T_x(d_x) = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x + d_x \\ y \\ z \\ 1 \end{bmatrix}$$

3D *Translation in y*

$$T_y(d_y) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y+d_y \\ z \\ 1 \end{bmatrix}$$

3D *Translation in z*

$$T_z(d_z) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z+d_z \\ 1 \end{bmatrix}$$

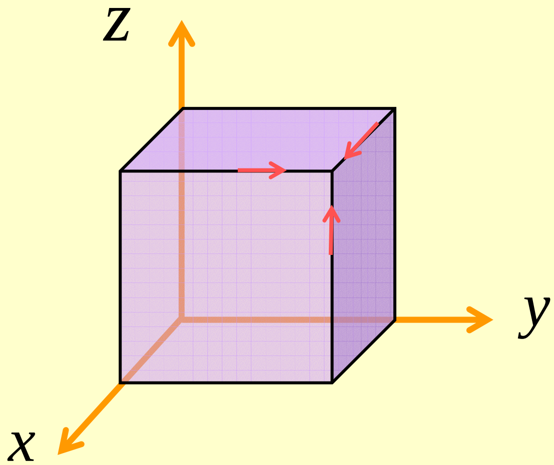
3D *Shear in x -direction*

$$Sh_x(a) = \begin{bmatrix} 1 & a & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x+ay \\ y \\ z \\ 1 \end{bmatrix}$$

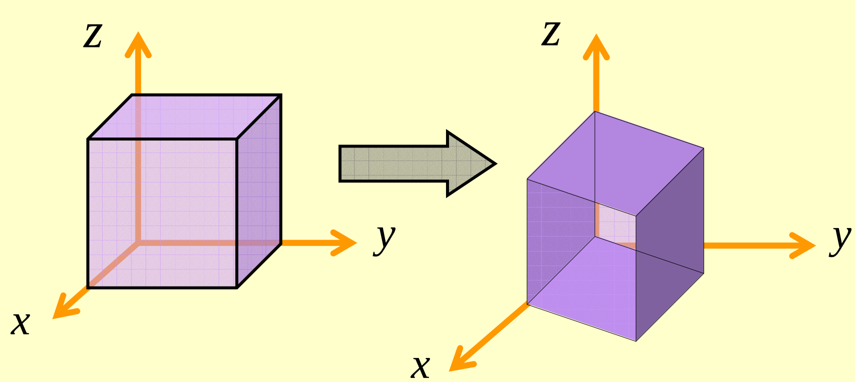
3D *Shear in x -direction*

$$Sh_x(b) = \begin{bmatrix} 1 & 0 & b & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x+bz \\ y \\ z \\ 1 \end{bmatrix}$$

3D *Shears*: Clamp a *Principal Plane*, shear in other 2 *DoFs*



$$Sh_x(a) = \begin{bmatrix} 1 & a & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x+ay \\ y \\ z \\ 1 \end{bmatrix}$$



3D *Shear in x -direction*

$$Sh_x(b) = \begin{bmatrix} 1 & 0 & b & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x+bz \\ y \\ z \\ 1 \end{bmatrix}$$

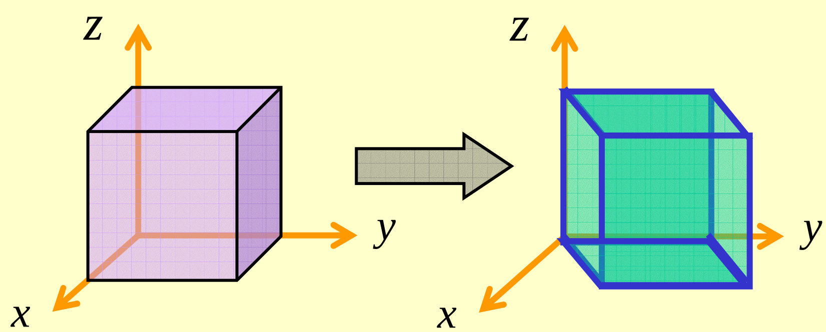
3D *Shear in y -direction*

$$Sh_y(c) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & c & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y+cz \\ z \\ 1 \end{bmatrix}$$

3D *Shear in y -direction*

$$Sh_y(d) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ d & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ dx+y \\ z \\ 1 \end{bmatrix}$$

$$Sh_y(d) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ d & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ dx+y \\ z \\ 1 \end{bmatrix}$$



3D *Shear in y -direction*

$$Sh_y(c) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & c & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y+cz \\ z \\ 1 \end{bmatrix}$$

3D *Shear in z-direction*

$$Sh_z(e) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ e & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ ex+z \\ 1 \end{bmatrix}$$

3D *Shear in z*

$$Sh_z(e) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ e & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ ex+z \\ 1 \end{bmatrix}$$

3D *Shear in z*

$$Sh_z(f) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & f & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ fy+z \\ 1 \end{bmatrix}$$

What About Elementary *Inverses*?

- *Scale*
- *Shear*
- *Rotation*
- *Translation*

Scale Inverse

$$\begin{bmatrix} \lambda & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\lambda & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \lambda & 0 \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1/\lambda & 0 \\ 0 & 1 \end{bmatrix}$$

Shear Inverse

$$\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -a \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -b & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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Shear Inverse

$$\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -a \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ -b & 1 \end{bmatrix}$$

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Rotation Inverse

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Rotation Inverse

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} =$$

$$= \begin{bmatrix} (\cos^2\theta + \sin^2\theta) & (\cos\theta\sin\theta - \cos\theta\sin\theta) \\ (\cos\theta\sin\theta - \cos\theta\sin\theta) & (\sin^2\theta + \cos^2\theta) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Rotation Inverse

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Translation Inverse

$$\begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -d_x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & (-d_x+d_x) \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Translation Inverse

$$\begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & -d_x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Double Shear in *x* and *y*

$$\underbrace{\begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix}}_{Sh_y(b)} \underbrace{\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}}_{Sh_x(a)} = \begin{bmatrix} 1 & a \\ b & (1+ab) \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}}_{Sh_x(a)} \underbrace{\begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix}}_{Sh_y(b)} = \begin{bmatrix} (1+ab) & a \\ b & 1 \end{bmatrix}$$

Ex: $a=0.5$ and $b=1.0$

$$\overbrace{\begin{bmatrix}1 & 0\\ b & 1\end{bmatrix}}^{Sh_y(b)}\overbrace{\begin{bmatrix}1 & a\\ 0 & 1\end{bmatrix}}^{Sh_x(a)}=\begin{bmatrix}1 & a\\ b & (1+ab)\end{bmatrix}$$

$$\underbrace{\begin{bmatrix}1 & a\\ 0 & 1\end{bmatrix}}_{Sh_x(a)}\underbrace{\begin{bmatrix}1 & 0\\ b & 1\end{bmatrix}}_{Sh_y(b)}=\begin{bmatrix}(1+ab) & a\\ b & 1\end{bmatrix}$$

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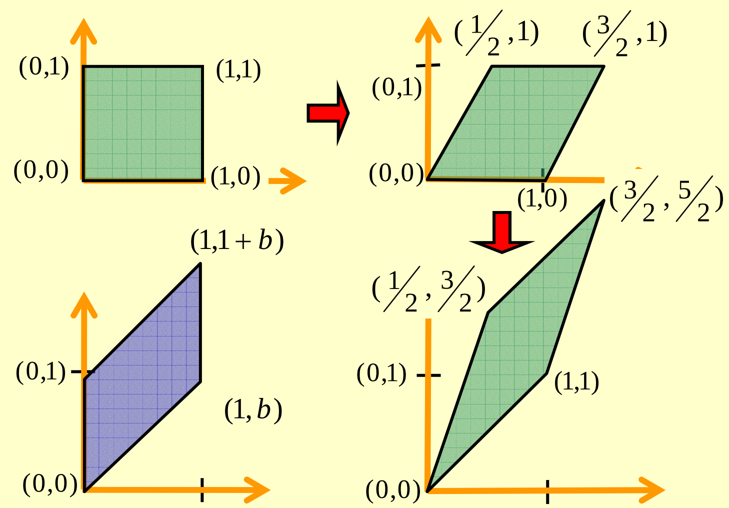
Shear in *x* then, *y*:

$$\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}1 & \frac{1}{2}\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & \frac{1}{2}\\ 1 & \frac{3}{2}\end{bmatrix}$$



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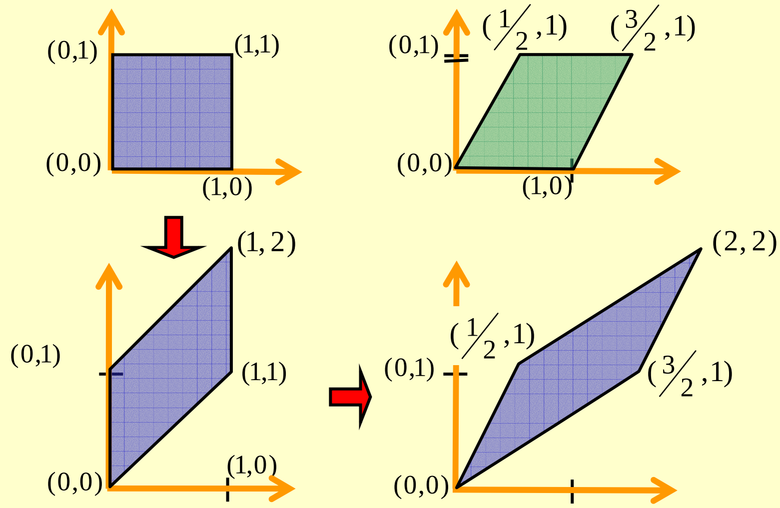
Shear in *y* then, *x* :



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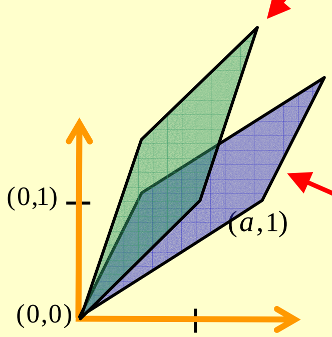
Shears in x and y do not Commute

Sh in x $_{(a=½)}$, then *Sh y* $_{(b=1)}$



Sh in y $_{(b=1)}$, then *Sh in x* $_{(a=½)}$

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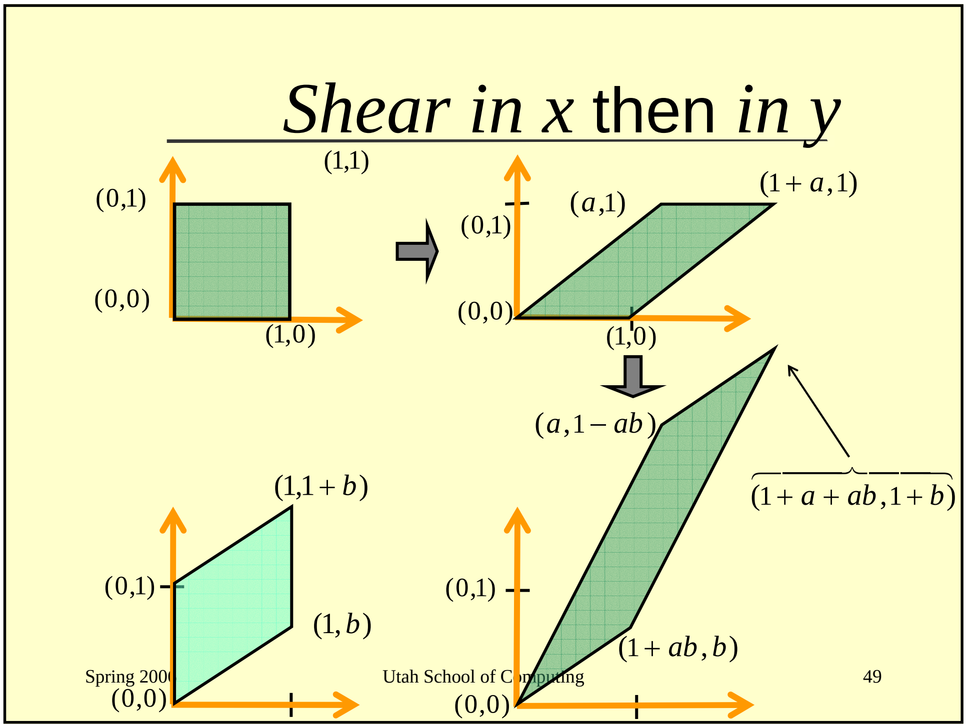
Shear in x then in y
(1,1)
(0,1)
(0,0)
(1,0)
(a,1)
(1+a,1)
(0,1)
(0,0)
(1,0)
(a,1−ab)
(1+a+ab,1+b)
(1,1+b)
(0,1)
(1,b)
(0,1)
(1+ab,b)
(0,0)
(0,0)
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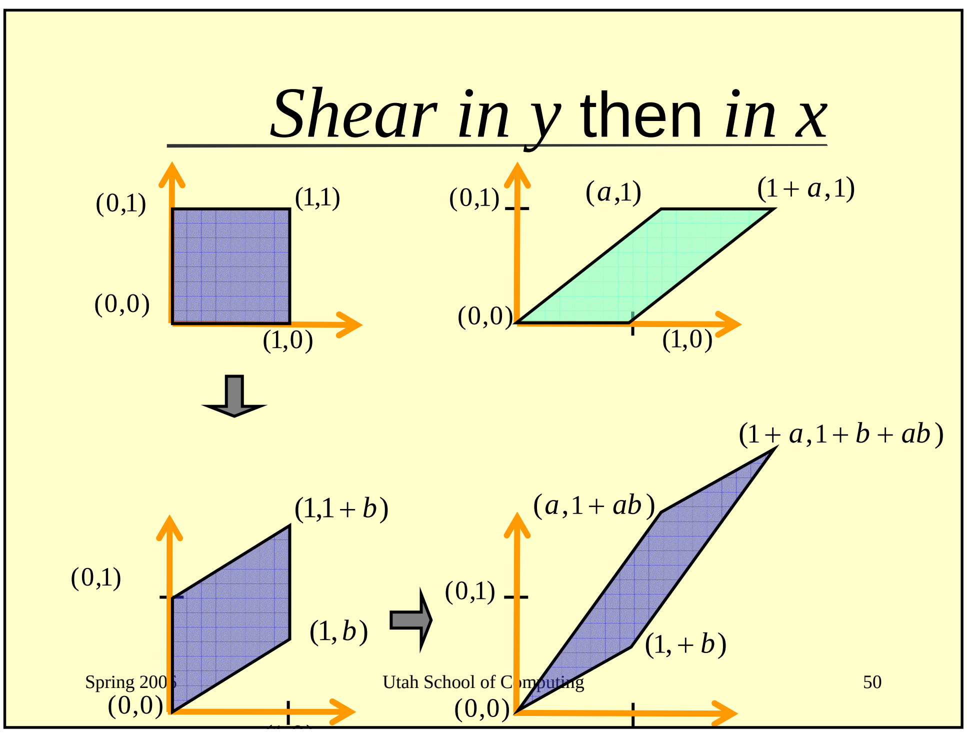
Shear in y then in x
(0,1)
(1,1)
(0,0)
(1,0)
(0,1)
(a,1)
(1+a,1)
(0,0)
(1,0)
(1+a,1+b+ab)
(1,1+b)
(a,1+ab)
(0,1)
(1,b)
(0,1)
(1,+b)
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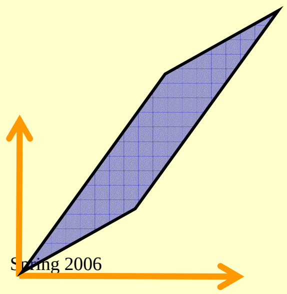
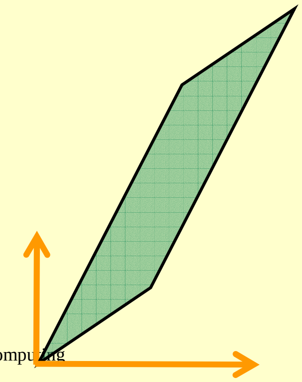
(0,0)
(0,0)

Results Are Different

y then *x*:



x then *y*:



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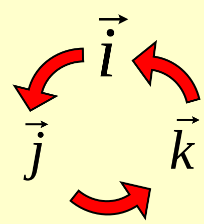
Want the *RHR* to Work

$$\vec{i} \times \vec{j} = \vec{k}$$

$$\vec{j} \times \vec{k} = \vec{i}$$

$$\vec{k} \times \vec{i} = \vec{j}$$



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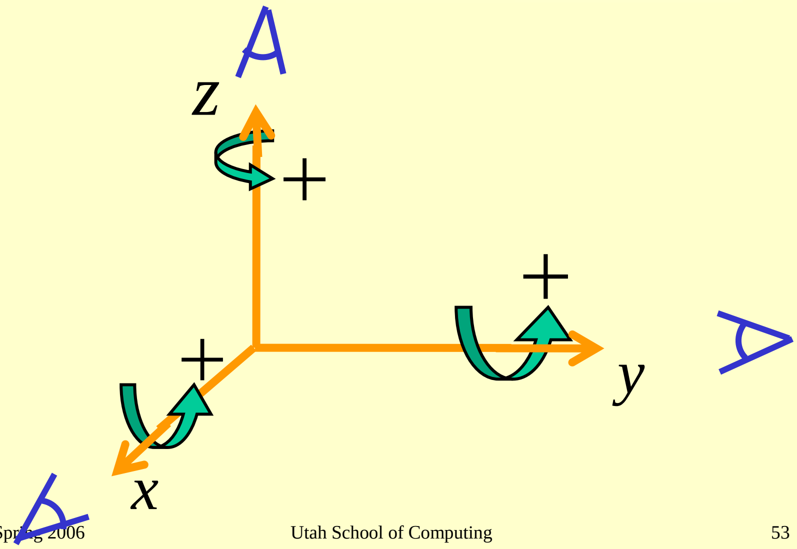
3D Positive *Rotations*



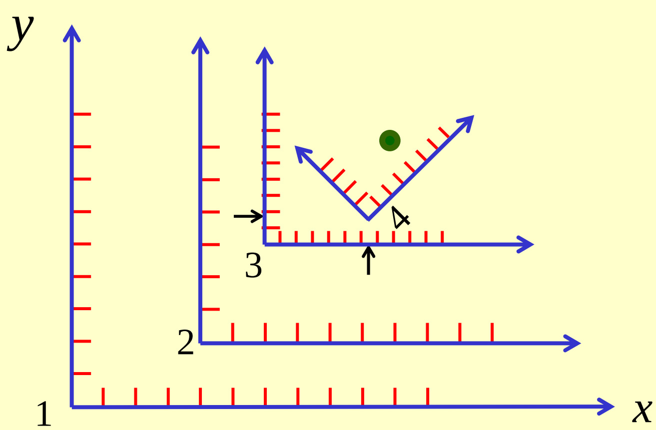
Stop here

- Pen attention

Transf's as Change in Coordinate Sys

- Useful in many situations
- Use most natural coordination system locally
- Tie things together in a global system

Example



Example

$M_{i \leftarrow j}$ is the transformation that takes a point $p^{(j)}$ in coordinate system *j* and converts it to a point $p^{(i)}$ in coordinate system *i*

Example

- $p^{(i)} = M_{i \leftarrow j} \; p^{(j)}$
- $p^{(j)} = M_{j \leftarrow k} \; p^{(k)}$
- $M_{i \leftarrow k} = M_{i \leftarrow j} M_{j \leftarrow k}$

Example

- $M_{1 \leftarrow 2} = T(4,2)$
- $M_{2 \leftarrow 3} = S(2,2) \bullet T(2,3)$
- $M_{3 \leftarrow 4} = R(-45^\circ) \bullet T(6.7,1.8)$

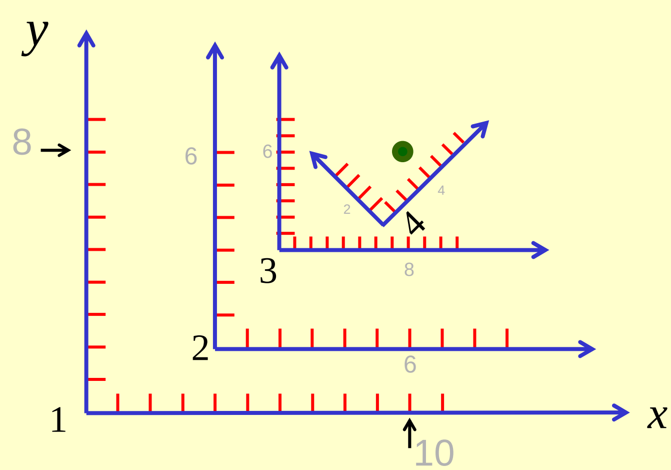
Recall the Following

$$(AB)^{-1} = B^{-1} A^{-1}$$

Since $M_{i \leftarrow j}^{-1} = M_{j \leftarrow i}$

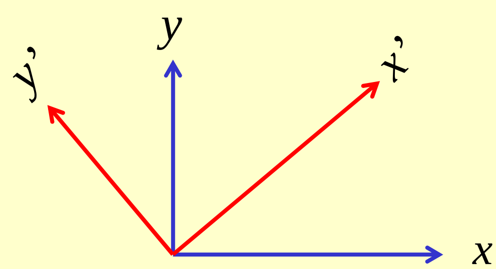
- $M_{2 \leftarrow 1} = T(-4, -2)$
- $M_{3 \leftarrow 2} = T(-2, -3) \bullet S(\frac{1}{2}, \frac{1}{2})$
- $M_{4 \leftarrow 3} = T(-6.7, -1.8) \bullet R(+45^{\circ})$

Example



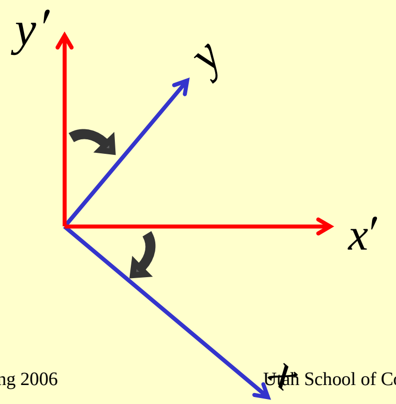
Change of Coordinate System

- Describe the *old* coordinate system in terms of the *new* one.



Change of Coordinate System

Move to the *new* coordinate system and describe the one *old*.



Old is a negative rotation of the *new*.