Cost of Substitution

```
(interp {with {x 1}
          {with {y 2}
            {+ 100 {+ 99 {+ 98 ... {+ y x}}}}}})
```

⇒

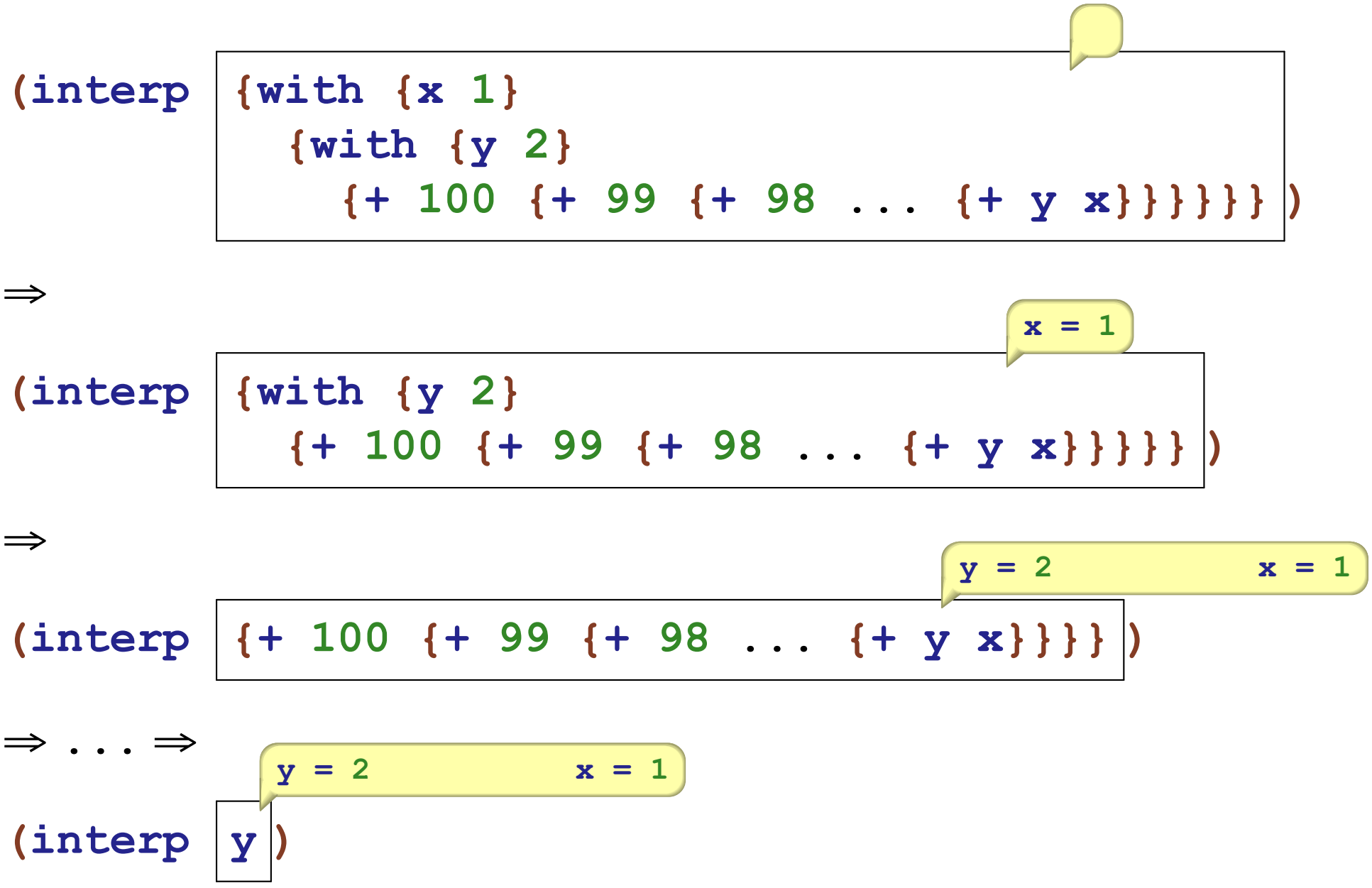
```
(interp {with {y 2}
          {+ 100 {+ 99 {+ 98 ... {+ y 1}}}}})
```

⇒

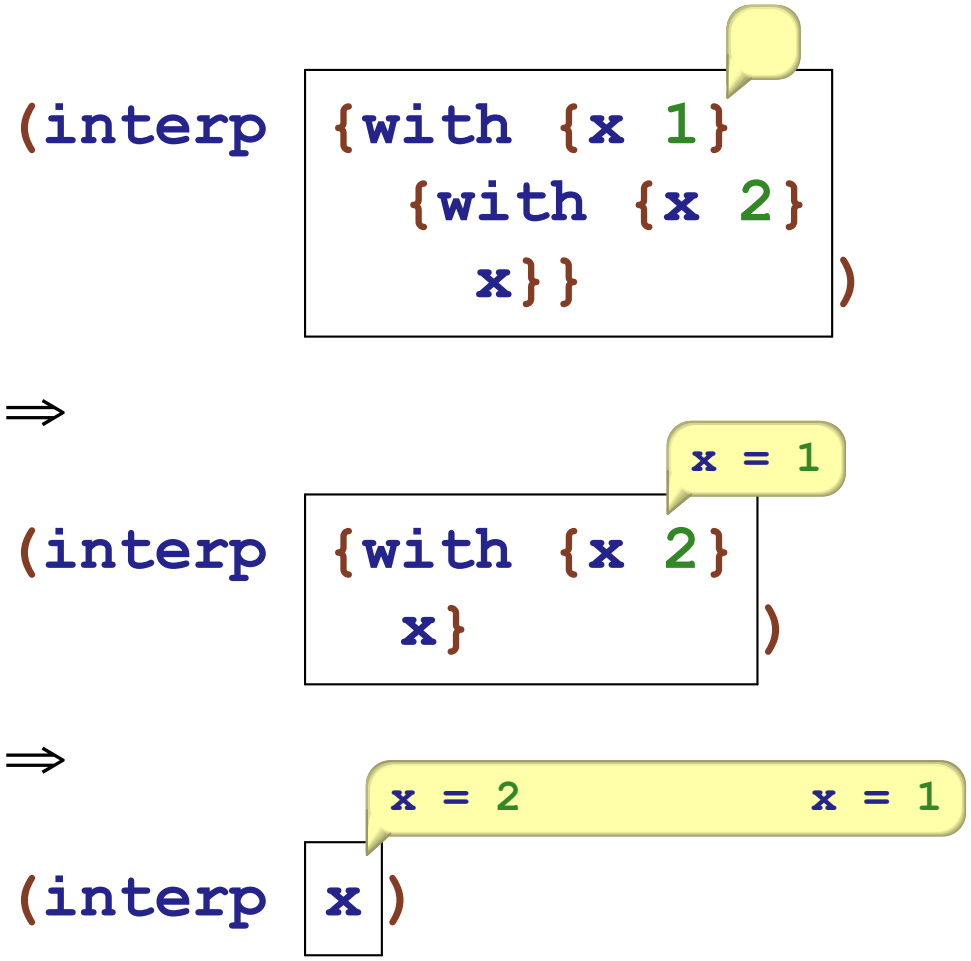
```
(interp {+ 100 {+ 99 {+ 98 ... {+ 2 1}}}})
```

With **n** variables, evaluation will take $O(\mathbf{n}^2)$ time!

Deferring Substitution



Deferring Substitution with the Same Identifier



Always add to start, then always check from start

Representing Deferred Substitution

Change

```
; interp : WAE -> num
```

to

```
; interp : WAE DefrdSub -> num
```

```
(define-type DefrdSub
  [mtSub]
  [aSub (name symbol?)
        (value number?)
        (rest DefrdSub?)])
```

Interp with DefrdSub

```
(interp {with {x 1}
          {with {y 2}
            {+ 100 {+ 99 {+ 98 ... {+ y x}}}}}}
        (mtSub))
```

```
⇒ (interp {with {y 2}
            {+ 100 {+ 99 {+ 98 ... {+ y x}}}}}
          (aSub 'x 1 (mtSub)))
```

```
⇒ (interp {+ 100 {+ 99 {+ 98 ... {+ y x}}}}
          (aSub 'y 2 (aSub 'x 1 (mtSub))))
```

```
⇒ ...
```

```
⇒ (interp y (aSub 'y 2 (aSub 'x 1 (mtSub))))
```

WAE Interpreter with Deferred Substitutions

```
; interp : WAE DefrdSub -> num
(define (interp a-wae ds)
  (type-case WAE a-wae
    [num (n) n]
    [add (l r) (+ (interp l ds) (interp r ds))]
    [sub (l r) (- (interp l ds) (interp r ds))]
    [with (bound-id named-expr body-expr)
      ...]
    [id (name) ...]))
```

WAE Interpreter with Deferred Substitutions

```
; interp : WAE DefrdSub -> num
(define (interp a-wae ds)
  (type-case WAE a-wae
    [num (n) n]
    [add (l r) (+ (interp l ds) (interp r ds))]
    [sub (l r) (- (interp l ds) (interp r ds))]
    [with (bound-id named-expr body-expr)
      ...]
    [id (name) (lookup name ds)]))
```

WAE Interpreter with Deferred Substitutions

```
; lookup : symbol DefrdSub -> num
(define (lookup name ds)
  (type-case DefrdSub ds
    [mtSub () (error 'lookup "free variable")]
    [aSub (sub-name num rest-ds)
          (if (symbol=? sub-name name)
              num
              (lookup name rest-ds))]))
```

WAE Interpreter with Deferred Substitutions

```
; interp : WAE DefrdSub -> num
(define (interp a-wae ds)
  (type-case WAE a-wae
    [num (n) n]
    [add (l r) (+ (interp l ds) (interp r ds))]
    [sub (l r) (- (interp l ds) (interp r ds))]
    [with (bound-id named-expr body-expr)
      ...]
    [id (name) (lookup name ds)]))
```

WAE Interpreter with Deferred Substitutions

```
; interp : WAE DefrdSub -> num
(define (interp a-wae ds)
  (type-case WAE a-wae
    [num (n) n]
    [add (l r) (+ (interp l ds) (interp r ds))]
    [sub (l r) (- (interp l ds) (interp r ds))]
    [with (bound-id named-expr body-expr)
      ... (interp named-expr ds) ...]
    [id (name) (lookup name ds)]))
```

WAE Interpreter with Deferred Substitutions

```
; interp : WAE DefrdSub -> num
(define (interp a-wae ds)
  (type-case WAE a-wae
    [num (n) n]
    [add (l r) (+ (interp l ds) (interp r ds))]
    [sub (l r) (- (interp l ds) (interp r ds))]
    [with (bound-id named-expr body-expr)
      ...
      (aSub bound-id (interp named-expr ds) ds)
      ...]
    [id (name) (lookup name ds)]))
```

WAE Interpreter with Deferred Substitutions

```
; interp : WAE DefrdSub -> num
(define (interp a-wae ds)
  (type-case WAE a-wae
    [num (n) n]
    [add (l r) (+ (interp l ds) (interp r ds))]
    [sub (l r) (- (interp l ds) (interp r ds))]
    [with (bound-id named-expr body-expr)
      (interp
       body-expr
        (aSub bound-id (interp named-expr ds) ds))]
    [id (name) (lookup name ds)]))
```

Function Calls

```
{deffun {f x} {+ 1 x}}
```

```
(interp {with {y 2}
          {f 10}})
```

⇒

```
(interp {f 10})
```

y = 2

⇒

```
(interp {+ 1 x})
```

...

Function Calls

```
{deffun {f x} {+ 1 x}}

(interp {with {y 2}
          {f 10}})
```

⇒

```
(interp {f 10})
```

y = 2

⇒

```
(interp {+ 1 x})
```

x = 10

Interpreting function body starts with only one substitution

F1WAE Interpreter with Deferred Substitutions

```
; interp : F1WAE list-of-FunDef DefrdSub -> num
(define (interp a-f1wae fundefs ds)
  (type-case F1WAE a-f1wae
    ...
    [app (name arg-expr)
         ...]))
```

F1WAE Interpreter with Deferred Substitutions

```
; interp : F1WAE list-of-FunDef DefrdSub -> num
(define (interp a-f1wae fundefs ds)
  (type-case F1WAE a-f1wae
    ...
    [app (name arg-expr)
         (local [(define a-fundef
                   (lookup-fundef name fundefs))]
           (interp (fundef-body a-fundef)
                   fundefs
                   ...
                   (interp arg-expr fundefs ds)
                   ...))]))
```

F1WAE Interpreter with Deferred Substitutions

```
; interp : F1WAE list-of-FunDef DefrdSub -> num
(define (interp a-f1wae fundefs ds)
  (type-case F1WAE a-f1wae
    ...
    [app (name arg-expr)
         (local [(define a-fundef
                   (lookup-fundef name fundefs))]
           (interp (fundef-body a-fundef)
                   fundefs
                   (aSub (fundef-arg-name a-fundef)
                         (interp arg-expr fundefs ds)
                         (mtSub))))]))
```