

Factorial

```
(local [(define fac
          (lambda (n)
            (if (zero? n)
                1
                (* n (fac (- n 1))))))]
  (fac 10))
```

`local` binds both in the body expression and in the binding expression

Factorial

```
(let ([fac  
      (lambda (n)  
        (if (zero? n)  
            1  
            (* n (fac (- n 1))))))] )  
  (fac 10))
```

Doesn't work: `let` is like `with`

Still, at the point that we call `fac`, obviously we have a binding for `fac`...

... so pass it as an argument!

Factorial

```
(let ([facX  
      (lambda (facX n)  
        (if (zero? n)  
            1  
            (* n (facX facX (- n 1))))))] )  
(facX facX 10))
```

Wrap this to get `fac` back...

Factorial

```
(let ([fac
      (lambda (n)
        (let ([facX
              (lambda (facX n)
                (if (zero? n)
                    1
                    (* n (facX facX (- n 1))))))]
          (facX facX n)))]])
  (fac 10))
```

Try this in the **HtDP Intermediate with Lambda** language, click **Step**

But the language we implement has only single-argument functions...

From Multi-Argument to Single-Argument

```
(define f
  (lambda (x y z)
    (list z y x)))

(f 1 2 3)
```

⇒

```
(define f
  (lambda (x)
    (lambda (y)
      (lambda (z)
        (list z y x))))))

(((f 1) 2) 3)
```

Factorial

```
(let ([fac
      (lambda (n)
        (let ([facX
              (lambda (facX)
                (lambda (n)
                  (if (zero? n)
                      1
                      (* n ((facX facX) (- n 1))))))]
          ((facX facX) n))))])
  (fac 10))
```

Simplify: $(\text{lambda } (n) (\text{let } ([f \dots]) ((f f) n)))$
 $\Rightarrow (\text{let } ([f \dots]) (f f)) \dots$

Factorial

```
(let ([fac
      (let ([facX
            (lambda (facX)
              (lambda (n)
                (if (zero? n)
                    1
                    (* n ((facX facX) (- n 1))))))]
            (facX facX))])
  (fac 10))
```

Factorial

```
(let ([fac
      (let ([facX
            (lambda (facX)
              ; Almost looks like original fac:
              (lambda (n)
                (if (zero? n)
                    1
                    (* n ((facX facX) (- n 1))))))]
          (facX facX))])
  (fac 10))
```

More like original: introduce a local binding for
(facX facX)...

Factorial

```
(let ([fac
      (let ([facX
            (lambda (facX)
              (let ([fac (facX facX)])
                ; Exactly like original fac:
                (lambda (n)
                  (if (zero? n)
                      1
                      (* n (fac (- n 1)))))))]))
      (facX facX))])
(fac 10))
```

Oops! – this is an infinite loop

We used to evaluate `(facX facX)` only when `n` is non-zero

Delay `(facX facX)`...

Factorial

```
(let ([fac
      (let ([facX
            (lambda (facX)
              (let ([fac (lambda (x)
                          ((facX facX) x))])
                ; Exactly like original fac:
                (lambda (n)
                  (if (zero? n)
                      1
                      (* n (fac (- n 1)))))))]
          (facX facX)))]
  (fac 10))
```

Now, what about `fib`, `sum`, etc.?

Abstract over the `fac`-specific part...

Make-Recursive and Factorial

```
(define (mk-rec body-proc)
  (let ([fX
        (lambda (fX)
          (let ([f (lambda (x)
                    ((fX fX) x))])
            (body-proc f)))]])
    (fX fX)))

(let ([fac (mk-rec
           (lambda (fac)
             ; Exactly like original fac:
             (lambda (n)
               (if (zero? n)
                   1
                   (* n (fac (- n 1))))))]])
  (fac 10))
```

Fibonnaci

```
(let ([fib
      (mk-rec
       (lambda (fib)
         ; Usual fib:
         (lambda (n)
           (if (or (= n 0) (= n 1))
               1
               (+ (fib (- n 1))
                  (fib (- n 2)))))))]))
(fib 5))
```

Sum

```
(let ([sum
      (mk-rec
       (lambda (sum)
         ; Usual sum:
         (lambda (l)
           (if (empty? l)
               0
               (+ (first l)
                  (sum (rest l))))))]
      (sum '(1 2 3 4))))
```

Implementing Recursion

```
{rec {fac {fun {n}
      {ifzero n
        1
        {* n
         {fac {- n 1}}}}}}}}
{fac 10}}
```

could be parsed the same as

```
{with {fac
      {mk-rec
       {fun {fac}
        {fun {n}
         {ifzero n
           1
           {* n
            {fac {- n 1}}}}}}}}}}
{fac 10}}
```

Implementing Recursion

```
{rec {<id>1 <FAE>1}  
      <FAE>2}
```

could be parsed the same as

```
{with {<id>1 {mk-rec {fun {<id>1} <FAE>1}}}}  
      <FAE>2}
```

which is really

```
{{fun {<id>1} <FAE>2}  
  {mk-rec {fun {<id>1} <FAE>1}}}}
```