Figure 2.10 An example of logic networks.
Figure 2.24 NOR-gate realization of the function in Example 2.4.
PROBLEMS

Answers to problems marked by an asterisk are given at the back of the book.

2.1 Use algebraic manipulation to prove that \( x + yz = (x + y) \cdot (x + z) \). Note that this is the distributive rule, as stated in identity 12b in section 2.5.

2.2 Use algebraic manipulation to prove that \((x + y) \cdot (x + \overline{y}) = x\).

2.3 Use algebraic manipulation to prove that \(xy + yz + \overline{x}z = xy + \overline{x}z\). Note that this is the consensus property 17a in section 2.5.

2.4 Use the Venn diagram to prove the identity in problem 2.1.

2.5 Use the Venn diagram to prove DeMorgan’s theorem, as given in expressions 15a and 15b in section 2.5.

2.6 Use the Venn diagram to prove that

\[
(x_1 + x_2 + x_3) \cdot (x_1 + x_2 + \overline{x_3}) = x_1 + x_2
\]

2.7 Determine whether or not the following expressions are valid, i.e., whether the left- and right-hand sides represent the same function.
(a) \( \overline{x_1}x_3 + x_1x_2\overline{x_3} + \overline{x_1}x_2 + x_1\overline{x_2} = \overline{x_2}x_3 + x_1\overline{x_3} + x_2\overline{x_3} + \overline{x_1}x_2x_3 \)
(b) \( x_1\overline{x_3} + x_2x_3 + \overline{x_2}\overline{x_3} = (x_1 + \overline{x_2} + x_3)(x_1 + x_2 + \overline{x_3})(\overline{x_1} + x_2 + \overline{x_3}) \)
(c) \( (x_1 + x_2)(\overline{x_1} + x_2 + \overline{x_3})(\overline{x_1} + x_2) = (x_1 + x_2)(x_2 + x_3)(\overline{x_1} + \overline{x_3}) \)

2.8 Draw a timing diagram for the circuit in Figure 2.19a. Show the waveforms that can be observed on all wires in the circuit.

2.9 Repeat problem 2.8 for the circuit in Figure 2.19b.

2.10 Use algebraic manipulation to show that for three input variables \( x_1, x_2, \) and \( x_3 \)

\[
\sum m(1, 2, 3, 4, 5, 6, 7) = x_1 + x_2 + x_3
\]

2.11 Use algebraic manipulation to show that for three input variables \( x_1, x_2, \) and \( x_3 \)

\[
\Pi M(0, 1, 2, 3, 4, 5, 6) = x_1x_2x_3
\]

2.12 Use algebraic manipulation to find the minimum sum-of-products expression for the function \( f = x_1x_3 + x_1\overline{x_2} + \overline{x_1}x_2x_3 + \overline{x_1}x_2\overline{x_3} \).

2.13 Use algebraic manipulation to find the minimum sum-of-products expression for the function \( f = x_1\overline{x_2}\overline{x_3} + x_1x_2x_4 + x_1\overline{x_2}x_3\overline{x_4} \).

2.14 Use algebraic manipulation to find the minimum product-of-sums expression for the function \( f = (x_1 + x_3 + \overline{x_4}) \cdot (x_1 + \overline{x_2} + x_3) \cdot (x_1 + \overline{x_2} + \overline{x_3} + x_4) \).

2.15 Use algebraic manipulation to find the minimum product-of-sums expression for the function \( f = (x_1 + x_2 + x_3) \cdot (x_1 + \overline{x_2} + x_3) \cdot (\overline{x_1} + \overline{x_2} + x_3) \cdot (x_1 + x_2 + \overline{x_3}) \).
2.21 Design the simplest sum-of-products circuit that implements the function \( f(x_1, x_2, x_3) = \sum m(1, 3, 4, 6, 7) \).

2.22 Design the simplest product-of-sums circuit that implements the function \( f(x_1, x_2, x_3) = \Pi M(0, 2, 5) \).

*2.23 Design the simplest product-of-sums expression for the function \( f(x_1, x_2, x_3) = \Pi M(0, 1, 5, 7) \).

2.24 Derive the simplest sum-of-products expression for the function \( f(x_1, x_2, x_3, x_4) = x_1\bar{x}_3\bar{x}_4 + x_2\bar{x}_3x_4 + x_1\bar{x}_2x_3 \).

2.25 Derive the simplest sum-of-products expression for the function \( f(x_1, x_2, x_3, x_4, x_5) = \bar{x}_1\bar{x}_3\bar{x}_5 + \bar{x}_1\bar{x}_3x_4 + \bar{x}_1x_4x_5 + x_1\bar{x}_2x_3x_5 \). (Hint: Use the consensus property 17a.)

2.26 Derive the simplest product-of-sums expression for the function \( f(x_1, x_2, x_3, x_4) = (\bar{x}_1 + x_3 + \bar{x}_4)(\bar{x}_2 + \bar{x}_3 + x_4)(x_1 + \bar{x}_2 + \bar{x}_3) \). (Hint: Use the consensus property 17b.)

2.27 Derive the simplest product-of-sums expression for the function \( f(x_1, x_2, x_3, x_4, x_5) = (\bar{x}_2 + x_3 + x_4)(x_1 + \bar{x}_3 + x_5)(x_1 + x_2 + x_5)(x_1 + \bar{x}_4 + \bar{x}_5) \). (Hint: Use the consensus property 17b.)

*2.28 Design the simplest circuit that has three inputs, \( x_1, x_2, \) and \( x_3 \), which produces an output value of 1 whenever two or more of the input variables have the value 1; otherwise, the output has to be 0.

2.29 Design the simplest circuit that has three inputs, \( x_1, x_2, \) and \( x_3 \), which produces an output value of 1 whenever exactly one or two of the input variables have the value 1; otherwise, the output has to be 0.

2.30 Design the simplest circuit that has four inputs, \( x_1, x_2, x_3, \) and \( x_4 \), which produces an output value of 1 whenever three or more of the input variables have the value 1; otherwise, the output has to be 0.

2.31 For the timing diagram in Figure P2.3, synthesize the function \( f(x_1, x_2, x_3) \) in the simplest sum-of-products form.

![Figure P2.3](image-url)  
A timing diagram representing a logic function.
**2.32** For the timing diagram in Figure P2.3, synthesize the function \( f(x_1, x_2, x_3) \) in the simplest product-of-sums form.

**2.33** For the timing diagram in Figure P2.4, synthesize the function \( f(x_1, x_2, x_3) \) in the simplest sum-of-products form.

**2.34** For the timing diagram in Figure P2.4, synthesize the function \( f(x_1, x_2, x_3) \) in the simplest product-of-sums form.

**2.35** Design a circuit with output \( f \) and inputs \( x_1, x_0, y_1, \) and \( y_0 \). Let \( X = x_1 \cdot x_0 \) be a number, where the four possible values of \( X \), namely, 00, 01, 10, and 11, represent the four numbers 0, 1, 2, and 3, respectively. (We discuss representation of numbers in Chapter 5.) Similarly, let \( Y = y_1 \cdot y_0 \) represent another number with the same four possible values. The output \( f \) should be 1 if the numbers represented by \( X \) and \( Y \) are equal. Otherwise, \( f \) should be 0.
   (a) Show the truth table for \( f \).
   (b) Synthesize the simplest possible product-of-sums expression for \( f \).

**2.36** Repeat problem 2.35 for the case where \( f \) should be 1 only if \( X \geq Y \).
   (a) Show the truth table for \( f \).
   (b) Show the canonical sum-of-products expression for \( f \).
   (c) Show the simplest possible sum-of-products expression for \( f \).

**2.37** Implement the function in Figure 2.26 using only NAND gates.

**2.38** Implement the function in Figure 2.26 using only NOR gates.

**2.39** Implement the circuit in Figure 2.33 using NAND and NOR gates.

**2.40** Design the simplest circuit that implements the function \( f(x_1, x_2, x_3) = \sum m(3, 4, 6, 7) \) using NAND gates.

**2.41** Design the simplest circuit that implements the function \( f(x_1, x_2, x_3) = \sum m(1, 3, 4, 6, 7) \) using NAND gates.

**2.42** Repeat problem 2.40 using NOR gates.

**2.43** Repeat problem 2.41 using NOR gates.
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2.44 (a) Use a schematic capture tool to draw schematics for the following functions

\[ f_1 = x_2 \bar{x}_3 \bar{x}_4 + \bar{x}_1 x_2 x_4 + \bar{x}_1 x_2 x_3 + x_1 x_2 x_3 \]
\[ f_2 = x_2 \bar{x}_4 + \bar{x}_1 x_2 + x_2 x_3 \]

(b) Use functional simulation to prove that \( f_1 = f_2 \).

2.45 (a) Use a schematic capture tool to draw schematics for the following functions

\[ f_1 = (x_1 + x_2 + \bar{x}_4) \cdot (\bar{x}_2 + x_3 + \bar{x}_4) \cdot (\bar{x}_1 + x_3 + \bar{x}_4) \cdot (\bar{x}_1 + \bar{x}_3 + \bar{x}_4) \]
\[ f_2 = (x_2 + \bar{x}_4) \cdot (x_3 + \bar{x}_4) \cdot (\bar{x}_1 + \bar{x}_4) \]

(b) Use functional simulation to prove that \( f_1 = f_2 \).

2.46 Write Verilog code to implement the circuit in Figure 2.27a using the gate level primitives.

2.47 Repeat problem 2.46 for the circuit in Figure 2.27b.

2.48 Write Verilog code to implement the function \( f(x_1, x_2, x_3) = \sum m(1, 2, 3, 4, 5, 6) \) using the gate level primitives. Ensure that the resulting circuit is as simple as possible.

2.49 Write Verilog code to implement the function \( f(x_1, x_2, x_3) = \sum m(0, 1, 3, 4, 5, 6) \) using the continuous assignment.

2.50 (a) Write Verilog code to describe the following functions

\[ f_1 = x_1 \bar{x}_3 + x_2 \bar{x}_3 + \bar{x}_3 \bar{x}_4 + x_1 x_2 + x_1 \bar{x}_4 \]
\[ f_2 = (x_1 + \bar{x}_3) \cdot (x_1 + x_2 + \bar{x}_4) \cdot (x_2 + \bar{x}_3 + \bar{x}_4) \]

(b) Use functional simulation to prove that \( f_1 = f_2 \).

2.51 Consider the following Verilog statements

\[ f_1 = (x_1 & x_3) | (\sim x_1 & \sim x_3) | (x_2 & x_4) | (\sim x_2 & \sim x_4); \]
\[ f_2 = (x_1 & x_2 & \sim x_3 & \sim x_4) | (\sim x_1 & \sim x_2 & x_3 & x_4) | \]
\[ (x_1 & \sim x_2 & \sim x_3 & x_4) | (\sim x_1 & x_2 & x_3 & \sim x_4); \]

(a) Write complete Verilog code to implement \( f_1 \) and \( f_2 \).
(b) Use functional simulation to prove that \( f_1 = \overline{f_2} \).

REFERENCES