Notes: Discrete Random Variables

CS 3130/ECE 3530: Probability and Statistics for Engineers

September 14, 2017

Random Variables:
A **random variable** is a function from a sample space to the real numbers. The mathematical notation for a random variable X on a sample space Ω looks like this:

$$X : \Omega \to \mathbb{R}$$

A random variable defines some *feature* of the sample space that is more interesting than the raw sample space outcomes.

Example: Sum of dice (see book)
Sample space: $\Omega = \{(i, j) : i, j \in \{1, \dots, 6\}\}$, Random variable: $S(i, j) = i + j$

Defining Events using Random Variables:
We can define events using random variables. The notation $\{X = a\}$ defines the event of all elements in our sample space for which the random variable X evaluates to a. In set notation

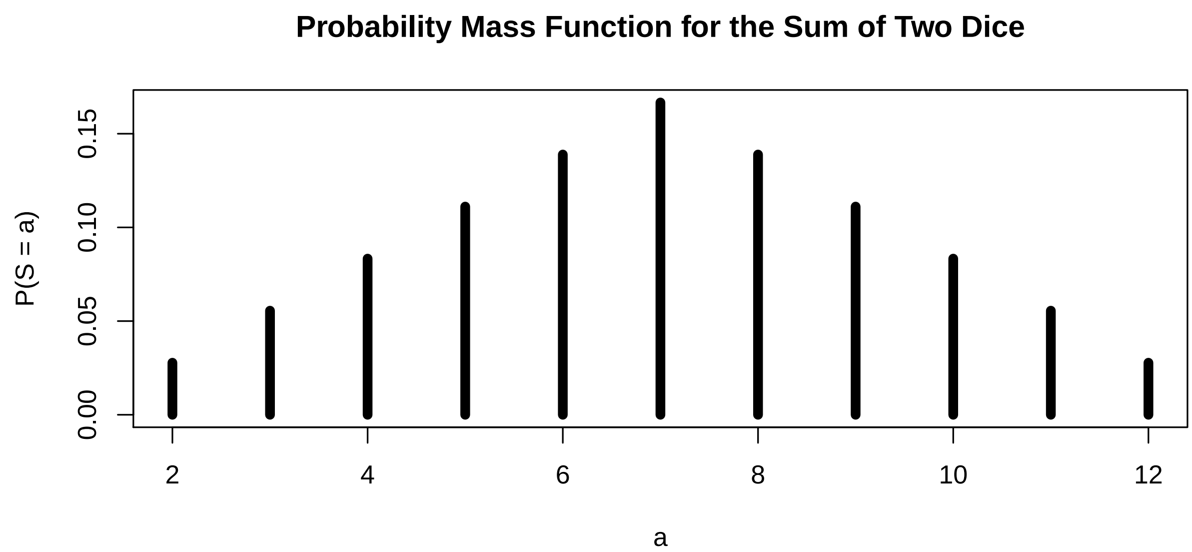
$$\{X = a\} = \{\omega \in \Omega : X(\omega) = a\}$$

The probability of this event is denoted $P(X = a)$.

Example: Sum of dice
What is $\{S = 5\}$? What is $P(S = 5)$? How about for $\{S = 7\}$?

In-class Exercise: Also for the two dice experiment, define the random variable $X(i, j) = i \times j$, i.e., X is the product of the two dice values. For $a = 3, 4, 12, 14$, what are the events $\{X = a\}$ and the probabilities $P(X = a)$?

Probability mass function:
The **probability mass function (pmf)** for a random variable X is a function $p : \mathbb{R} \to [0, 1]$ defined by $p(a) = P(X = a)$. Notice this function is zero for values of a that are not possible outcomes. Sometimes we'll also call a pmf a **probability density function (pdf)** or just a **density**.



Cumulative distribution function:
The **cumulative distribution function (cdf)** for a random variable X is a function $F : \mathbb{R} \to [0, 1]$ defined by $F(a) = P(X \leq a)$.