Homework 5: Expectation and Variance

Instructions: Submit a single R Markdown file (.Rmd) of your work on Canvas by 11:59pm on the due date. You may also submit diagrams, drawings, etc. as image files (.png, .jpg, .gif)—they must be formatted into your .Rmd document (we won't look at them separately). **Be sure to show all the work involved in deriving your answers! If you just give a final answer without explanation, you may not receive credit for that question.**

You may discuss the concepts with your classmates, but write up the answers entirely on your own. Do not look at another student's answers, and do not show your answers to anyone.

1. You are playing a game where you roll a die and win 0 jellybeans for rolling a one or two; 1 jellybean for rolling a three or four; and 2 jellybeans for rolling a five or six. Each time you play the game, you must pay 1 jellybean.

   (a) What is the expected number of jellybeans you win each round?

   (b) What is the variance of the number of jellybeans that you win?

   (c) If you like jellybeans, is this a game you want to play?

2. Using R, simulate the expected value in Problem 1.

3. You roll a die until you have seen the numbers 1, 2, and 3, not necessarily in that order. On average, how many rolls will this take?

4. Consider a continuous random variable X with the following pdf:

$$f(x) = \begin{cases} 2 - 2x & \text{for } 0 \le x \le c, \\ 0 & \text{otherwise,} \end{cases}$$

   where c is an unknown constant.

   (a) What value of c makes f a valid pdf?

   (b) What is $E[X]$?

   (c) What is $Var(X)$? (Hint: first think about which variance formula is easiest to apply.)

   (d) What is $E[3X + 2]$?

   (e) What is $Var(3X + 2)$?

5. Let $X \sim Unif(0, \pi)$. What is $E[\cos(X)]$?

6. Using R, simulate the expected value in Problem 5. **Hint:** The R function `runif` generates random values from a uniform distribution. The function `mean` computes the average of a list of numbers.