Homework 2: Total Probability, Independence, and Bayes' Rule

Instructions: Submit a single R Markdown file (.Rmd) of your work on Canvas by 11:59pm on the due date. You may also submit diagrams, drawings, etc. as image files (.png, .jpg, .gif)—they must be formatted into your .Rmd document (we won't look at them separately). **Be sure to show all the work involved in deriving your answers! If you just give a final answer without explanation, you may not receive credit for that question.**

You may discuss the concepts with your classmates, but write up the answers entirely on your own. Do not look at another student's answers, and do not show your answers to anyone.

1. Independence: **Show the formula you use to check these!**

   (a) You roll a six-sided die. Are the following two events independent?

   $$A = \text{``the number is greater or equal to 3''},$$
   $$B = \text{``the number is odd''}.$$

   (b) You flip three coins. Are the following two events independent?

   $$A = \text{``you flipped repeats (HH or TT appears in the sequence)''},$$
   $$B = \text{``you flipped at least one heads''}.$$

   Hint: In both of these problems, you can enumerate all of the elements in each of the A and B sets.

2. According to the American Lung Association, there is a 0.13% chance to develop lung cancer. Of the people who have lung cancer, 90% of them are smokers. In the population of people who do not have lung cancer, 16.9% are smokers.

   (a) If you are a smoker, what is your probability to develop lung cancer?
   (b) If you are not a smoker, what is your probability to develop lung cancer?

3. You have a bucket of 10 red balls and 10 green balls. I remove one ball from the bucket. I make a note of the color of the ball, but do not tell you. Next, I replace the ball along with 10 new balls with the same color as the ball that I selected. After all of this, you select a ball from the bucket.

   (a) Draw a probability tree diagram, where the first level is the color I select, and the second level is the color that you select. **Be sure to give all branches of the tree and their probabilities, as well as the joint probabilities at the end of the branches!**
   (b) Using your diagram, what is the probability that the ball you selected is red?
   (c) Consider the two events $A = $ "I pick green" and $B = $ "you pick red". Are these events independent? (Don't just answer yes or no, show your work!)
   (d) If you pick a red ball, what is the probability that I picked a red ball?

4. Exercise 3.16 from the book.