CS 3130 / ECE 3530: Probability and Statistics for Engineers

More Practice

1. You have 4 red socks and 2 blue socks in your drawer. If you pull two socks out at random, what is the probability that they match?

2. Let X be a random variable for an n-sided die. The possible values you can roll are $1, 2, 3, \ldots, n$, each with equal probability. The expected value of a die roll is $E[X] = 9.5$. What is n? **Hint:** The following formula will be helpful: $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$.

3. A Bernoulli random variable, $X \sim Ber(\theta)$, has variance $\mathrm{Var}(X) = 0.21$. What are the possible values for the probability θ? **Hint:** There are **two** possible values θ could have.

4. The bus line that you take in to school claims that it is on average only 5 minutes late. Based on how many times you have been late to class, you think the bus is actually later than that. So you record the bus arrival times over 9 days. On average you find the bus is $\bar{x}_9 = 6$ minutes late, with a sample standard deviation of $s_9 = 1.5$ minutes. You now want to perform a hypothesis test on this data, to see if the bus really is later than their claim.

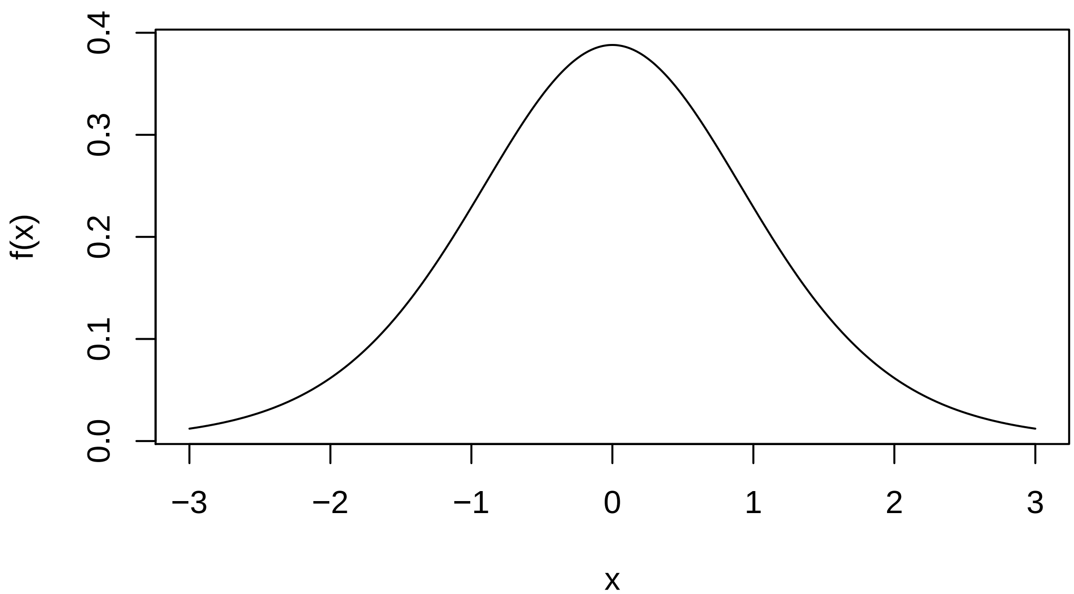
   (a) What is the null hypothesis, H_0, and the alternate hypothesis, H_1?

   (b) What type of statistic would you compute to test this hypothesis? What is the value of it? (Hint: it's a simple number)

(c) Say you choose a significance level of $\alpha = 0.05$. Below is a graph of the pdf for the sample statistic in part (b). Label it with the following information:

  i. The critical value for this test comes out to either -2.26 or +2.26. Pick the correct one, and mark it on the x-axis of the graph.

  ii. Draw on the graph how the p-value would be computed from your test statistic in part (b). (Hint: I'm looking for you to shade an area of the graph.)



(d) Would you reject the null hypothesis? (Just answer yes or no.)

(e) Now, instead of a hypothesis test, compute a 99% confidence interval of the average. Let F denote the cdf for the appropriate Student's t distribution. You will need one of the following values:

$$F(0.99) = 0.826 \quad F(0.995) = 0.827 \quad F^{-1}(0.99) = 2.82 \quad F^{-1}(0.995) = 3.25$$

Hint: Your confidence interval should be symmetric about the sample mean, and you don't need to do the arithmetic to simplify the final answer.

5. Say you are given a random sample, $Z_1, Z_2, \ldots, Z_n$, where each random variable is defined as $Z_i = \frac{1}{2}X_i^2 + \frac{1}{2}Y_i^2$, with both $X_i \sim N(\mu, 1)$ and $Y_i \sim N(\mu, 1)$.

   (a) What is the expectation $E[X_i^2]$? **Hint:** Use the formula for variance of a random variable, and the fact that you know $E[X_i]$ and $\text{Var}(X_i)$ because $X_i \sim N(\mu, 1)$.

   (b) What is $E[Z_i]$? **Hint:** Use part (a), and the fact that $E[X_i^2] = E[Y_i^2]$.

   (c) Say you want to estimate μ^2 with the mean statistic: $\hat{\mu}^2 = \bar{Z}_n$. What is the bias of this statistic? **Hint:** Use part (b), you should get a simple number.