10

P1

Key

Class avg	52.9
stdev	12.67
Max	70
Above 60	22
Above 50	32
Median	54.5

CS 3100 – Models of Computation – Fall 2010

Open Book/Notes Midterm Exam #1

Total Points : 70 (one point a minute)

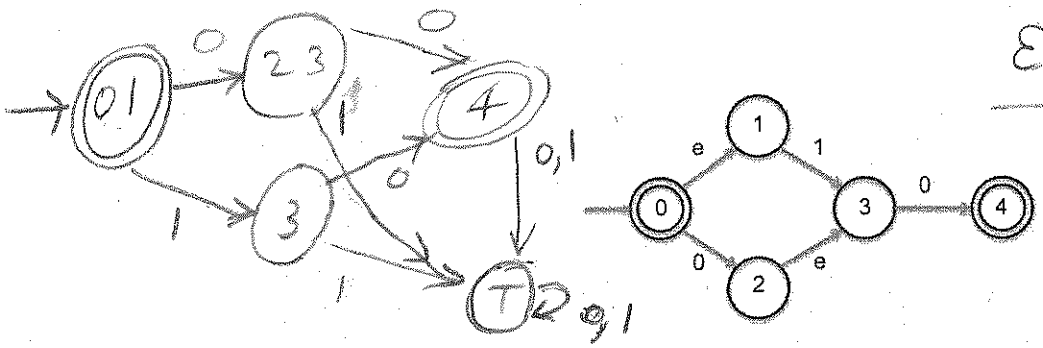
Answer in space provided, attaching additional sheets of paper as needed.

Clearly number all your questions.

Please write your name and UNID HERE:

1. **(10 points, split as follows)**

   (a) **(1 point)** What is the language of the NFA given below?



$\varepsilon + (0+1)0$

   (b) **(7 points)** Convert this NFA into an equivalent DFA following the subset construction method. You can label a DFA state as "23" to mean state $\{2, 3\}$, as "3" to mean state $\{3\}$, and so on.

   (c) Argue that this DFA also accepts the same language as the NFA. Show this by

      i. **(1 point)** Taking two strings accepted by your NFA and showing that your DFA accepts the same. Show the strings and write one sentence of explanation.

10 , ε

      ii. **(1 point)** Taking two strings, one of length two and one of length four, that are rejected by your NFA, and showing that these strings are rejected by your DFA. Show the strings and write one sentence of explanation.

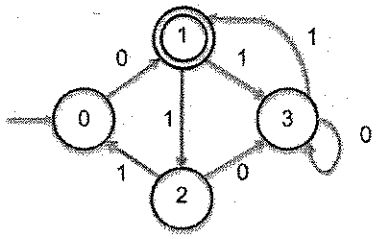
01

1111

20

P2

2. **(20 pts, split as follows)** Consider the DFA given below



(a) **(0.5 points)** What is one string in the language of this DFA of length more than two obtained by taking the self loop of 0 on the right (these are called "test strings")?

0101

(b) **(0.5 points)** What is another test string in the language obtained by following the vertical down arrow?

01001

(c) **(0.5 points)** What is one test string **not** in the language of this DFA of length more than two obtained by taking the self loop of 0 on the right?

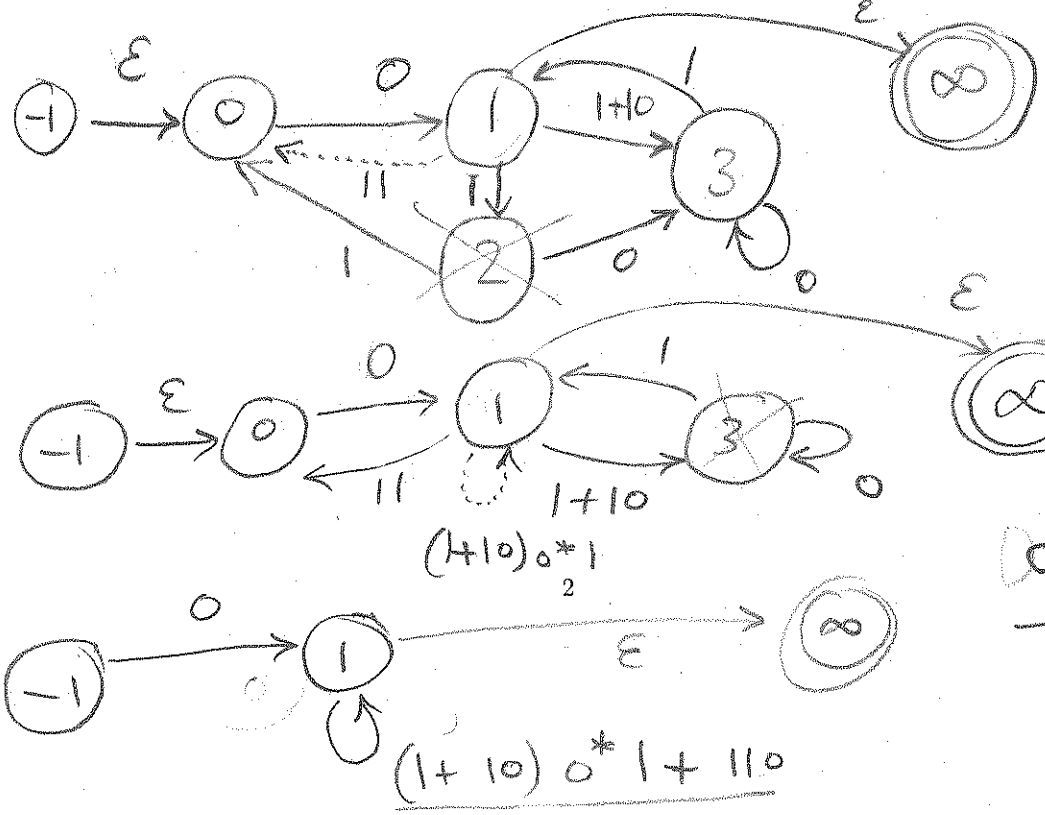
01000

(d) **(0.5 points)** What is another test string **not** in the language obtained by following the vertical down arrow?

01101

(e) **(4 points for each state elimination step)** Extract a regular expression eliminating states in the order **2, 3, 0, and finally 1**. It is important that you follow this order for two reasons: (i) ease of grading, (ii) the RE will be smaller this way. Write down the final RE you obtain.

(f) **(0.5 points for each of the test strings)** Show that the above test strings you wrote down are correct with respect to this RE (accepts the right test strings, rejects the right ones).



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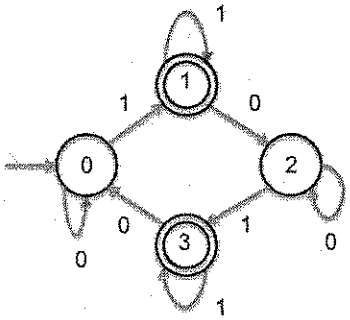
P3

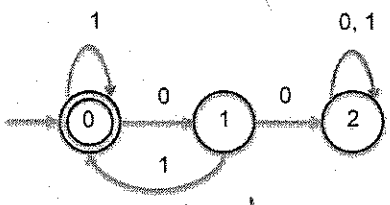
3. **(10 pts, split as follows)**

(a) **(0.5 pt)** Describe the language of this DFA, D_1

Strings ending in 1



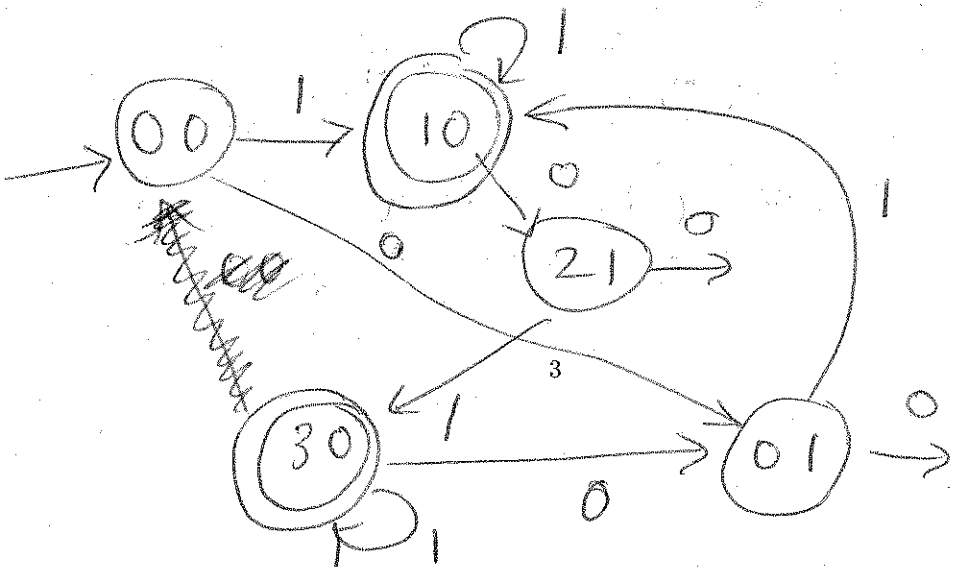
(b) **(0.5 pt)** Describe the language of this DFA, D_2



No two 0's in sequence and does not end in 0.

(c) **(8 pts)** Compute the product (intersection) of these two DFAs. There is no need to draw the "trap constellations." A trap constellation is a collection of non-accepting states such that if you get into one of those, you will keep cycling among them, never getting out. (They are a generalization of "trap states which are singleton trap constellations.)

(d) **(1 pt)** Show that your DFA product (intersection) is correct by writing explanations in English (1-2 sentences).



2 0's in a row ⇒ reject and trap.

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4. (1 pt)
What is the result of the following set intersection operation:

$\{a, aaa, \varepsilon, abab\} \cap \{aa, \varepsilon, baba\}$

$\{\varepsilon\}$

5. (1 pt)
Draw an NFA for the language $\{\varepsilon\}$

6. (1 pt)
List all strings in the language $\{a, aba\}\ \{\varepsilon, ba\}$. Here juxtaposition of two languages denotes conjunction.

a, aba, ababa

7. (2 pts)
List four of the shortest strings in the language $\{a, bbbb\}^*$

$\{\varepsilon, a, aaa, aaaa\}$

8. (2 pts)
Suppose language L over the alphabet $\{0, 1\}$ is

$$L = \{x \mid x \textit{ begins with } 0 \textit{ and ends with a } 1\}$$

Write down a regular expression for L. $0(0+1)^*1$

9. (3 pts)
Consider the language L of Question 8. Describe the complement of L in English. Recall that the complementation must be done with respect to $\{0, 1\}^*$.

strings that dont begin with 0
or dont end with 1
(includes ε)

10 pts

10. (2 pts) Write down a regular expression for the complement of the language L defined by Question 8.

$\varepsilon + 1(0+1)^* + (0+1)^*0$

11. (3 pts)

What is this set S defined as follows? Here, *Odd* are odd numbers 1, 3, etc. Also $prime(x)$ asserts that x is a prime number.

$$S = \{x \in Odd \mid if\ (x > 3)\ then\ prime(x+1)\}$$

$\{1, 3\}$

12. **(5 pts), split as follows.** Show using the Pumping lemma that the language of **all odd length palindromes** Palindromes L_{palOdd} over the alphabet $\{0, 1\}$ is not regular.

(a) (1 pt) Clearly describe how you initially choose the string in L_{palOdd}.

pick $0^k 1 0^k$

(b) (1 pt) Clearly show the u, v, w chosen.

u in 0^k
$|v| > 0$
$v \in 0^+$
w is the rest

(c) (1 pt) Show how you pump (i value).

$i = 0$ (pump down)

(d) (2 pts) Show there is a contradiction.

$0^{k-i|v|} 1 0^k \notin L_{palodd}$

Many of you Assumed $v = 0$'s And you know is $v \in 0^+$

Most common error: you chose a specific string like 00100

10 P6

or $(0+1)^*$ [~~***01(0+1)*~~ + *1*1 + 0*0* + *1*0* + 01** + 0**1] $(0+1)^*$

where $* = (0+1)$.

13. **(5 pts)** We are interested in capturing all two-bit errors with respect to the language $L_{0101} = (0+1)^*0101(0+1)^*$. That is, we must define a new language L_{2berr} such that L_{2berr} can be off by at-most two bits with respect to L_{0101}. Write down a regular expression for L_{2berr}

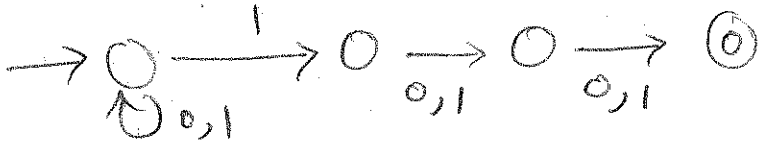
$(0+1)^*$ (1001 + 1111 + 1100 + 0011 + 0000 + 0110 + 1101 + 0001 + 0111 + 0100) $(0+1)^*$

14. We are interested in writing down a regular expression for the language of all strings over $\{0,1\}$ that end in $1(0+1)^n$.

(a) **(2 pts)** Write down an RE (as simple as possible) for $n = 2$.

$(0+1)^*1(0+1)(0+1)$

(b) **(1 pt)** Draw an NFA for this language.



(c) **(2 pts) split as follows** Suppose a DFA for this language has 32 states for $n = 5$.

i. (1 pt) How many states will the DFA have for $n = 6$? Explain in one sentence.

64

ii. (1 pt) For $n = 8$? Explain in one sentence.

256

exp growth