P_1
10/18

1. In a DFA long paths are loopy paths.

2. In any DFA of N states, consider an accepting path p of some length. Consider a sub-path q of p of length N.

p describes some string x

q describes some string y

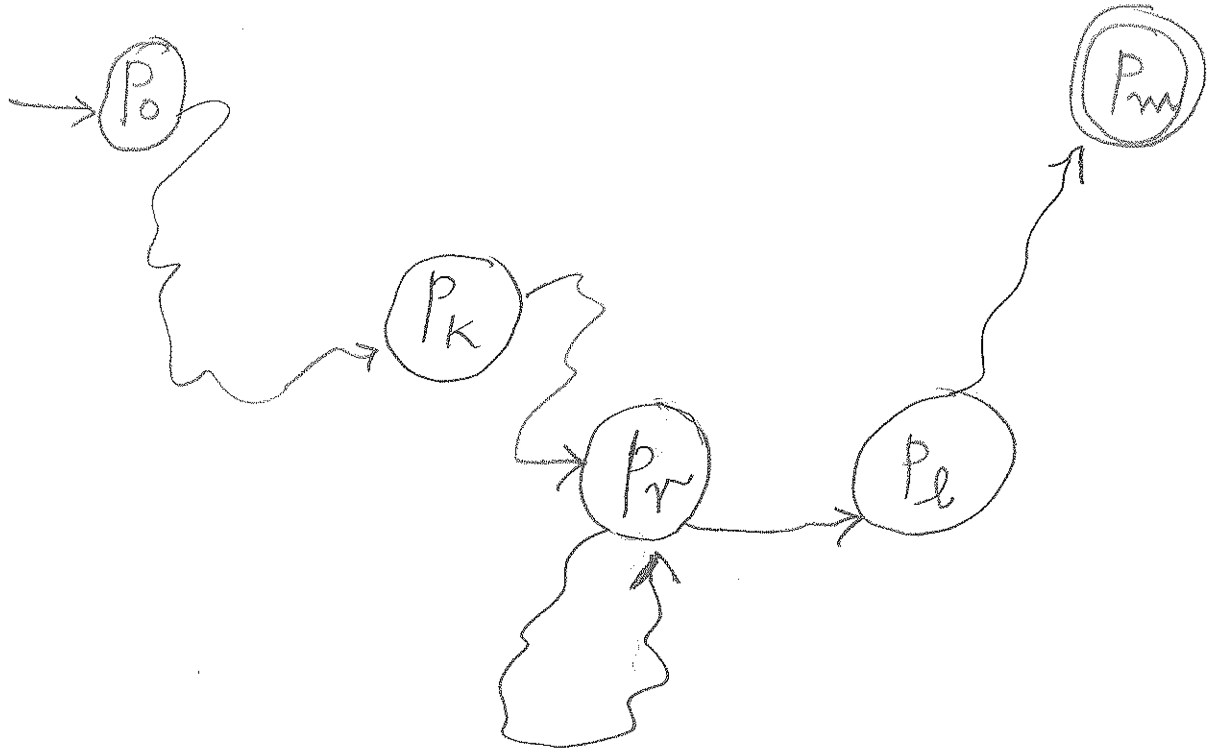
$$p = p_0 \xrightarrow{x_1} p_1 \rightarrow \cdots\cdots \; p_{m-1} \xrightarrow{x_m} p_m$$

$$q = p_k \xrightarrow{y_1} p_{k+1} \;\cdots\; p_{l-1} \xrightarrow{y_N} p_l$$

Then one of $p_k \cdots p_l$ are the same state!

call it p_r

The big picture:



Call the portion leading FIRST up to $P_r = u$

Call the FIRST P_r to P_r loop $= v$

Call the first P_r REVISIT to $P_m = w$

Then
- $uw \in L$
- $uvw \in L$ because $x \in L$
- $uv^2w \in L$ ...
- $uv^iw \in L$ for $i \geq 0$.

Consider

$$L_{2p} = \{ \varepsilon, (), (()) \}$$

$$L_{3p} = \{ \varepsilon, (), (()), ((())) \}$$

$$L_{100p} = \{ \varepsilon, (), \dots, (^{100})^{100} \}$$

All of these have DFAs!

But suppose someone claims that

$L_{\infty p}$ has a DFA — $D_{\infty p}$

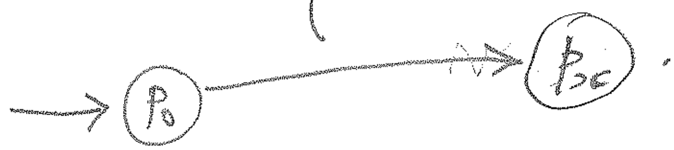
$$L_{\infty p} = \{ (^n)^n \mid n \geq 0 \}.$$

Then merely ask that person "how many states in that DFA"? Suppose they say "N".

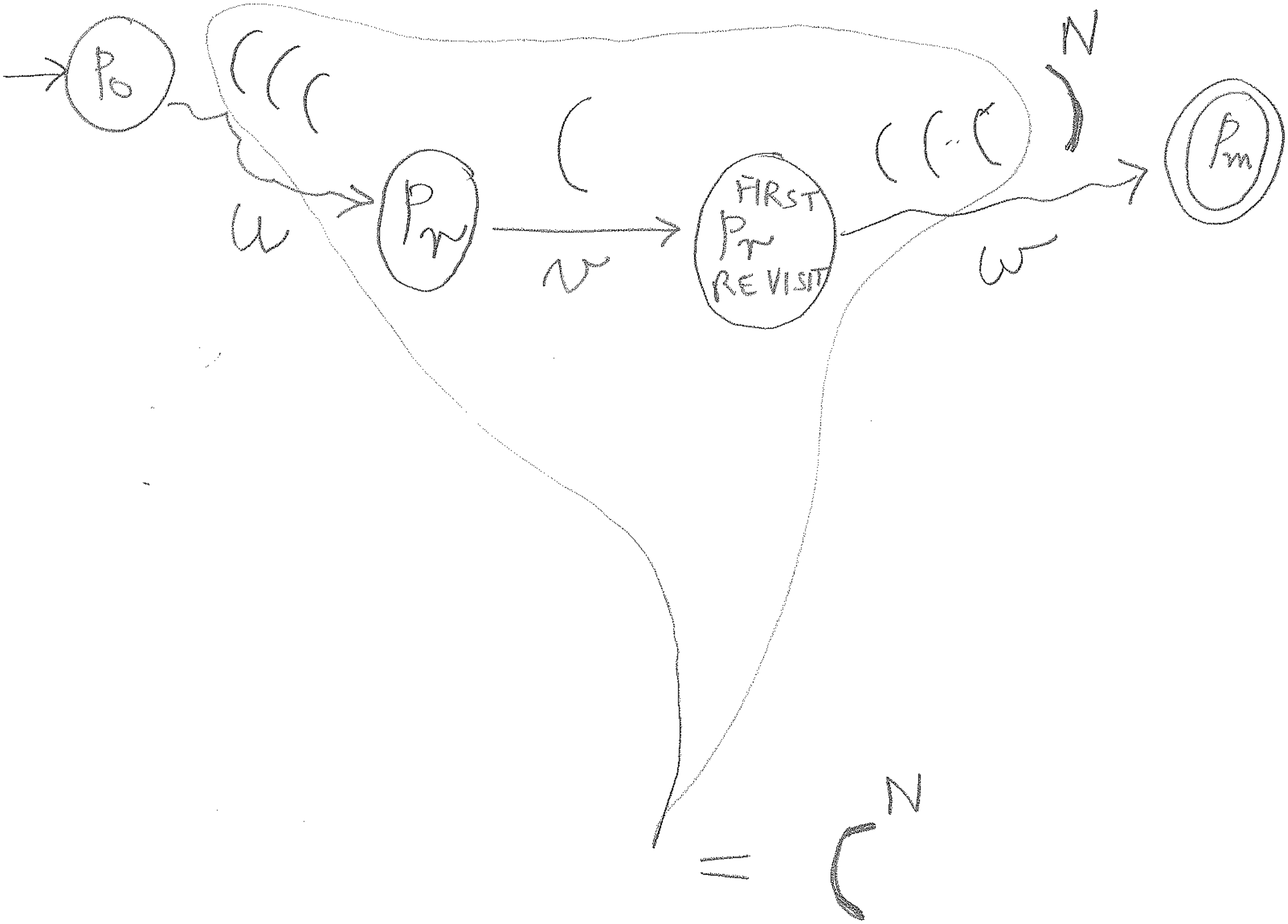
Then consider $(^N)^N$

Clearly $(^N)^N \in L(D_{\infty p})$.

Consider $(^N$



clearly $\exists$ a p_r state in this path.
We don't know what exactly things look like — EXCEPT:



$u = (^i$ for some $i \geq 0$

$v = (^j$ for some $j \geq 1$

$w =$ whocares! some (but then $)^N$.

But now

A1. $uvw \in L(D_{\infty p})$

A2. $u\,v\,v\,w \in L(D_{\infty p})$

A3. $u\,w \in L(D_{\infty p})$

A1 is fine.

A2 is a contradiction!

If you want more, then

A_3 is a contradiction!

So $D_{\infty p}$ can't exist.

"(If it exists, it can't be for language $L_{\infty p}$!!)"

Treating (= 0 and) = 1, we see that

$L_{0^n1^n} = \{0^n 1^n \mid n \geq 0\}$ not regular, as well!

(*) $L_{eq01} = \{ w \mid w \in \{0,1\}^*$ and $\#_0(w) = \#_1(w) \}$

L_{eq01} not regular!

<u>Proof 1:</u>

Suppose L_{eq01} reg.

Then $\exists$ a D_{eq01}

Clearly $\exists$ a $D_{0^*1^*}$

But then $D_{eq01} \cap D_{0^*1^*} = D_{0^n1^n}$

CONTRA!

<u>Proof 2:</u> Choose $0^n 1^n$ in L_{eq01}.

Proceed to Contradiction.

(*) $L_{abc} = \{ a^i b^j c^k \mid i, j, k \geq 0$ and if odd(i) then $j = k \}$

hmm!

Consider now Rev(L_{abc})