Adding to a Sorted Sequence

What if you need to frequently find and insert ordered items?

• Array: can find in $O(\log n)$ time, but takes $O(n)$ time to insert into the middle

• Doubly-linked list: can insert in $O(1)$ time, but takes $O(n)$ time to find position

A binary search tree can make both find and insert $O(\log n)$ time
Binary Search Tree
Binary Search Tree

; An X-tree is either
;   - empty
;   - (make-node X X-tree X-tree)
(define-struct node (value left right))

(define (leaf v) (make-node v empty empty))
(define (branch v l r) (make-node v l r))

(define num-tree
  (branch 5
    (branch 3
      (branch 1 empty (leaf 2))
      (leaf 4))
    (branch 10
      (branch 7 (leaf 6) empty)
      (leaf 11))))
Binary Search Tree

; A dir is either 'too-big, 'too-small, or 'same

; btsearch X-tree (X -> dir) -> X-or-false
(define (btsearch t check)
  (cond
    [(empty? t) false]
    [else
     (define d (check (node-value t)))
     (cond
      [(eq? d 'too-big)
       (btsearch (node-left t) check)]
      [(eq? d 'too-small)
       (btsearch (node-right t) check)]
      [else (node-value t)]))]))
Binary Search Tree

See `btsearch` in `btsearch.c`
Binary Search Tree Inserts

; btinsert X-tree X (X -> dir) -> X-tree
(define (btinsert t v check)
  (cond
   [(empty? t) (leaf v)]
   [else
    (define d (check (node-value t)))
    (cond
     [(eq? d 'too-big)
      (branch (node-value t)
       (btinsert (node-left t) v check)
       (node-right t))]
     [(eq? d 'too-small)
      (branch (node-value t)
       (node-left t)
       (btinsert (node-right t) v check))]
     [else t]]))))
Binary Search Tree Inserts

See `btinsert` in `btsearch.c`
Unbalanced Tree
Unbalanced Tree
Unbalanced Tree
Unbalanced Tree
Balancing a Tree
Balancing a Tree
Balancing a Tree
Balancing a Tree
Balancing a Tree
Balancing a Tree
Balancing a Tree
Balancing a Tree
Balancing a Tree
AVL Trees

An **AVL tree** uses a particular balancing strategy

Define *balance* at $N$ as

$$\text{height}(\text{A}) - \text{height}(\text{B})$$

After insert, a balance of $\pm 2$ triggers rotations
AVL Trees

picture based on http://en.wikipedia.org/wiki/AVL_tree
AVL Trees

Whenever balance at 5 is +2

picture based on http://en.wikipedia.org/wiki/AVL_tree
AVL Trees

If balance at 3 is < 0

Whenever balance at 5 is +2

picture based on http://en.wikipedia.org/wiki/AVL_tree
AVL Trees

picture based on http://en.wikipedia.org/wiki/AVL_tree
AVL Trees

Whenever balance at 3 is -2

picture based on http://en.wikipedia.org/wiki/AWL_tree
AVL Trees

If balance at 5 is > 0

Whenever balance at 3 is -2
AVL Trees

See `avl.c`
if (get_balance(t) == 2) {
    /* need to rotate right */
    tree left = t->left;
    if (get_balance(left) < 0) {
        /* double right rotation */
        tree left_right = left->right;
        left->right = left_right->left;
        left_right->left = left;
        fix_height(left);
        left = left_right;
    }
    t->left = left->right;
    left->right = t;
    fix_height(t);
    fix_height(left);
    return left;
}
if (get_balance(t) == 2) {
    /* need to rotate right */
    tree left = t->left;
    if (get_balance(left) < 0) {
        /* double right rotation */
        tree left_right = left->right;
        left->right = left_right->right;
        left_right->right = left;
        left_right->left = left;
        fix_height(left);
        left = left_right;
    }
    t->left = left->right;
    left->right = t;
    fix_height(t);
    fix_height(left);
    return left;
}
JFYI: Red–Black Trees

A red-black tree uses a similar but different rebalancing strategy.

It is often implemented with for loops instead of recursion, which is/was useful in some settings.
JFYI: Splay Trees

A *splay tree* uses another balancing approach. Instead of rebalancing after an insert, a splay tree rotates all lookups and inserts to the root.