ECE 6130 Impedance and Multiport networks

Text Sections 4.1

Portfolio Question: Explain the concept of "impedance" as it relates to microwave transmission lines and waveguides.

Microwave Network Theory

Circuit Network (Low Frequency) : Elements can be modeled as lumped elements. Given a set of R,L,C components at low frequencies, you can find voltage and current and impedance at any points BETWEEN these elements using circuit theory (KVL, KCL, etc.) You do not find the values of voltage, field, etc. in points INSIDE the elements. They are usually not of interest.

Microwave Network (High Frequency): Elements cannot be modeled as lumped elements. Elements are “large” compared to wavelength.
In Microwave network analysis:

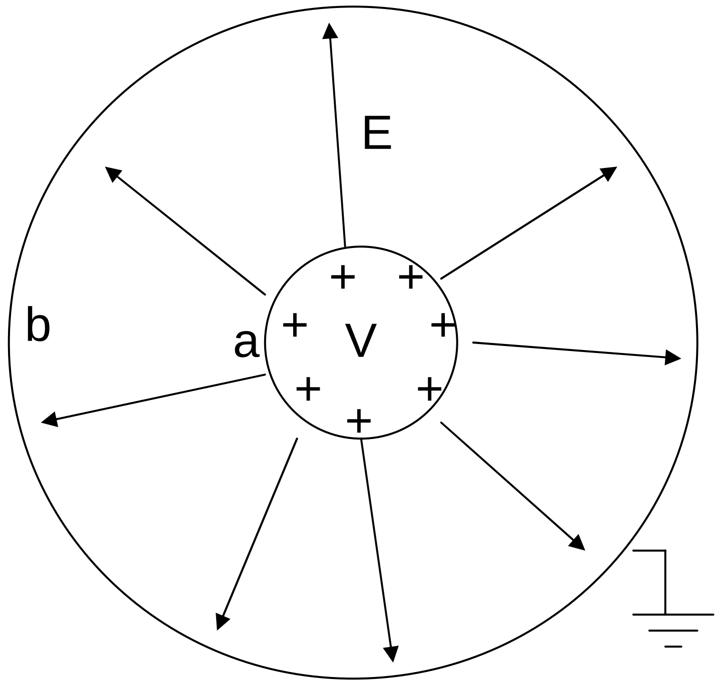
(a) Field theory (Maxwell’s equations) is used to find the complete field distribution in each element.
(b) Determine equivalent circuit properties of each element (Like Zo and Zin for transmission lines) from complete fields.
(c) Equivalent circuit models are then analyzed with Circuit Theory.

Example: Coaxial Transmission line analysis.

(a) Use Maxwell’s equations and boundary conditions to find E and H everywhere in the line (Text section 3.5)
(b) Use E and H to find R,L,G,C parameters (text section 2.2)
(c) Analyze R,L,G,C parameters using circuit theory (text section 2.1).
(d) One step further: Combine RLGC parameters to give V,I,Z relations. Use these to analyze multiple lines.

So .. what we need for any case is a method to determine V,I,Z values so we can combine effects of multiple elements using circuit theory.

Impedance



For 2-conductor transmission line, V is voltage between the conductors

$$V = \int_{+}^{-} \overline{E} \bullet d\ell = \int_{a}^{b} \frac{V}{r \ln(b/a)} \bullet dr = V \frac{\ln(b/a)}{\ln(b/a)} = V$$

I is current flowing on the positive conductor

$$I = \oint \overline{H} \bullet d\ell = \int_{\phi=0}^{2\pi} \frac{I}{2\pi r} r d\phi = I \frac{2\pi}{2\pi} = I$$

Characteristic impedance of the line is defined:

Zo = V/I (ohms)

BUT for a single-conductor transmission line, V and I are not well-defined.

Start with a rectangular waveguide…

Table 3.2 shows the fields of a rectangular waveguide.
Sketch the TE10 and TM10 modes.

TE_{10} mode of rectangular waveguide:
Ey (x,y,z) = (jωμa / π) A sin(π x /a) exp (-jβz)
Hx(x,y,z) = (jβa / π) A sin(π x /a) exp (-jβz)

Voltage between top and bottom of waveguide is:

V = integral E •dl = (-jwua / pi) A sin(pi x /a) integral dy = (-jwua / pi) A sin(pi x /a) b

This depends on x … which value of x should we take?? (There is no unique answer to this.)

Uses of impedance:

- Wave impedance $Z_w = E_t / H_t$: Depends on size and shape of guide, frequency, type of wave (Z_{TEM}, Z_{TE} , Z_{TM})
- Intrinsic Impedance of the material: $\eta=\sqrt{\mu/\varepsilon}$
- Characteristic Impedance: $Zo = V^+ / I^+$, unique for TEM waves, nonunique for TE and TM waves.

So how do we decide on definitions of impedance for TE and TM modes?

1) Voltage and current are defined for a particular mode. Choose voltage proportional to $E_{transverse}$ and current proportional to $H_{transverse}$
2) Choose V and I so that (V)(I) = power flow of the mode
3) For a single traveling wave (+ or -) V / I = Zo, which is usually chosen to = wave impedance of line or normalized to 1.

Example:Rectangular Waveguide

1) Voltage and current are defined for a particular mode. Choose voltage proportional to

$$\overline{E_t}(x,y,z) = \bar{e}(x,y)(A^+e^{-j\beta z} + A^-e^{+j\beta z}) = \bar{e}(x,y)\frac{1}{C_1}(V^+e^{-j\beta z} + V^-e^{+j\beta z})$$

$$and \quad current \quad proportional \quad to$$

$$\overline{H_t}(x,y,z) = \bar{h}(x,y)(A^+e^{-j\beta z} - A^-e^{+j\beta z}) = \bar{h}(x,y)\frac{1}{C_2}(I^+e^{-j\beta z} - I^-e^{+j\beta z})$$

$$where$$

$$\bar{e}(x,y) = transverseE = \sin(\pi x/a)\hat{y}$$

$$\bar{h}(x,y) = transverseH = \frac{-1}{Z_{TE}}\sin(\pi x/a)\hat{x}$$

$$also:$$

$$\bar{h}(x,y) = \frac{\hat{z}\times\bar{e}(x,y)}{Z_w}$$

$$and$$

$$Z_o = V^+/I^+ = V^-/I^-$$

E_t and current proportional to H_t

2) Choose V and I so that (V)(I) = power flow of the mode.

This is going to partially define C_1 and C_2.

Power flowing in positive-traveling field wave (only):

$$P^{+} = \frac{-1}{2}\int_S \overline{E} \times \overline{H^{*}} \bullet d\overline{S} = \frac{-1}{2}\int_S E_y H_x dxdy = \frac{ab\,|A|^2}{4Z_{TE}}$$

Power flowing in positive-traveling voltage and current wave (only):

$P^{+} = (1/2)\ V^{+}I^{+}$

Setting these two powers equal gives:

$$\frac{ab\,|A|^2}{4Z_{TE}} = \frac{1}{2}V^{+}I^{+}$$

Recall from above: $V^{+} = AC_1$ and $I^{+} = AC_2$

3) Set ratio of V / I for single traveling wave to be wave impedance for that mode

$$\frac{V^{+}}{I^{+}} = \frac{C_1}{C_2} = Z_{TE}$$

Solving :

$$C_1 = \sqrt{\frac{ab}{2}} \quad C_2 = \frac{1}{Z_{TE}}\sqrt{\frac{ab}{2}}$$