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Network Impedance Transformations

10.1 *L*-TYPE MATCHING CIRCUITS

The advantages of matching impedances at a junction between two circuits or media have been pointed out in Chap. 9 and elsewhere in this book and need no further explanation here.

One of the most commonly used and generally satisfactory impedance transforming networks for radio frequency applications is the half-section *L*-type circuit employing two essentially pure reactance elements [19, 149]. At a single frequency and, for most practical purposes, embracing at least the sideband frequencies of a radio telephone transmitter, the simple *L*-type circuit may be used effectively to transform any load impedance to any desired pure input resistance value. Conversely, the *L*-type circuit may be employed to accomplish the reverse transformation, that is, to transform any load resistance to any desired complex input impedance value. It is necessary to consider only the former type of transformation, however, since the circuit can always be reversed to make the transformation in the opposite direction. This simplifies the presentation of design information.

It will be seen by referring to Fig. 10.1 that there is a total of eight possible combinations of reactance types, i.e., inductive and capacitive, in an *L*-type circuit. Each of these eight circuits is capable of transforming a restricted range of complex load impedance values to a given pure resistance value. The transformable impedance values associated with each circuit can conveniently be represented by the impedances within a bounded area on a SMITH CHART. A set of eight such representations will therefore completely outline the capability and limitations of the eight possible reactance combinations, and will furnish a comprehensive outline of the impedance transforming capability of each reactance combination.

For radio-frequency applications, the losses in an *L*-type circuit are usually small in comparison to the power which is being conducted through the circuit. Thus, the circuit losses generally will not limit to any serious extent the range of load impedance values which can be transformed to a desired resistance, nor will they ordinarily have a major effect upon the reactance values for the circuit elements which are theoretically required on the assumption that they are lossless. The design charts to be described are, therefore, plotted for the idealized case of lossless circuits. Having selected a suitable lossless circuit and having obtained the reactance values required in such a circuit from the charts, the probable resistance of the circuit elements which must be used and the

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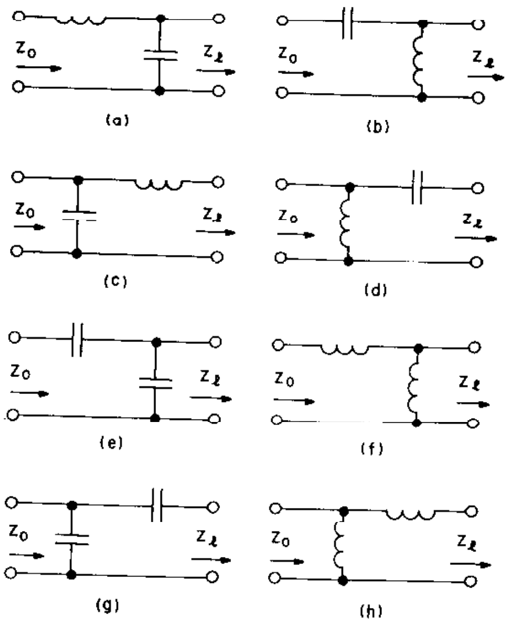

Fig. 10.1 Eight possible L-type circuits for transforming a complex load impedance Z_ℓ to a pure resistance Z_0

resulting losses will be more readily determinable.

10.1.1 Choice of Reactance Combinations

The eight SMITH CHART overlays in Fig. 10.2 summarize the matching capabilities of the eight possible L-type circuit combinations of Fig. 10.1, and serve as a guide to the selection of a suitable L-type matching circuit for any particular impedance transformation.

A shaded area is shown on each of the eight diagrams in Fig. 10.2. This is to indicate that any load impedance vector whose extremity falls anywhere within this "forbidden" area cannot be transformed to Z_0 (the desired input resistance value) with the specific circuit to which the diagram applies, and that in this case one of the seven other L-type circuits must be selected. If the extremity of the load impedance vector falls anywhere inside of the unshaded area, the circuit is capable of performing the desired impedance transformation.

In cases where the impedance transforming capabilities of two or more L-type circuits overlap, the particular circuit which calls for the more practical circuit constants should, of course, be chosen. It is of interest to note that the circuits shown in Fig. 10.2(e) and (g) are each capable of transforming the same range of load impedances, although each accomplishes a given transformation with different reactance values. The circuits in Fig. 10.2(f) and (h) are also both capable of making the same impedance transformations.

10.1.2 SMITH CHART Representation of Circuit Element Variations

On each of the eight diagrams shown in Fig. 10.2 an example of the function of each element of the circuit is illustrated using an assumed load impedance vector Z_ℓ. The influence of each of the circuit elements upon Z_ℓ may be regarded as forcing the latter to move along an "impedance path" on a SMITH CHART from its initial position to a position along the R axis, with its extremity at position Z_0, This impedance path followed by a single vector is illustrated on each of the eight diagrams of Fig. 10.2 by a heavy line and an accompanying arrow.

For example, refer to Fig. 10.2(a). Here, any load impedance (such as Z_ℓ) whose extremity falls in the unshaded area may be selected to be transformed with an L-type circuit of the type indicated on this diagram to a chosen value of pure resistance Z_0. In this case it will be noted that the effect of the shunt capacitive reactance on the impedance vector Z_ℓ is to rotate its extremity clockwise around a circular path leading to the point Z_1. This path is always along a circle tangent to the X axis at $X = 0$, and centered on the R axis of the SMITH CHART. Z_1 represents the extremity of a second impedance vector, the resistance component of which is equal to Z_0. (To simplify the diagrams, only the extremities of the vectors are indicated.) The capacitive reactance component of the impedance vector Z_1 is then canceled by the reactance of the series inductance element of the L-type circuit, which moves the vector along the path to position Z_0, thus completing the transformation.

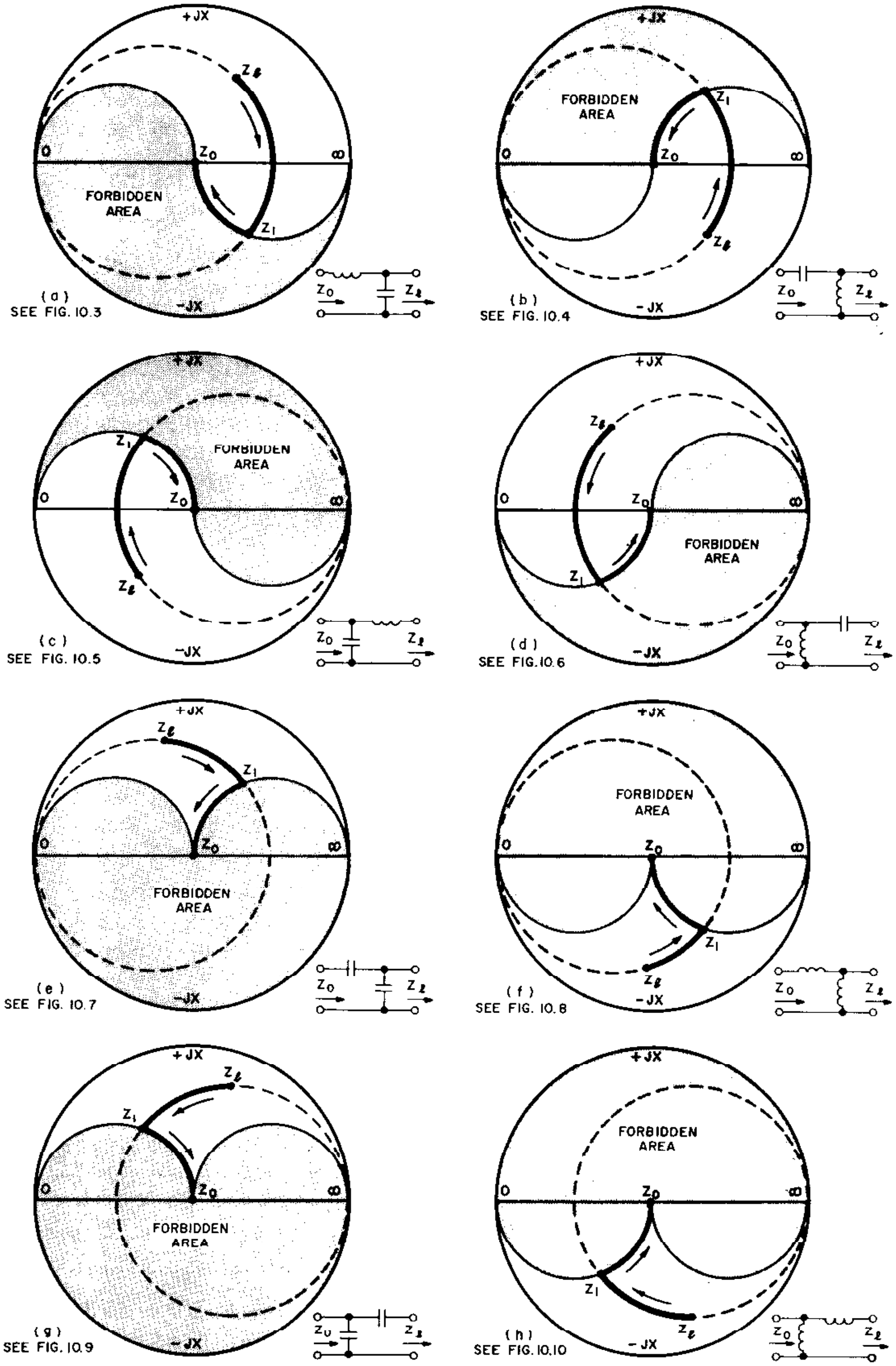

Fig. 10.2 Impedances in unshaded areas of SMITH CHARTS, represented by eight circular boundaries, are transformable to a pure resistance Z_0 with specific L-type circuits indicated (transforming effect of each circuit element is indicated by a heavy line with arrows).

The required inductive and capacitive reactances of the *L*-type circuit elements are not shown in Fig. 10.2, which, as explained, is useful primarily to compare the matching capabilities of the various circuits.

10.1.3 Determination of L-type Circuit Constants with a SMITH CHART

To obtain the proper value of the inductive and capacitive reactances required in a given *L*-type circuit to transform a given complex load impedance Z_ℓ to a given pure resistance Z_0, select the design curves which have been plotted for the particular circuit chosen. These are plotted in Figs. 10.3 through 10.10 for each reactance element of each of the eight possible circuits. The applicable circuit can be identified by referring to the small schematic diagram associated with each. Each of these families of design curves is used as an overlay for SMITH CHART A, in the cover envelope, with which it must be accurately aligned.

The load impedance, indicated at the extremity of the load impedance vector, should be spotted on the SMITH CHART coordinates which are superimposed on the appropriate design chart. The required circuit reactance values are then obtained from the design chart by interpolating between the indicated values on the nearest reactance curves plotted thereon.

Problem 10-1 which follows, further illustrates this use of the design charts.

10-1

(a) Select an *L*-type circuit which will transform a load impedance of 140 + *j* 60 ohms to a pure resistance of 50 ohms.

Solution:

From the foregoing we may write:

$$Z_0 = 50$$
$$Z = 140 + j60 = 2.8Z_0 + j1.2Z_0$$

Refer to SMITH CHART A and Fig. 10.2 and observe that the above load impedance vector Z_ℓ falls within the unshaded (transformable) area of diagrams a and b, and within the "forbidden" area of diagrams c to h inclusive. A choice of two circuits is therefore available for this transformation. Select one-for example, that of diagram b.

(b) Determine the reactance value of each element of the circuit selected.

Solution:

On SMITH CHART A locate the above impedance value; then superimpose this chart on the design curves of Fig. 10.4 (as directed on diagram b of Fig. 10.2). Next determine the correct reactance values for X_L and X_C by noting that the extremity of this load impedance vector Z_ℓ, as plotted on the SMITH CHART, falls at the intersection of the design curves labeled $X_L/Z_0 = 3.0$ and $X_C/Z = 1.5$. Since $Z_0 = 50$ ohms, $X_L = 3.0 \times 50 = 150$ ohms, and $X_C = 1.5 \times 50 = 75$ ohms.

If a complex load impedance is not known exactly but can be estimated within certain limits, these limits may be mapped directly on the SMITH CHART and the range of circuit reactances required can thus be completely bracketed.

This feature will be most appreciated when an *L*-type circuit must be designed to accommodate any one of a range of possible load impedance values. The design of a circuit to match the input impedance of a radio antenna, which is usually not definitely known in advance of its construction, to the characteristic impedance of a transmission line is readily accomplished with this type of diagram. In such cases, the limitations of a given circuit establish limiting requirements for the circuit elements. Problem 10-2, which follows, illustrates this case.

10-2

(a) Select an *L*-type circuit which can be adjusted to match any load impedance falling within the range 25 to 75 ohms resistance and 0 to 50 ohms positive reactance to a pure resistance of 100 ohms.

Solution:

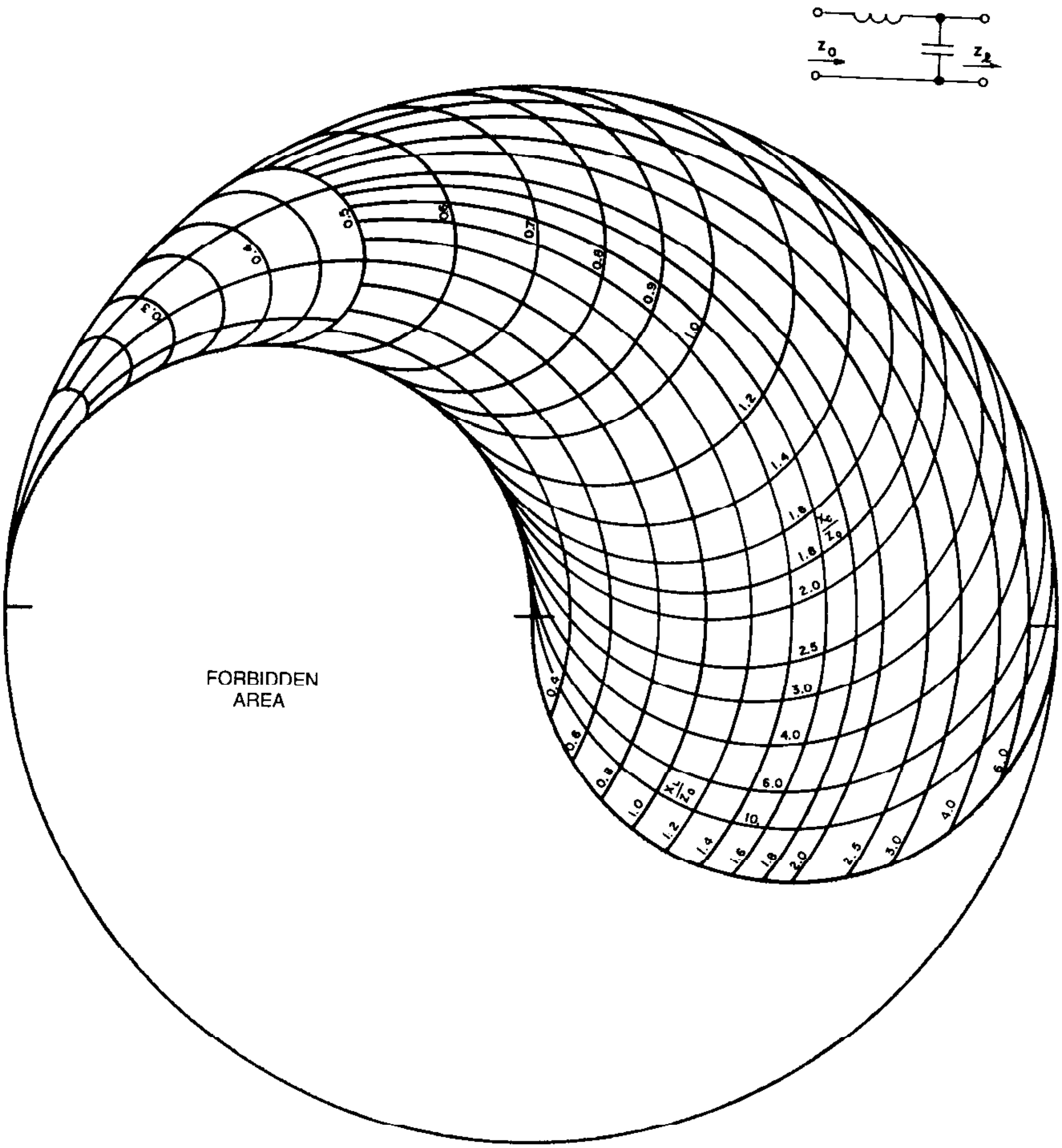

Fig. 10.3 Normalized reactances of L-circuit elements required to transform Z_ℓ to Z_0 (overlay for Charts A, B, or C in cover envelope).

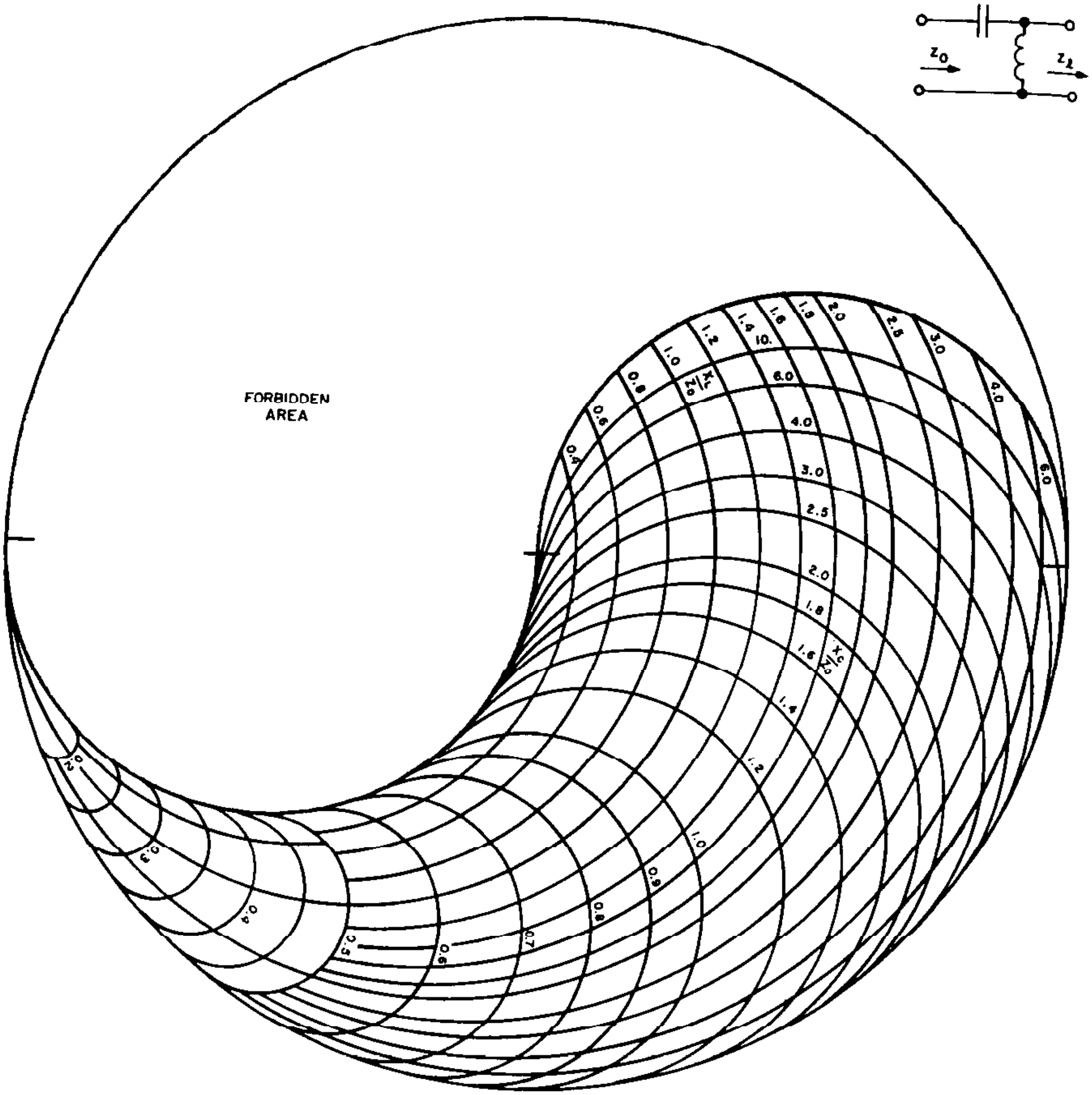

Fig. 10.4 Normalized reactances of *L*-circuit elements required to transform Z_ℓ to Z_0 (overlay for Charts A, B, or C in cover envelope).

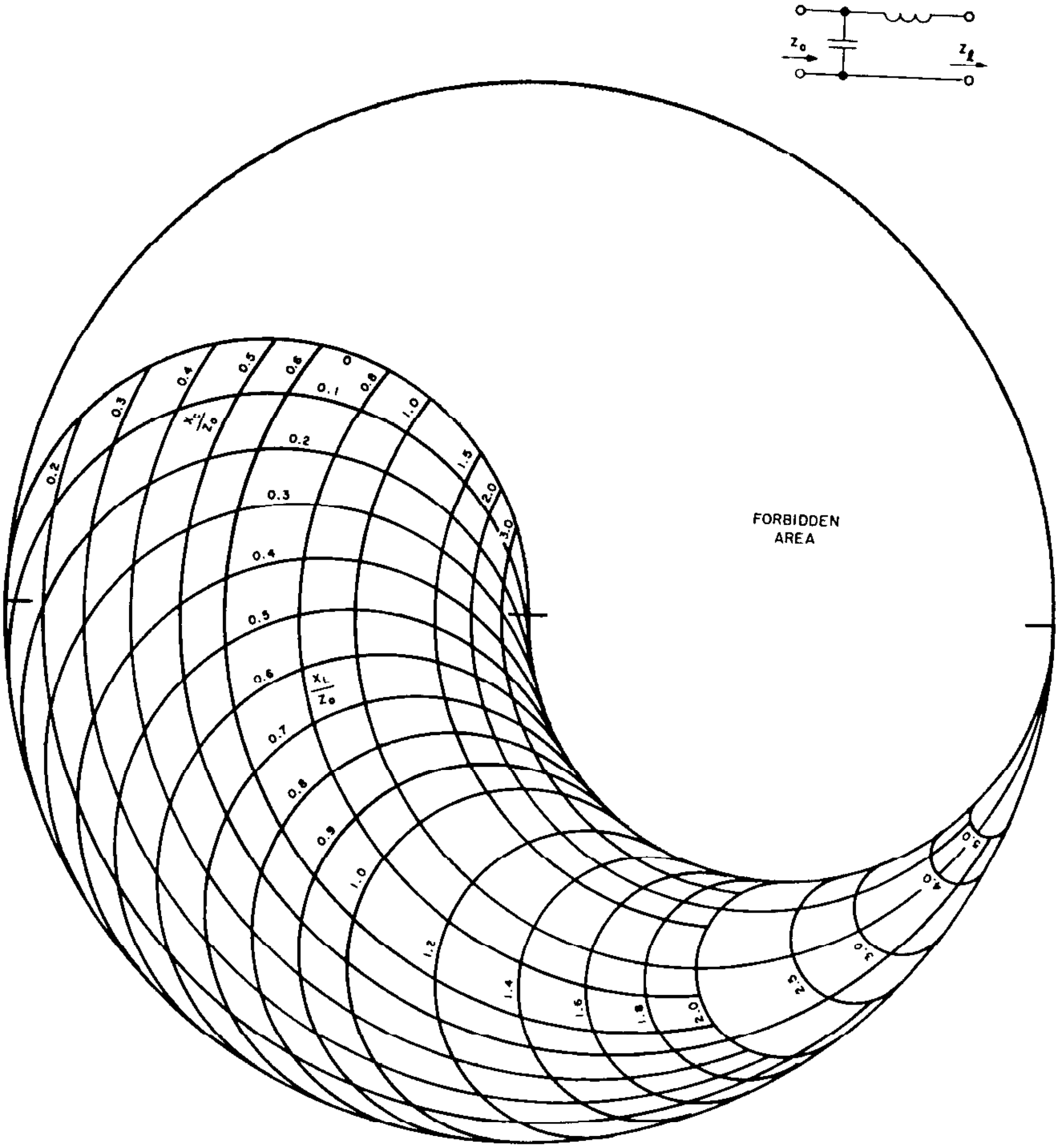

Fig. 10.5 Normalized reactances of L-circuit elements required to transform Z_ℓ to Z_0 (overlay for Charts A, B, or C, in cover envelope).

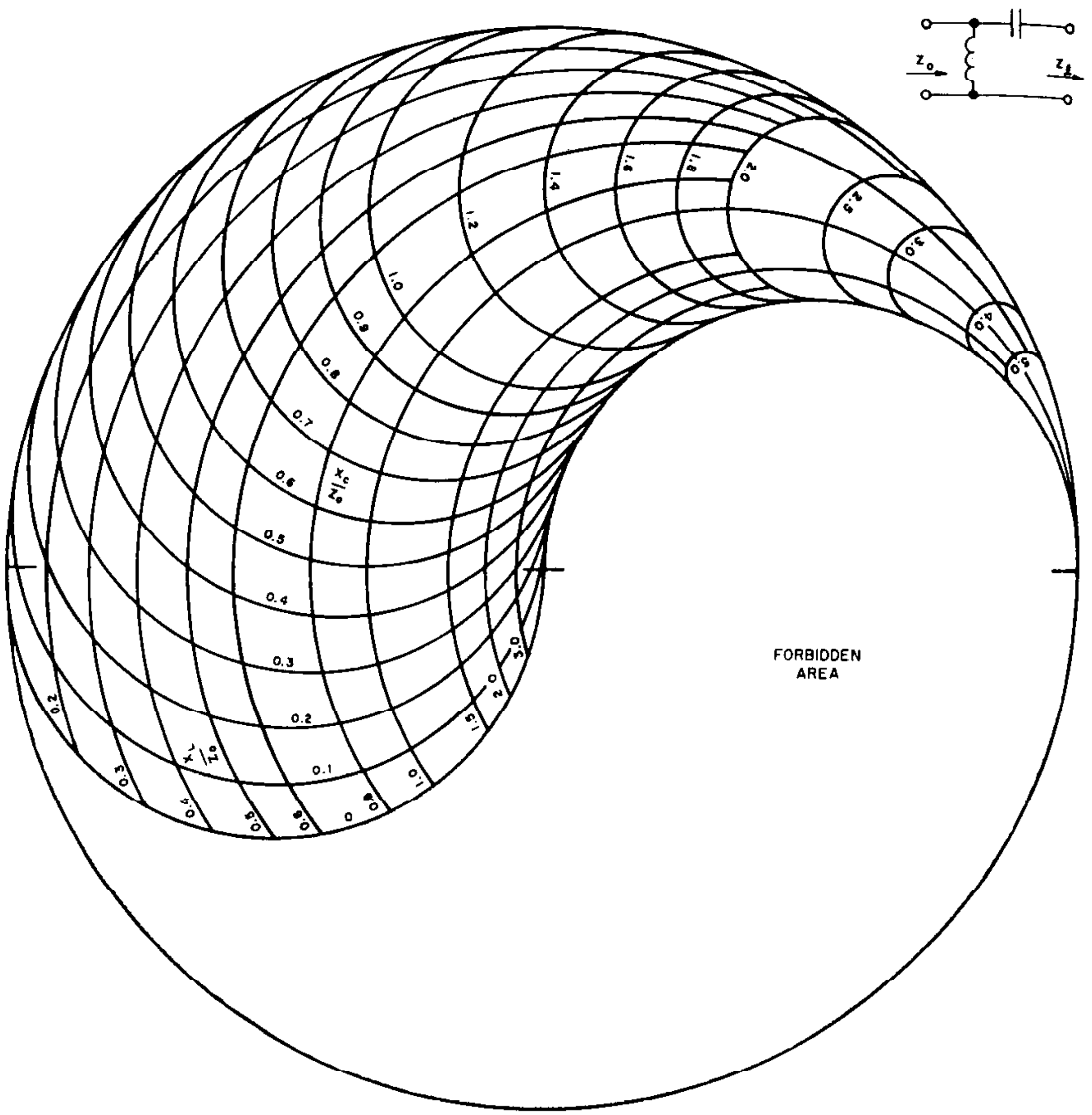

Fig. 10.6 Normalized reactances of *L*-circuit elements required to transform Z_ℓ to Z_0 (overlay for Charts A, B, or C, in cover envelope).

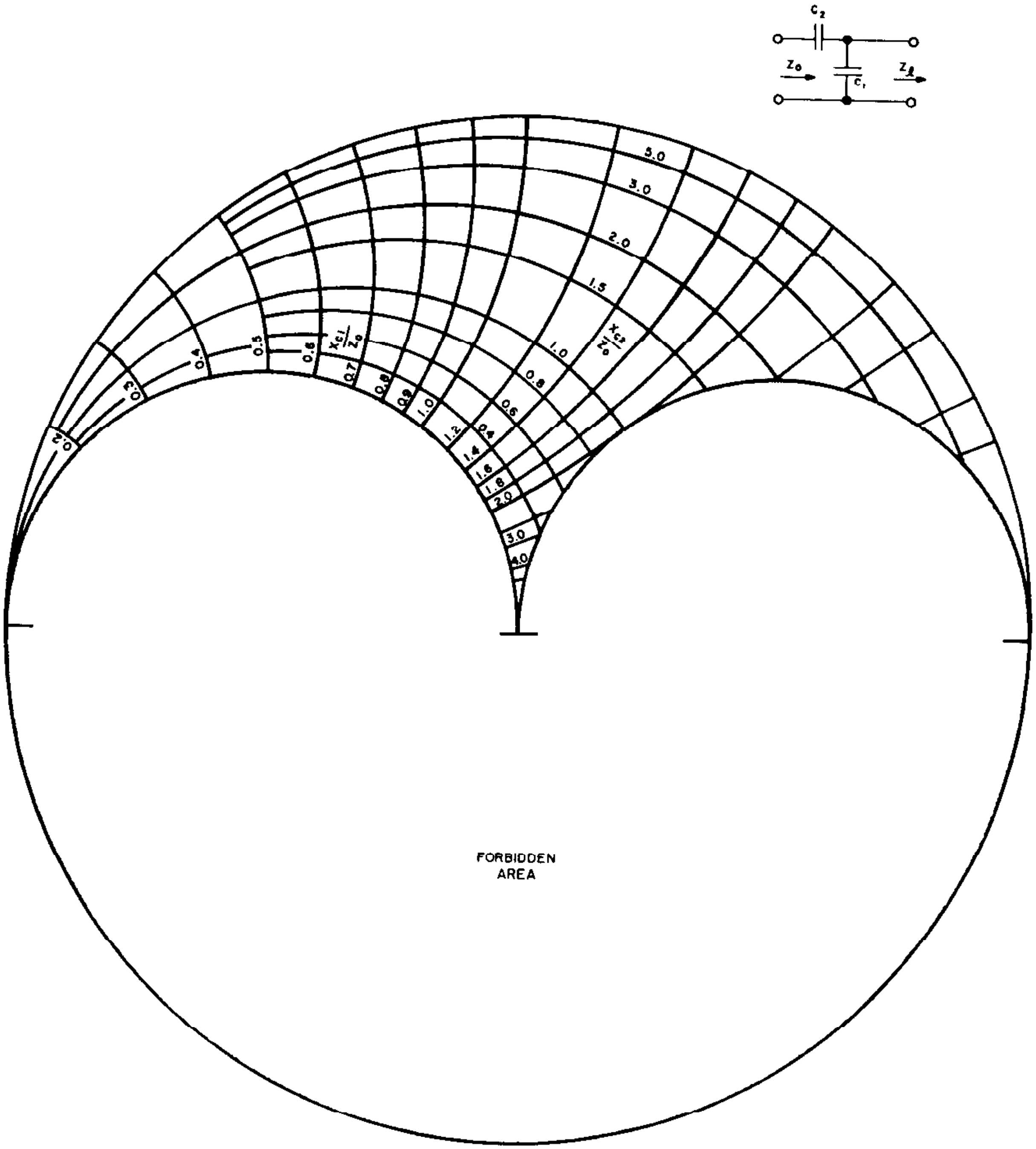

Fig. 10.7 Normalized reactances of *L*-circuit elements required to transform Z_l to Z_0 (overlay for Charts A, B, or C, in cover envelope).

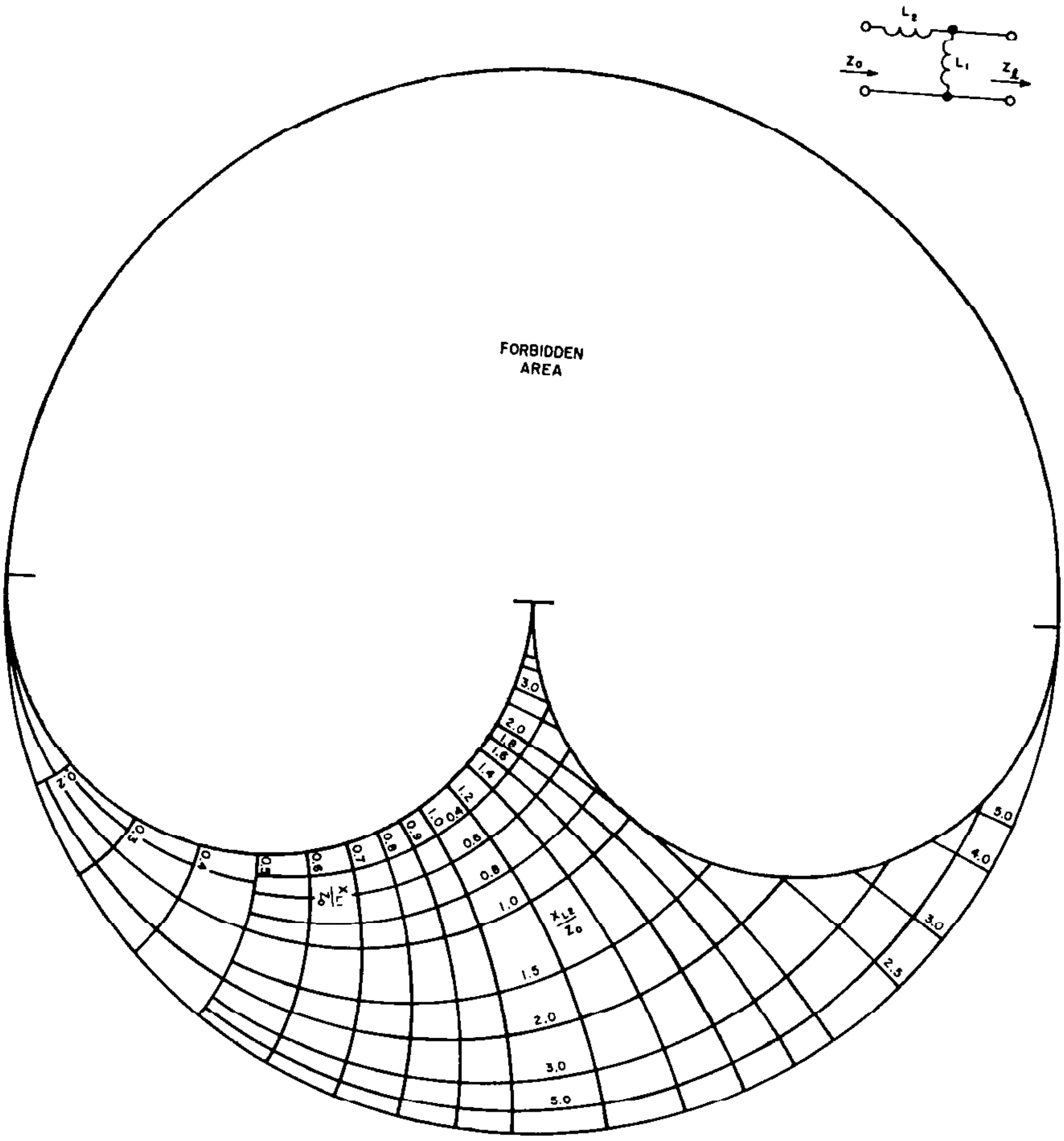

Fig. 10.8 Normalized reactances of L-circuit elements required to transform Z_ℓ to Z_0 (overlay for Charts A, B, or C, in cover envelope).

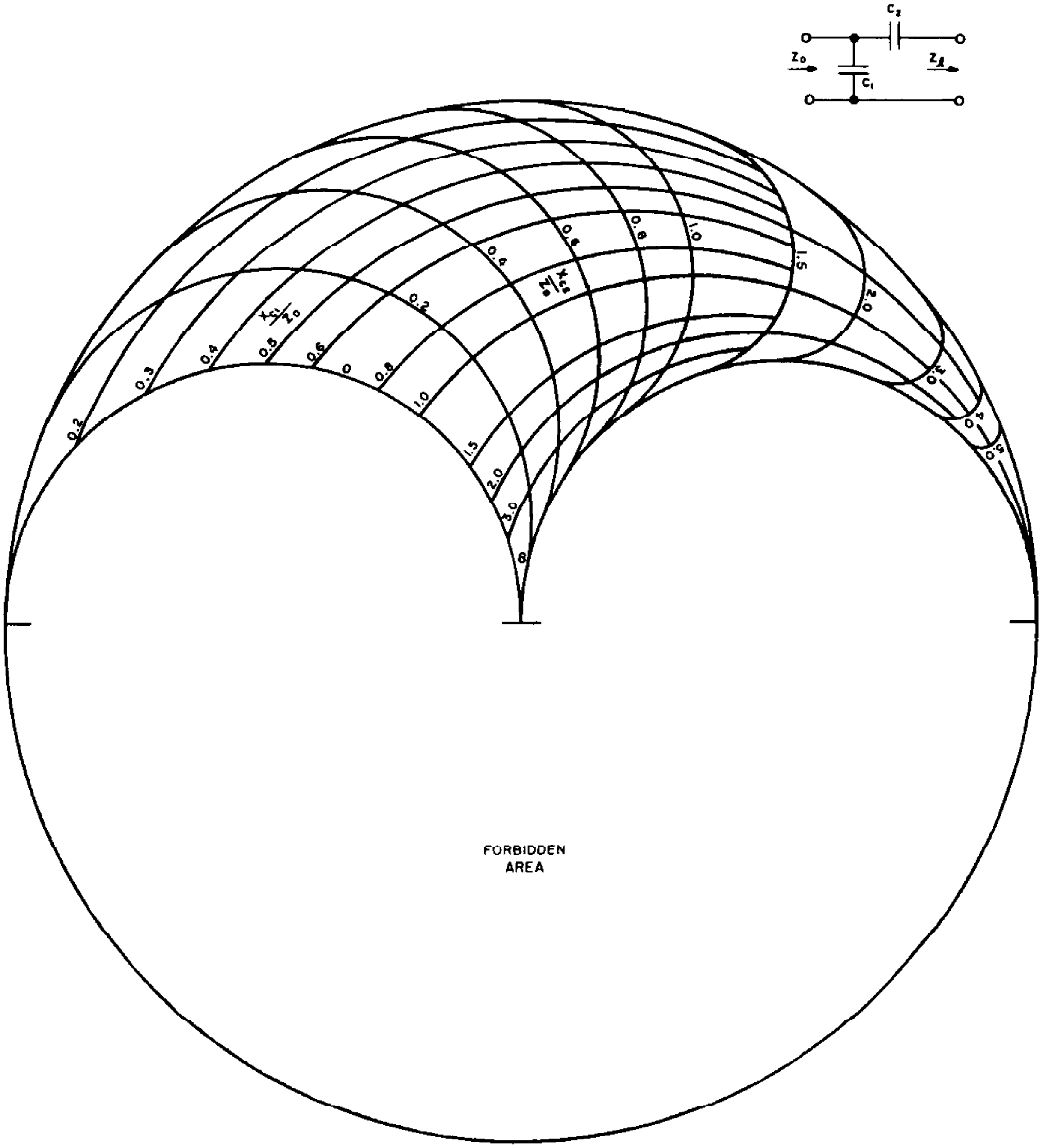

Fig. 10.9 Normalized reactances of *L*-circuit elements required to transform Z_ℓ to Z_0 (overlay for Charts A, B, or C, in cover envelope).

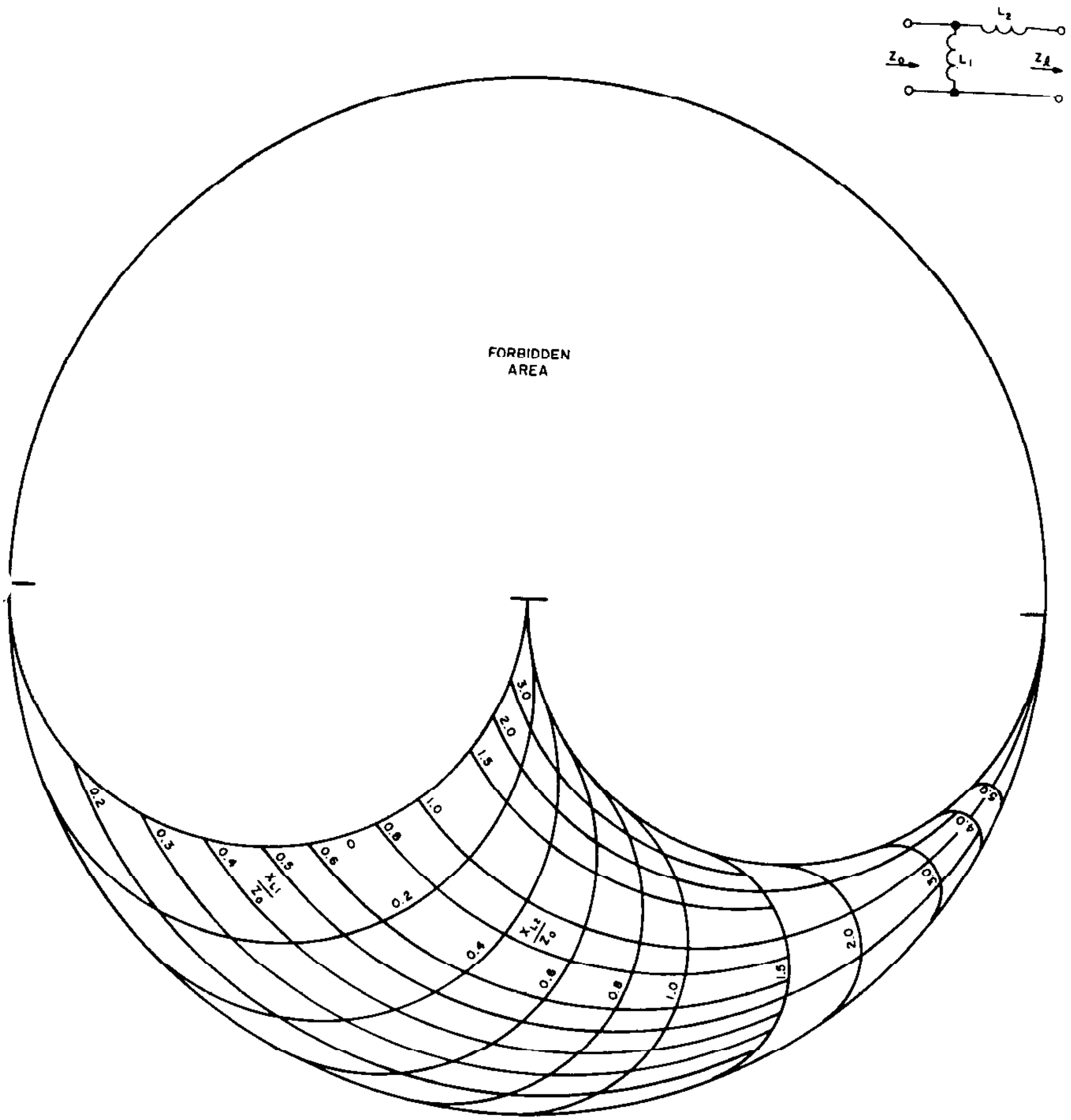

Fig. 10.10 Normalized reactances of *L*-circuit elements required to transform Z_l to Z_0 (overlay for Charts A, B, or C, in cover envelope).

From the foregoing we may write

$$Z_0 = 0$$
$$Z = (20 \text{ to } 80) + j(0 \text{ to } 60) \text{ ohms}$$
$$= (0.20Z_0 \text{ to } 0.80Z_0)$$
$$+ j(0 \text{ to } 0.60Z_0) \text{ ohms}$$

From Figs. 10.3 to 10.10 and SMITH CHART A select a diagram upon which the above "block" of impedance values all fall within a transformable (unshaded) area. The L-type circuit of Fig. 10.6 is found to be the only suitable circuit for this case.

(b) Determine the limiting reactance values of each of the two circuit elements.

Solution:

On SMITH CHART A outline the above range of impedance values, then superimpose this chart on the design curves of Fig. 10.6. Determine the limiting values for X_L and X_C from the curves which just touch the edges of the outlined area. The following limiting values will be observed:

$$X_L = 0.5Z_0 \text{ to } 2.0Z_0 = 50 \text{ to } 200 \text{ ohms}$$
$$X_C = 0.5Z_0 \text{ to } 1.1Z_0 = 50 \text{ to } 110 \text{ ohms}$$

10.2 *T*-TYPE MATCHING CIRCUITS

By the addition of a third reactance element in series with the chosen input resistance obtained with an L-type impedance matching circuit, thus forming a T-type circuit, any complex load impedance value can be transformed to any desired complex input impedance value. The overlay charts of Figs. 10.3 to 10.10 inclusive are applicable in this case also. The reactance required in the third element depends upon the value of input reactance desired. If a circuit is chosen which already includes a series reactance element in the input side, such as one of the circuits shown in diagrams a, b, e, and f of Fig. 10.1, the "third" reactance required would be combined algebraically with the former, resulting in a single net reactance value in this position.

10.3 BALANCED *L*- OR BALANCED *T*-TYPE CIRCUITS

If the input impedance of a matching circuit must be balanced with respect to ground, the L-type circuit design curves can be used to design a suitable impedance matching circuit by treating the problem as an unbalanced one. The required series reactance obtained from the diagrams is simply divided into two parts, each having one-half of the value called for on the charts. These two halves of the necessary total series reactance are then connected in series with each side of the circuit to preserve the balanced-to-ground arrangement.