Dynamic Ductile Fracture with Uintah

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Fracture in 4340 steel.

Approaches.
  - Particle state change.
  - Maximum stress and J-integral.
  - Strong discontinuity.
  - Extended FEM.
  - Cohesive zone.

Level sets.

Remarks.
Examples of Fracture in Steel

Ductile fracture - Void Coalescence

Fragmentation - Adiabatic Shear Bands - Void Coalescence
Fracture in 1-D

**Quasistatic.**

**Temperature-Dependence.**

- Strain at fracture depends on strain-rate.
- Fracture depends on temperature (not obvious from plots).

High Strain Rate.
Simulating Fracture

- Voids + Fracture Strain ≡ TEPLA.
  \[
  (f/f_c)^2 + \left(\frac{\varepsilon_p}{\varepsilon_p^f}\right)^2 = 1
  \]
  (Johnson and Addessio, 1988, J. Appl. Phys., 64(12), 6699-6712).

- Loss of Hyperbolicity.
  \[c = \sqrt{E/\rho}\]
  Negative E  \(\implies\) Imaginary wave speed.

  - In 3D  \(\det(n \cdot M \cdot n + n \cdot \sigma \cdot (n_1)) \leq 0\)  where  \(M\) is the incremental tangent modulus tensor, and  \(n\) is the normal to the localization band.

Current Approach

■ **Identify failed particle.**

■ **Modify particle state.**
  - Zero stress.
  - Allow only compressive stresses.
  - Change particle material but keep state. Allow particle to interact with others via contact.
  - Remove particle mass.

■ **Problems?**
  - No clear cut fracture surface.
  - Mesh dependent.
  - Not easily justified.

■ **Advantage:** Appears to get the job done (to some extent).
Reasonable Predictions?

Unsymmetrical heating

Symmetric heating
Asymmetric impact on edge notched plate (ZRR expt.).

(Zhou, Rosakis, Ravichandran, 1996, JMPS, 44(6), p. 981.)

Simulation of ZRR experiment with Uintah.

We don’t get the right fracture behavior at the notch tip.
Option 1: CRAMP

**Crack Initiation.**
- Identify initial crack using failure criteria.
- Represent crack using separate velocity fields.
- Translate crack using center of mass velocity.
- Crack contact based on change in volume.
- Crack propagation using J-integral $\rightarrow$ Stress Intensity factor

Multiple Crack Initiation.

- Cracks have to be separated by at least two cells.
- No crack branching allowed.
- Cracks may not originate from particles (contact).
- Volume based contact fails for edge cracks.
- For plastic materials, strain energy release rate and stress concentration factors from J-integral not straightforward.
Option 2: Strong Discontinuity

- Velocity field.
  \[ \mathbf{v}(\mathbf{X}, t) = \mathbf{\bar{v}}(\mathbf{X}, t) + [\mathbf{v}] \mathcal{H}(\mathbf{X}, t) \]

- Deformation gradient.
  \[ \mathbf{F}(\mathbf{X}, t) = \mathbf{\bar{F}} + ([\varphi] \otimes \mathbf{N}) \delta \Gamma \]
  \[ = (\mathbf{1} + ([\varphi] \otimes \mathbf{\bar{n}}) \delta \Gamma) \bullet \mathbf{\bar{F}} \]

- Assume: Continuum model can predict failure and persists at failure.

- Tractions are bounded and rates are regular.

- Local softening and evolution rules derived from the continuum equations.

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Kinematics

(Simo et al., 1993, Computational Mech., 12, p. 277; Armero and Garikipati, 1996, IJSS, 33(20), p. 2863; Regueiro and Borja, 1999, FEAD, 33, p. 283; Oliver et al., 2002, EFM, 69, p. 113; Oliver et al., 2003, IJNME, 56, p. 1051)
Strong Discontinuity: FEM

\[ \mathbf{v}^e(X, t) = \mathbf{\bar{v}}^e(X, t) + [\mathbf{v}]^e(t) \mathcal{M}^e_{\Gamma}(X) \]

Mesh

Shape Functions (\( \mathcal{M}^e_{\Gamma}(X) \))
Is this really a crack tip?

- Conceptually simple and elegant.
- MPM needs to be re-worked to fit the approach.
- Void nucleation and growth must be incorporated into the constitutive model.
- Unclear how interaction with fluids can be done.
Option 3: Extended FEM

\[ u^h = \sum_{i=1}^{10} N_i u_i \]

\[ u^h = \sum_{i=1}^{8} N_i u_i + N_{11} u_{11} + \mathcal{H}(x) N_{11} b_{11} \]

\[ b_{11} = \frac{u_9 - u_{10}}{2} \]

General Form:

\[ u^h = \sum_i N_i u_i + \mathcal{H}(x) \sum_j N_j b_j \]

(Belytschko and Black, 1999, IJNME, 45, p. 601; Moes et al., 1999, IJNME, 46, p. 131.)
Extended FEM: Crack Tip

- Asymptotic crack tip displacements (linear elastic)

\[ F(r, \theta) \equiv \{ \sqrt{r} \sin(\theta/2), \]
\[ \sqrt{r} \cos(\theta/2), \]
\[ \sqrt{r} \sin(\theta/2) \sin(\theta), \]
\[ \sqrt{r} \cos(\theta/2) \sin(\theta) \} \]

- General Form:

\[ u^h = \sum_i N_i u_i + \mathcal{H}(x) \sum_j N_j b_j + \]
\[ \sum_k N_k \left( \sum_l c_k^l F_l(x) \right) \]

- Mode I and mode II only.
Extended FEM: Issues

- Governing equations are not modified.
- Asymptotic crack tip displacement unknown for rate-dependent plastic materials.
- Enrichment fields can get complicated for multiple cracks.
- Fluid-structure interaction has to be incorporated into enrichment (no node separation).
- Moving meshes (think MPM) need enriched nodes to be calculated every timestep.
Option 4: Cohesive Zone

The Fracture Process (Cohesive) Zone

(Dugdale, 1960, JMPS, 8, p. 100; Barenblatt, 1962, AAM, 7, p. 56)
Cohesive Laws

Normal traction-separation laws.

Shear traction-separation laws.
• Tractions are calculated along element boundaries.
• A maximum traction criterion is tested and nodes are split if necessary.
  ○ For extended FEM, the zero traction requirement along crack faces is replaced with a non-zero traction.
  ○ Extended FEM does not require the cohesive law to be applicable only along element boundaries.
• A radial return algorithm is used to determine the separation.
• A crack growth criterion is needed.
• MPM implementation will be similar to that for XFEM

(Belytschko et al., 2003, IJNME, 58, p. 1873)
• The cracks are segments of a family of curves within the vector field $T$ of tangents to the cracks $S_i$.

• The curves are level contours of an appropriate function $\theta(x)$ (i.e., $\theta(x) = \theta_{S_i}$).

\[ S_i = \{ x \in \Omega | \theta(x) = \theta_{S_i} \} \]
Computing $\theta(x)$

- **Goal:** Find a function $\theta(x)$ whose level sets are curves of the vector field $T$.
- **Governing Equation:** $T \cdot \nabla \theta = \frac{\partial \theta}{\partial T} = 0$ in $\Omega$.
- **Equivalent Boundary Value Problem:** Find $\theta(x)$ such that
  \[
  \nabla \cdot q = 0 \quad \forall \ x \in \Omega \\
  q = -K \cdot \nabla \theta \quad \forall \ x \in \Omega \\
  q \cdot n = 0 \quad \forall \ x \in \partial q \Omega \\
  \theta = \hat{\theta} \quad \forall \ x \in \partial \theta \Omega
  \]
- The problem is equivalent to steady state heat conduction and can be solved on the grid.
- The direction of the crack is incorporated in $K$.

\[
K = T \otimes T \equiv \begin{bmatrix}
T_x T_x & T_x T_y \\
T_x T_y & T_y T_y
\end{bmatrix}
\]
Locating the crack

- Locate the root material point $r_i$ for each crack.
- Interpolate the $\theta_i$ from the grid to this particle. This is the value of $\theta_{S_i}$.
- The crack can now be located inside any grid cell using the sign change of $\theta_{S_i} - \theta_k$ where $k$ is a node index.
- The points of intersection with the grid can be obtained by interpolation.
Level Set Issues

- Normal to crack easily computed.
- Need two level sets if crack tips are to be tracked.
- Need more level sets if there is crack branching.
- Does CRAMP benefit in any way?
- Alternative approaches exist:
  - Advection type equation:
    \[ \theta_t + F |\nabla \theta| = 0 \]
  - \( \theta(x) \) is a minimum distance function (from crack)
  - Needs certain computations in reference configuration.
Conclusions/Future Work

- **CRAMP.**
  - Multiple velocity field and crack contact can be used.
  - Crack propagation needs to be replaced.
  - Could use level sets to track crack.

- **Strong discontinuity.**
  - Needs reformulation in MPM to reflect change in continuum description.
  - Fits into MPM paradigm.
  - Can’t do much at the crack tip.

- **Extended FEM.**
  - Crack enrichment functions need to be rethought with CRAMP in mind.
  - Easy to add cohesive laws at crack tip.
Need the best from each approach and a way for cracks to see fluids.
Questions?