LES LECTURE ...

- Start with presentation schedule
- Last Time we started talking about Particle models.

* These are models that track individual fluid particle elements (passive tracers).

* They are commonly used for scalar dispersion studies.

* majority of development in context of RANS

- fluid elements are tracked based on their velocity.

$$\Rightarrow \quad \frac{d\vec{x}}{dt} = \vec{u}$$

in RANS we then use $\vec{u} = \langle \vec{u} \rangle + u'$

The goal of any particle model is to specify u'

- <u>Langevin models</u> we discussed two types of models. Langevin and Random displacement.

? what is the main difference?

- Basic idea, we assume the particle trajectory follows a Markov process ~~and can be characterized by a~~

Markov process? ⇒ given current condition future and past are independent.

⇒ result particles move in a "smooth" but non-differentiable ~~path~~ path.

<u>Langevin model</u> follows the Langevin Equation

Wiener process

$$\circledast \quad dv_i = a_i(\vec{x}, \vec{u}, t) + b_{ij}(\vec{x}, \vec{u}, t)\, dW_j$$

"white"

$$\langle dW_i(t)\, dW_j(t+\tau) \rangle = \delta_{ij}\, \delta(\tau)\, dt\, d\tau \Rightarrow$$ ~~white~~ Gaussian noise uncorrelated with other components and in time

* Typically

$a_i \equiv$ drift term

$b_{ij} \equiv$ Random term or diffusion term (recall def of Weiner Process)

* Recall that (X) is a stochastic differential equation to find our position we need to solve for du_i and then integrate in time.

⇒ Specifying the terms.

The random term is usually the easiest to specify: b_{ij}

- The Lagrangian Structure function is of the form

$$\langle dv_i \, dv_j \rangle = \delta_{ij} C_0 \varepsilon \, dt$$

constant!

for inertial subrange turbulence (recall K41 discussion at beginning of class)

- using this we can determine that to have a diffusion term consistent with K41 we need.

$$b_{ij} = \sqrt{C_0 \varepsilon} \, \delta_{ij}$$

⇒ determined by universal small scale properties and independent of the large scales.

* for a_i (drift term) the specification is a little more difficult.

- To find a_i we can use what Thomson (1987) called the well-mixed criteria.

idea: particles that are well mixed should stay well mixed.

⇒ Thomson showed that this was equivalent to saying that the Langevin equation should be consistent with the Fokker-Planck equation

$$\frac{\partial a_i P_E}{\partial u_i} = -\frac{\partial P_E}{\partial t} - \frac{\partial u_i P_E}{\partial x_i} + \frac{1}{2} C_0 \varepsilon \frac{\partial^2 P_E}{\partial u_i \partial u_j}$$

P_E → Eulerian velocity PDF

- This is the Eulerian version of the Langevin equation and is basically the same as Popes Generalized Langevin Equation we saw when we talked about the velocity PDF equation.

- Ie our criteria is that statistics should be the same in a Lagrangian or Eulerian Ref. frame.

- Unfortunately, there isn't a general solution for a_i in more than 1 direction except for isotropic turbulence.

- for stationary isotropic turbulence with the assumption that P_E is a Gaussian ie

$$P_E = \frac{1}{(2\pi)^{3/2}(\det\langle \tau_{ij} \rangle)^{1/2}} \exp\left[-\frac{1}{2}(u_i - \langle u_i \rangle)\langle \lambda_{ij} \rangle (u_j - \langle u_j \rangle)\right]$$

$\langle \lambda_{ij} \rangle$ → inverse of $\langle \tau_{ij} \rangle$

we get

$$dU_i = \underbrace{-\frac{C_0 \varepsilon}{2\sigma_u^2} U_i}_{a_i} dt + \sqrt{C_0 \varepsilon}\, dW_i$$

σ_u → standard dev.

for isotropic but non-stationary (grid turbulence) we have

$$a_i = \left(-\frac{1}{2}\frac{C_0 \varepsilon}{\sigma_u^2} - \frac{1}{\sigma_u}\frac{\partial \sigma_u}{\partial t}\right) U_i$$

for the general 3D case with P_E still a Gaussian Thomson found the most general form of a_i to be,

$$a_i = -\frac{C_0\varepsilon}{2}\langle\lambda_{ik}\rangle(u_k - \langle u_k\rangle) + \frac{\phi_i}{P_E}$$

where

$$\frac{\phi_i}{P_E} = \frac{1}{2}\frac{\partial\langle\tau_{il}\rangle}{\partial x_l} + \frac{\partial\langle u_i\rangle}{\partial t} + \langle u_l\rangle\frac{\partial\langle u_i\rangle}{\partial x_l} +$$

$$\left[\frac{1}{2}\langle\lambda_{lj}\rangle\left(\frac{\partial\langle\tau_{il}\rangle}{\partial t} + \langle u_m\rangle\frac{\partial\langle\tau_{il}\rangle}{\partial x_m}\right) + \frac{\partial\langle u_i\rangle}{\partial x_j}\right](u_j - \langle u_j\rangle)$$

$$+ \frac{1}{2}\langle\lambda_{lj}\rangle\frac{\partial\langle\tau_{il}\rangle}{\partial x_k}(u_j - \langle u_j\rangle)(u_k - \langle u_k\rangle)$$

(See Rodean 1996 for derivation)

- So what about LES?
- Several researchers starting with Lamb (1978) have use LES with Lag. part. models.

idea: break the velocity up to get:

$$\underbrace{\vec{u}_L}_{\text{Lagrangian}} = \underbrace{\vec{u}_R}_{\text{Resolved}} + \underbrace{\vec{u}_S}_{\text{SFS}} \quad \text{and use in } \frac{d\vec{x}}{dt} = \vec{u}_L$$

- So far really only 1 paper has examined this issue specifically for LES (how to formulate u_s)

Weil et al (2004)

re formulated the model by replacing $\langle u_i \rangle$ with u_r

and noting that ~~u_s~~ $u_i - u_r = u_s \rightarrow$ SFS

and using the instantaneous τ_{ij}

with this

$$\frac{\phi_i}{\rho_f} = \frac{1}{2}\frac{\partial \tau_{il}}{\partial x_l} + \frac{\partial u_{ri}}{\partial t} + \frac{1}{2}\lambda_{lj}\frac{\partial \tau_{il}}{\partial t}u_{sj}$$

and ~~f~~ the eqn for this our SFS eq velocity is

$$du_{si} = -\frac{f_s C_0 \epsilon}{2}\lambda_{ik}u_{sk}\,dt + \frac{1}{2}\left(\lambda_{lj}\frac{\partial \tau_{il}}{\partial t}u_{sj} + \frac{\partial \tau_{il}}{\partial x_l}\right)dt + (f_s C_0 \epsilon)^{1/2}\,dW_i$$