

# LES of Turbulent Flows: Lecture 12

## (ME EN 7960-008)

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# Turbulence modeling (alternative strategies)

- **Filtered Density Functions (FDF)**: Colucci et al., (Phys. of Fluids, 1998)

- In this method, the **evolution of the filtered probability density functions** is solved for (i.e., we solve for the evolution of the SFS general moments)

- Similar to general PDF transport methods 1<sup>st</sup> introduced by Lundgren (Phys. of Fluid, 1969) and outlined in detail in Pope chapter 12.

- Many applications use FDF for scalars in turbulent reacting flows while traditional (low-pass filtered N-S) equations are solved for momentum. For a more extensive discussion see Fox, 2002.

- This type of method is often employed for LES with Lagrangian particle models and for chemically reactive flows. In **Lagrangian particle models it leads to a form of the Langevin equations for SFS particle evolution** and in **chemically reactive flows** it has the advantage that the **reactions occur in closed form**

- We will return to these type of methods later in the class when we discuss combining LES with particle models.

# LES and numerical methods

- LES requires that the filtered equations of motion (see Lectures 5 and 6) be solved on a numerical grid.
- **In LES we need to accurately represent high wavenumber turbulent fluctuations** (small scale turbulence) this means either:
  - we use high-order schemes (e.g., spectral methods)
  - we use fine grids with low-order schemes (e.g., 2<sup>nd</sup> order central differences)
- High-order schemes are more expensive but for a given mesh they are more accurate (see Pope 571-579 for a discussion of resolving filtered fields)
- Low-order finite difference (or volume) schemes provide flexibility of geometry but give rise to complications when modeling small scales motions
- For **low-order FD schemes truncation errors can be on the order of SGS contributions** unless  $\Delta$  is considerably larger than the grid spacing (see Ghosal J. Comp. Phys, 1996)
- Note that although 2<sup>nd</sup> order schemes may have undesirable truncation errors (with respect to SGS model terms), **even order schemes are non-dissipative while SGS models (on average) are purely dissipative**, therefore, all hope is not lost!
- **The same is not true for dissipative schemes** common in compressible flow solutions For example in upwind schemes and TVD or FLS schemes (see Leveque 1992 for a review of this type of numerics)

# LES and numerical methods

- The filter applied in LES can be either **implicit** or **explicit**

-**Implicit filtering**: The grid (or numerical basis) is assumed to be the LES low-pass filter

- **Pro**: takes full advantage of the numerical grid resolution
- **Cons**: for some methods it is helpful to know the shape of the LES filter (this can be difficult to determine for some numerical methods). Truncation error (see above) can also become an issue.

-**Explicit filtering**: A filter (typically box or Gaussian) is applied to the numerically grid (i.e., explicitly to the discretized N-S equations)

- **Pros**: truncation error is reduced and the filter shape is well defined
- **Cons**: loss of resolution. The total simulation time goes up as  $\Delta_g^4$  (where  $\Delta_g$  is the grid spacing) so maintaining the same space resolution as an implicit filter with  $\Delta_g/\Delta=1/2$  will take  $2^4=16$  more grid points.

- For reviews of LES and numerics see Guerts chapter 5 and Sagaut chapter 8.2 and 8.3

# Subgrid-Scale Modeling

- One of the major hurdles to making LES a reliable tool for engineering and environmental applications is the formulation of SGS models and the specification of model coefficients.

- **Recall:** we can define 3 different “scale regions” in LES depicted in the figure to the right =>

-Resolved Scales, Resolved SFS, SGSs

- We can also decompose a general variable as:

$$\phi = \tilde{\phi} + \phi'$$

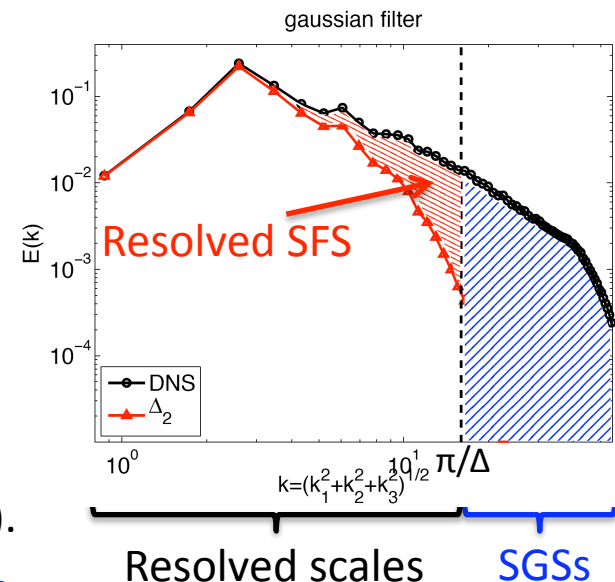
- When we talk about SGS models we are specifically talking about the scales below  $\Delta$  **NOT** the Resolved SFS.

- We will specifically discuss the Resolved SFSs when we talk about filter reconstruction later on (time permitting).

- Also, here we will focus on LES with explicit SGS models.

- A class of LES referred to as Implicit LES (ILES) also exists.

- ILES was 1<sup>st</sup> developed for compressible flow. It assumes the SGSs are purely dissipative and act in a similar way to dissipative numerical schemes (in general ILES uses monotonicity preserving numerical schemes). See handout Grinstein\_etal\_2007\_ch2.pdf for details.



# Modeling $\tau_{ij}$

- see Pope pgs. 582-583 or Sagaut pgs. 49-50, 59-60 (this will mostly follow Sagaut)
- we can decompose the nonlinear term as follows (using  $\phi = \tilde{\phi} + \phi'$ ):

$$\begin{aligned}\widetilde{u_i u_j} &= \overline{(\tilde{u}_i + u'_i)(\tilde{u}_j + u'_j)} \\ &= \widetilde{\tilde{u}_i \tilde{u}_j} + \widetilde{\tilde{u}_i u'_j} + \widetilde{\tilde{u}_j u'_i} + \widetilde{u'_i u'_j}\end{aligned}$$

- we now have the nonlinear term as a function of  $\tilde{u}_i$  and  $u'_i$ .
- two different basic forms of the decomposition (based on the above equation are prevalent)

-The 1<sup>st</sup> one is based on the idea that all terms appearing in the evolution of a filtered quantity should be filtered:

$$\tau_{ij} = C_{ij} + R_{ij} = \widetilde{u_i u_j} - \widetilde{\tilde{u}_i \tilde{u}_j}$$

where  $C_{ij} = \widetilde{\tilde{u}_i u'_j} + \widetilde{\tilde{u}_j u'_i} \Rightarrow$  interaction between resolved and SFSs

and  $R_{ij} = \widetilde{u'_i u'_j} \Rightarrow$  SFS “Reynold’s” stress

## Modeling $\tau_{ij}$ (continued)

-A 2<sup>nd</sup> definition can be obtained by further decomposition of  $\widetilde{\widetilde{u_i u_j}} \Rightarrow$

$$\widetilde{\widetilde{u_i u_j}} = \underbrace{\left( \widetilde{\widetilde{u_i u_j}} - \widetilde{u_i u_j} \right)}_{L_{ij}} + \widetilde{u_i u_j}$$

$L_{ij} \Rightarrow$  Leonard stress (the interaction among the smallest resolved scales)

Our total decomposition is now:

$$\tau_{ij} = L_{ij} + C_{ij} + R_{ij} = \widetilde{\widetilde{u_i u_j}} - \widetilde{u_i u_j}$$

(Leonard, Adv. in Geo., 1974)

- If our filter is a Reynolds operator (e.g., cutoff filter)  **$C_{ij}$  and  $L_{ij}$  vanish!**
- **NOTE:** while  $\tau_{ij}$  is invariant to Galilean transformations,  $L_{ij}$  and  $C_{ij}$  are NOT. Because of this, the decomposition given above (for the most part) is not used anymore (although we will see similar terms again in our SGS models)
- **A more rigorous decomposition was proposed by Germano into generalized filtered moments.** Under this framework the generalized moments look just like “Reynolds” moments, our original  $\tau_{ij}$ ! (see Germano, JFM, 1992 in the handouts section. This is also recommended reading for a discussion of filtering and the relationship between LES filters and Reynolds operators)