LES of Turbulent Flows: Lecture 9 (ME EN 7960-008)

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Homework #2

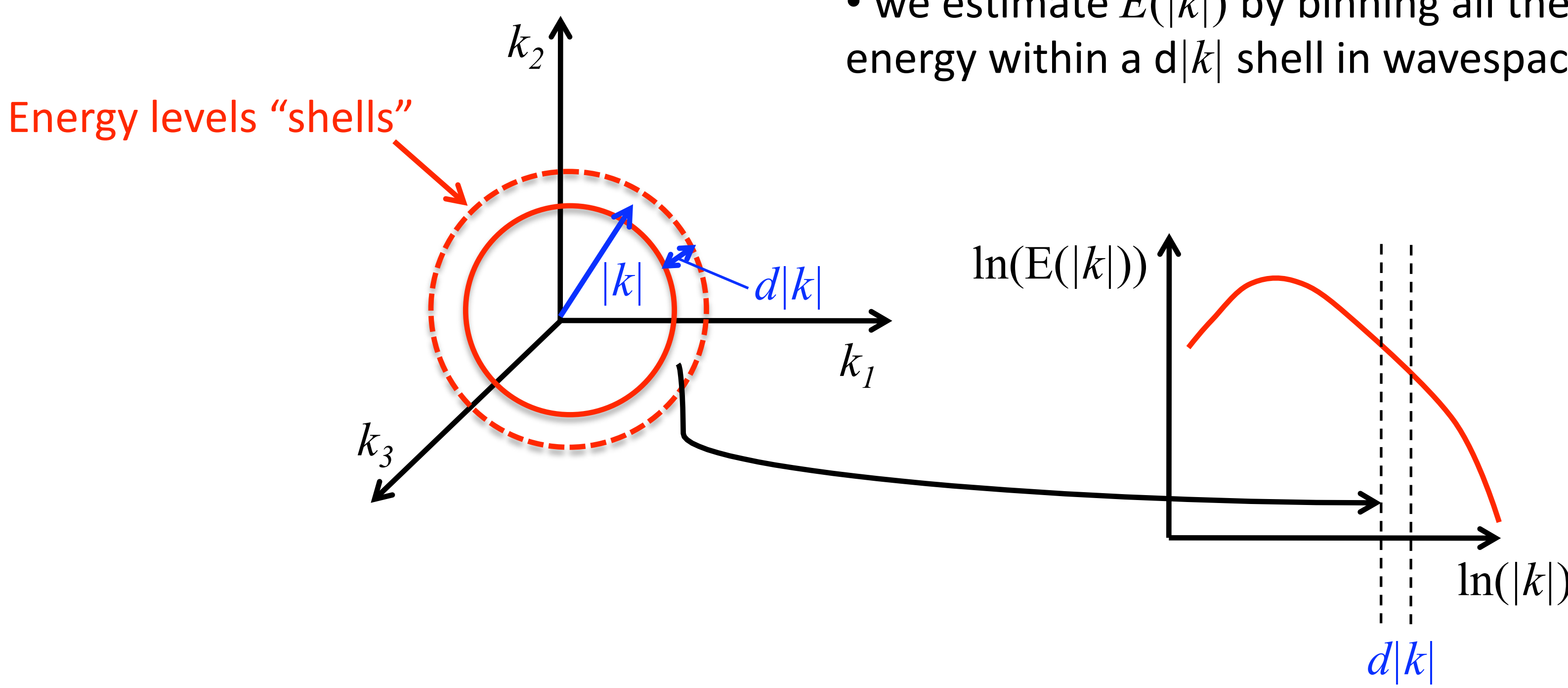
• Goal of homework assignment is to calculate 3D energy spectrum from isotropic data and to perform 3D filtering on a 3D isotropic turbulence dataset

• **Calculating the 3D energy spectrum**

• plot: $E(|k|)$ vs. $|k|=[k_1^2+k_1^2+k_1^2]^{½}$

• $E(|k|) = |\hat{u}_{\bar{k}}|^2 + |\hat{v}_{\bar{k}}|^2 + |\hat{w}_{\bar{k}}|^2$

• we estimate $E(|k|)$ by binning all the energy within a d$|k|$ shell in wavespace

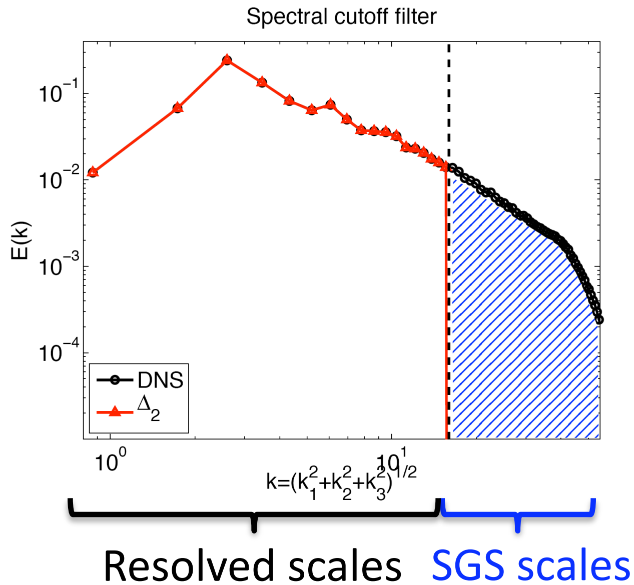


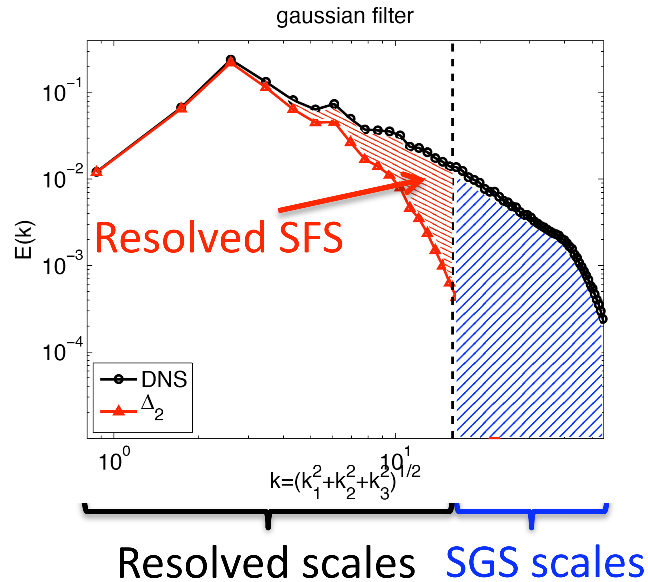
Decomposition of Turbulence for real filters

The LES filter can be used to decompose the velocity field into resolved and subfilter scale (SFS) components

$$\phi(\vec{x},t) = \widetilde{\phi}(\vec{x},t) + \phi'(\vec{x},t)$$

We can use our filtered DNS fields to look at how the choice of our filter kernel affects this separation in wavespace





The Gaussian filter (or box filter) does not have as compact of support in wavespace as the cutoff filter. This results in attenuation of energy at scales larger than the filter scale. The scales affected by this attenuation are referred to as **Resolved SFSs**.

Filtering the incompressible N-S equations

- What happens when we apply one of the above filters to the N-S equations?

-**Conservation of Mass:**

filtering both sides of the conservation of mass: $\widetilde{\frac{\partial u_i}{\partial x_i}} = 0 \Rightarrow \frac{\partial \tilde{u}_i}{\partial x_i} = 0$

where we have used the property of LES filters => $\frac{\widetilde{\partial \phi}}{\partial \vec{x}} = \frac{\partial \widetilde{\phi}}{\partial \vec{x}}$ and (~) denotes the filtering operation.

-Conservation of Momentum:

Using the filter properties $\widetilde{a} = a$, $\widetilde{\phi + \zeta} = \widetilde{\phi} + \widetilde{\zeta}$ and $\frac{\widetilde{\partial \phi}}{\partial \vec{x}} = \frac{\partial \widetilde{\phi}}{\partial \vec{x}}$ we can write the momentum equation as:

$$\frac{\partial \tilde{u}_i}{\partial t} + \frac{\partial \widetilde{u_i u_j}}{\partial x_j} = -\frac{\partial \tilde{p}}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 \tilde{u}_i}{\partial x_j^2} + F_i$$

The 2nd term on the LHS (convective term) now contains the unknown $\widetilde{u_i u_j}$ we can rewrite this term to obtain the standard LES equations for incompressible flow

Filtering the incompressible N-S equations

We can add and subtract $\tilde{u}_i\tilde{u}_j$ from the convective term:

$$\frac{\partial \widetilde{u_i u_j}}{\partial x_j} = \frac{\partial\left(\widetilde{u_i u_j} + \tilde{u}_i\tilde{u}_j - \tilde{u}_i\tilde{u}_j\right)}{\partial x_j} = \frac{\partial \tilde{u}_i\tilde{u}_j}{\partial x_j} + \frac{\partial\left(\widetilde{u_i u_j} - \tilde{u}_i\tilde{u}_j\right)}{\partial x_j}$$

Putting this back in the momentum equation and rearranging we have

$$\frac{\partial \tilde{u}_i}{\partial t} + \frac{\partial \tilde{u}_i\tilde{u}_j}{\partial x_j} = -\frac{\partial \tilde{p}}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 \tilde{u}_i}{\partial x_j^2} - \frac{\partial \tau_{ij}}{\partial x_j} + F_i$$

SFS force vector

where $\tau_{ij} = \widetilde{u_i u_j} - \tilde{u}_i\tilde{u}_j$ is the **subfilter scale (SFS) stress tensor**

- For the **scalar concentration** equation we can go through a similar process to obtain:

$$\frac{\partial \tilde{\theta}}{\partial t} + \frac{\partial \tilde{u}_i\tilde{\theta}}{\partial x_i} = \frac{1}{Sc\,\mathrm{Re}}\frac{\partial^2 \tilde{\theta}}{\partial x_i^2} - \frac{\partial q_i}{\partial x_i} + Q$$

Where $q_j = \widetilde{u_j\theta} - \tilde{u}_j\tilde{\theta}$ is the **SFS flux**