LES of Turbulent Flows: Lecture 2 (ME EN 7960-008)

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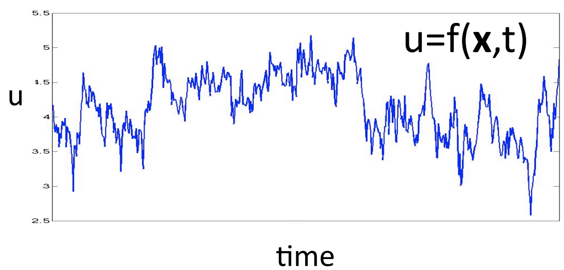
Turbulent Flow Properties

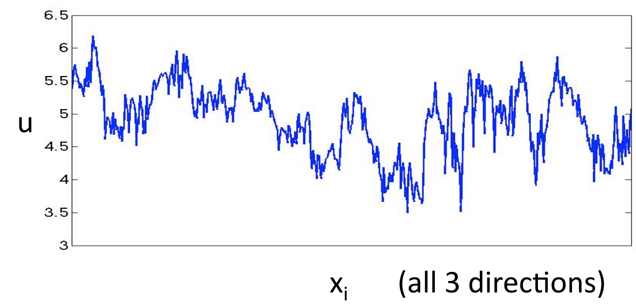
- Why study turbulence? Most real flows in engineering applications are turbulent.

Properties of Turbulent Flows:

1. Unsteadiness:



2. 3-dimensional



3. Vortex stretching → mechanism to increase the intensity of turbulence
   (we can measure the intensity of turbulence with the turbulence intensity => $\frac{\sigma_u}{\langle u \rangle}$)

   Vorticity: $\omega = \vec{\nabla} \times \vec{u}$ or $\omega_k = \epsilon_{ijk} \frac{\partial}{\partial x_i} u_j \hat{e}_k$

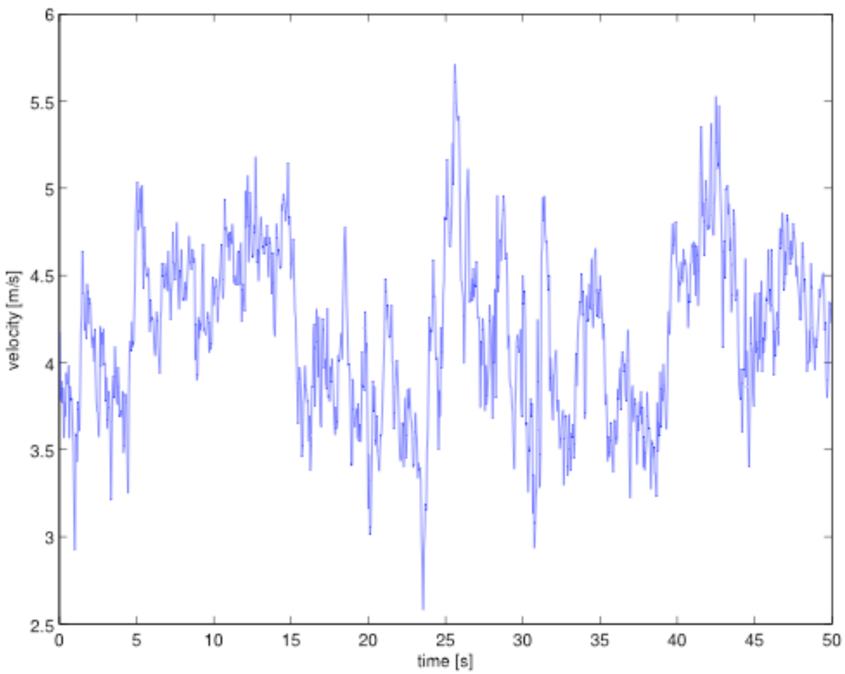
4. Mixing effect:
   Turbulence mixes quantities with the result that gradients are reduced (e.g. pollutants, chemicals, velocity components, etc.). This lowers the concentration of harmful scalars but increases drag.

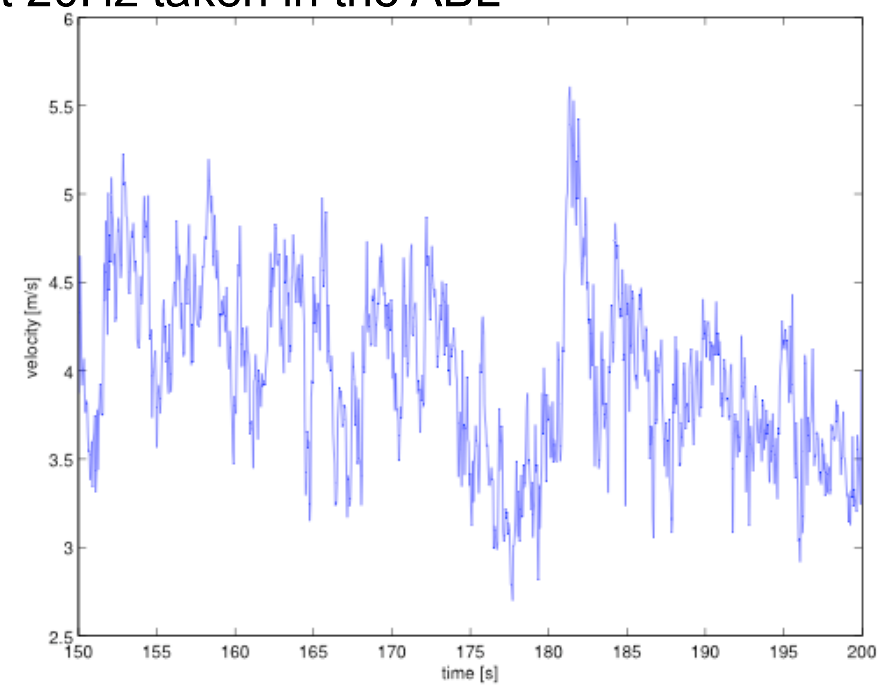
Velocity Series

• A common property in turbulent flow is their random nature
• Pope (2000) notes that using the term “random” means nothing more than that an event may or may not occur (it says nothing about the nature of the event)

• Lets examine the velocity fields given below:

-sonic anemometer data at 20Hz taken in the ABL-



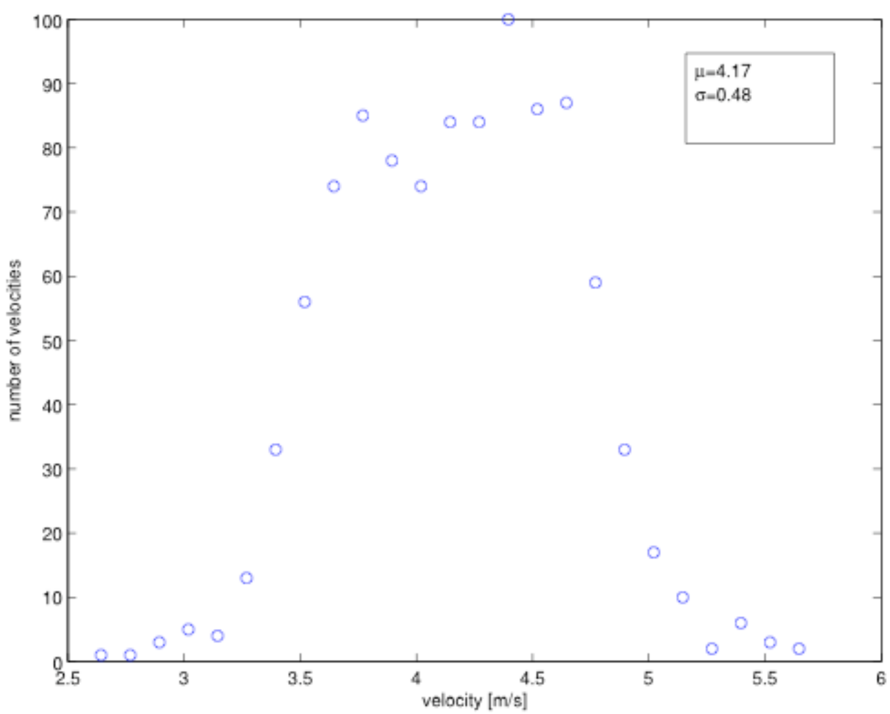


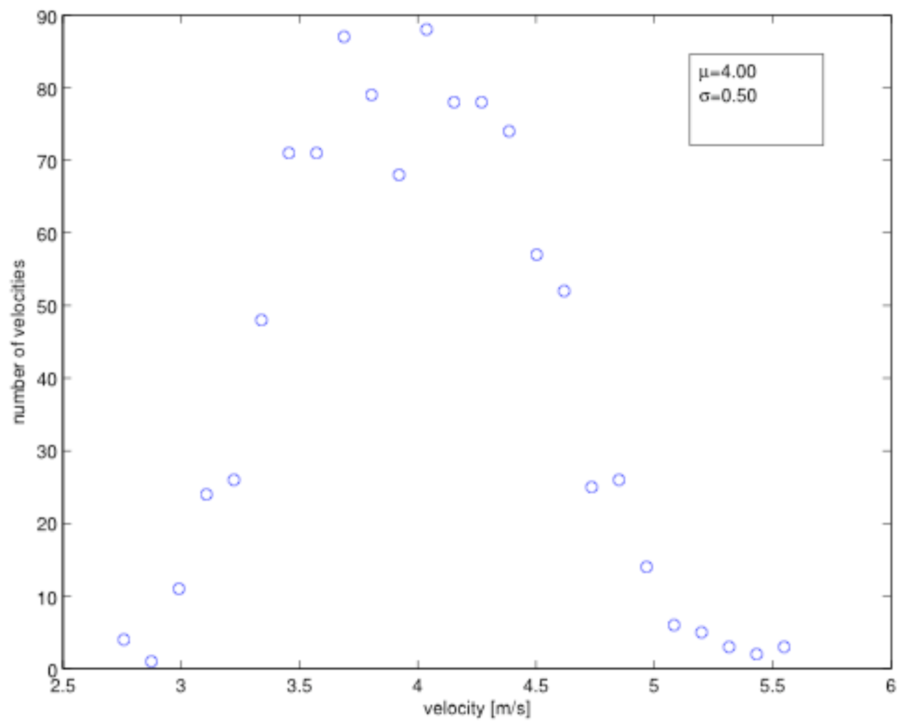
Velocity Series

- We can observe 3 things from these velocity fields:
  1. The signal is highly disorganized and has structure on a wide range of scales (that is also disorganized).
     - Examine the figure on the previous page, notice the small (fast) changes verse the longer timescale changes that appear in no certain order.
  2. The signal appears unpredictable
     - Compare the left plot with that on the right (taken ~100 sec later) basic aspects are the same but the details are completely different and from looking at the left signal it is impossible to predict the right signal.
  3. Some of the properties of the signal appear to be reproducible
     - The reproducible property isn't as obvious from the signal. Instead we need to look at the histogram on the next page.

Velocity Histograms

Notice that the histograms are similar with similar means and standard deviations.





The Random Velocity Field

- The random behavior observed in the time series can appear to contradict what we know about fluids from classical mechanics.
- The Navier-Stokes equations (more later) are a determanistic set of equations (they give us an exact mathematical description of the evolution of a Newtonian fluid).
- Question: Why the randomness?
    1. In any turbulent flow we have unavoidable perturbations in initial conditions, boundary conditions, forcing etc.
    2. Turbulent flows (and the Navier-Stokes equations) show an acute sensitivity to these perturbations
- This sensitivity to initial conditions has been explored extensively from the viewpoint of dynamical system (referred to many times as chaos theory) starting with the work on atmospheric turbulence and atmospheric predictability by Lorenz (1963).

Statistical Tools for Turbulent Flow

- A consequence of the random behavior of turbulence and the fact that it is the histogram that appears to be reproducible is that turbulence is that it is usually studied from a statistical viewpoint.

Probability:

Some event (value) V_b in the space V (e.g., our sample velocity field)

$$P = P(B) = P\{U < V_b\} \quad \text{for event } B \equiv \{U < V_b\}$$

this is the probablity (likely-hood) that U is les than V_b where P=0 means there is no chance and P=1 means we have certainty.

Cumulative density function (cdf):

$$F(V) \equiv P\{U < V\} \text{ where for a specific event } P(B) = P\{U < V_b\} = F(V_b)$$

$$\text{and if we have } P(C) \equiv P\{V_a \le U < V_b\} = F(V_b) - F(V_a)$$

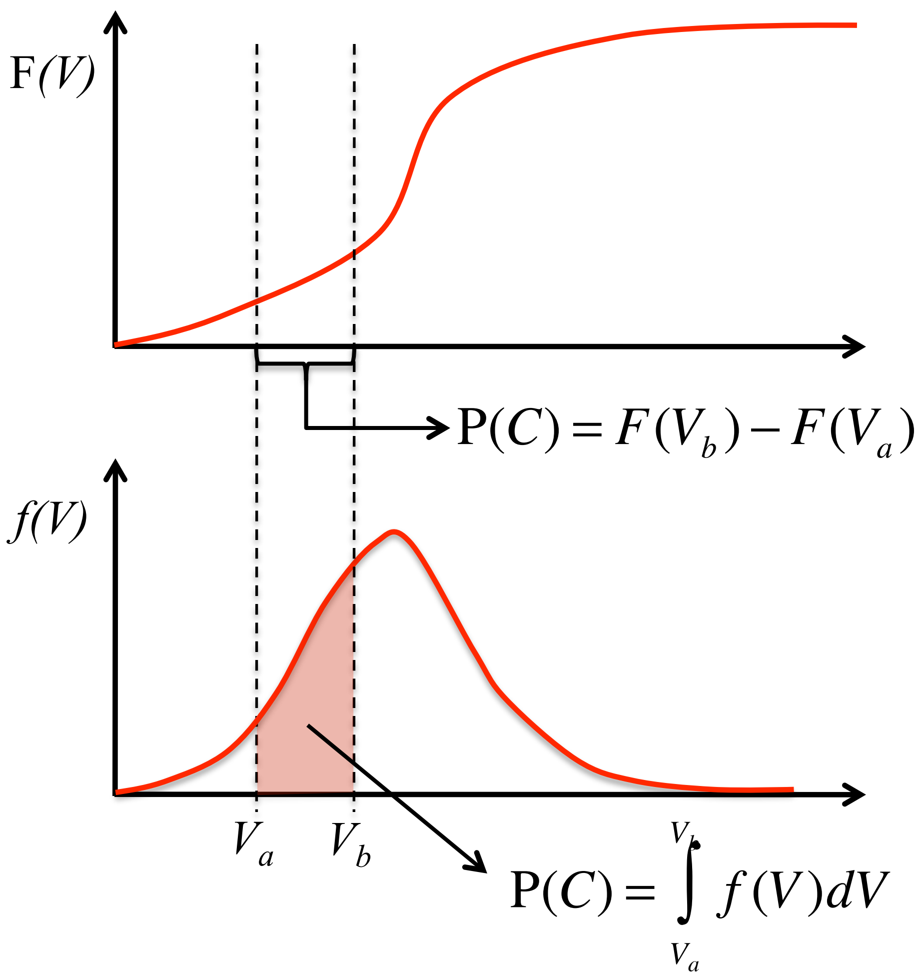
$$\text{this is bounded by: } \mathrm{F}(-\infty) = 0 \ (U < -\infty \text{ is impossible})$$

$$\mathrm{F}(\infty) = 1 \ (U < \infty \text{ is certain})$$

We also know that since P is non negative, F is non a non decreasing function

Statistical Tools for Turbulent Flow

pdf vs cdf



Means and Moments

- The pdf fully defines the statistics of a signal (random variable)
- If two signals have the same pdf, they are considered to be statistically identical
- We can also define a signal by its individual stats that collectively describe the pdf

  - <u>The mean (or expected value)</u>

$$\langle U \rangle \equiv \int_{-\infty}^{\infty} V f(V)\, dV \;\text{ in a discrete form we have }\; \frac{1}{N}\sum_{i=1}^{N} V_i$$

- The mean is the probability weighted sum of all possible values
- In general for and $Q(U)$ ➔ something that is a function of U

$$\langle Q(U) \rangle \equiv \int_{-\infty}^{\infty} Q(V) f(V)\, dV$$

and from this equation we can show that for constants a and b:

$$\langle aQ(U) + bR(U) \rangle = a\langle Q(U) \rangle + b\langle R(U) \rangle$$

Means and Moments

- We can also define a fluctuation from the mean by

$$u \equiv U - \langle U \rangle$$

- The variance is then the mean square fluctuation

$$\sigma_u^2 = \mathrm{var}(U) = \langle u^2 \rangle \equiv \int_{-\infty}^{\infty} \left(V - \langle U \rangle\right)^2 f(V)\,dV \ \text{ discretely } \ \sigma_u^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(V_i - \langle U \rangle\right)^2$$

- And the standard deviation (or rms) is simply the root of the variance

$$\sigma_u = sdev(U) = \langle u^2 \rangle^{1/2}$$

- We can define the n^{th} central moment as:

$$\mu_n \equiv \langle u^n \rangle = \int_{-\infty}^{\infty} \left(V - \langle U \rangle\right)^n f(V)\,dV$$

- Many times we prefer to express variables as standardized random variables

$$\hat{U} \equiv \frac{U - \langle U \rangle}{\sigma_u} \ \text{ a centered and scaled variable}$$

- The standardized moments are then

$$\hat{\mu}_n \equiv \frac{\mu_n}{\sigma_u^n} = \int_{-\infty}^{\infty} \hat{V}^n \hat{f}(\hat{V})\,d\hat{V} \ \text{ with } \hat{\mu}_0 = 1, \hat{\mu}_1 = 0 \text{ and } \hat{\mu}_2 = 1$$

Means and Moments

- The different moments each describe an aspect of the shape of the pdf

$$\mu_1 \Rightarrow \text{ mean or expected value}$$
$$\mu_2 \Rightarrow \text{ variance}$$
$$\mu_3 \Rightarrow \text{ skewness}$$
$$\mu_4 \Rightarrow \text{ kurtosis (or flatness)}$$

- In basic probability theory we have several different types of pdfs. Pope 3.3 gives a fairly extensive list of these the most important of which is the normal or Gaussian distribution