LES of Turbulent Flows: Lecture 16
(ME EN 7960-003)

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Statistical Conditions for a SGS model

- What conditions should a SGS model satisfy?

-Specifically we are interested in answering the question what **statistical** properties should τ_{ij} and τ_{ij}^{mod} share?

-We know a "good" model should adhere to our equations of motion:

- Invariance to translation, rotation, and reflection (in the absence of boundaries)
- Hopefully, invariance to Re
- Ideally, invariant to Δ

-To get more specific than this, we need to talk about **statistics of SGS models** (Meneveau, Physics of Fluids, 1994).

- **To obtain correct 1st and 2nd order moments** of our resolved field, our **model must** at least be able to **produce average modeled stresses** that match the real stresses everywhere.
- **This doesn't guarantee that our 2nd order moments** are correct it is only a **necessary condition**.
- **To produce 2nd order moments**, we need to have our model reproduce 2nd and 3rd order SGS stats including stresses and correlations (e.g. stresses with velocity or gradients). This includes **matching <Π> everywhere**.
- For even higher order moments we need to match higher order SGS stats…

Computing SGS quantities

• Procedurally, **How do we compute these SGS stats from data** (DNS or Experiments)?
Here is a "quick" list, also see the handout Project_apriori_study.pdf on the web.

-Select your data (after quality control) and identify missing velocity or gradient terms

-Separate the data into resolved and SGS scales by calculating $\tilde{u}_i$ and $\widetilde{u_i u_j}$ with an appropriate LES filter (see lecture 4 for the most common examples).

- At this point, a decision must be made: to **down-sample or not** (see Liu et al.,JFM 1994)
  - -Down-sampling means removing points from the field that are separated (spatially) by < our filter scale Δ (denoted by the ~). Effectively this means we keep less points than we started with (e.g. from 128^3 to 32^3) after filtering.
  - -**Pros:** we get a "true" representation of the effect of gradient estimates on our SGS models and avoid enhanced correlations due to filter overlap.
  - -**Cons:** we lose data points (important if we have limited data) and we now need to consider the above gradient estimation errors!

-Calculate <u>**local values**</u> of all the components of $\tau_{ij}^{\Delta} = \widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j$ and $\tilde{S}_{ij} = \frac{1}{2}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right)$ you can (you may need approximations here based on your data!)

-For some models you may need to calculate other parameters (e.g., mixed and nonlinear models) but the general procedure is the same

Computing SGS quantities

-Homework #2 implemented different types of filters

-Once you have these basic quantities calculated you can calculate model values $\tau_{ij}^{\Delta,M}$ and statistics of the actual (from data) and modeled SGS stresses including average values, correlation coefficients and variances (see project handout).

-We can also calculate other SGS statistics like $\langle \Pi^{\Delta} \rangle = -\langle \tau_{ij}^{\Delta} \tilde{S}_{ij} \rangle$ and $\langle \Pi^{\Delta,M} \rangle$ or any model coefficients of interest (see handout for an example).

• The following pages give some examples of SGS statistics and model coefficients calculated form various references (discussed in class).

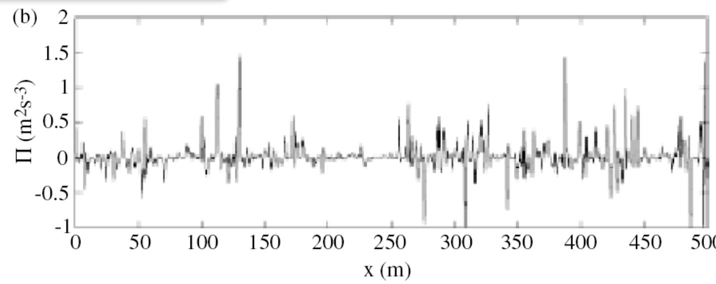
SGS Dissipation

- SGS Energy transfer from experiments in the Utah desert (Carper and Porté-Agel , 2004)

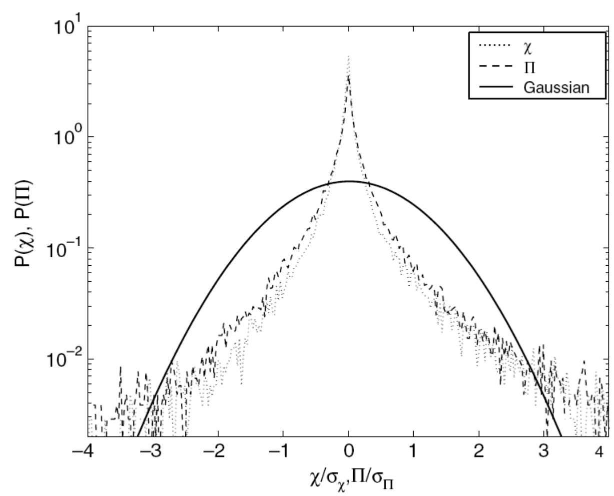
$$\Pi = -\tau_{ij}\tilde{S}_{ij}$$

$\langle\Pi\rangle \times 10^3 = 7.13\ m^2s^{-3}$
$\sigma_\Pi \times 10^2 = 18.02\ m^2s^{-3}$

Experimental setup



Example of time series of Π from the ABL (late afternoon)

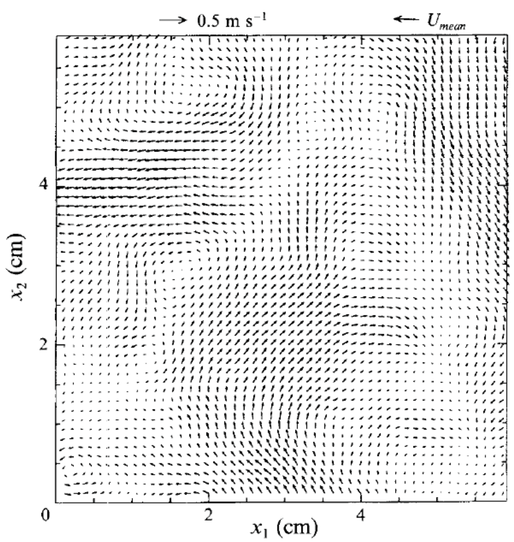


Example PDF of Π from the ABL (late afternoon)

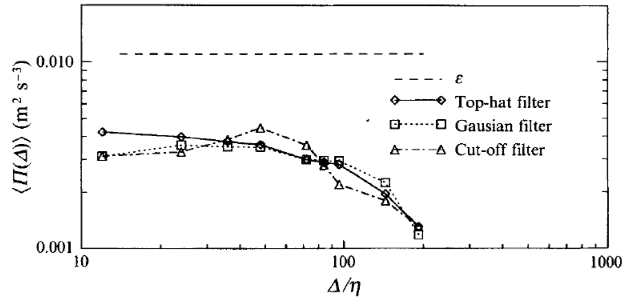
SGS Dissipation

- SGS Energy transfer from wind tunnel experiments in a round jet (Liu et al., 1994)

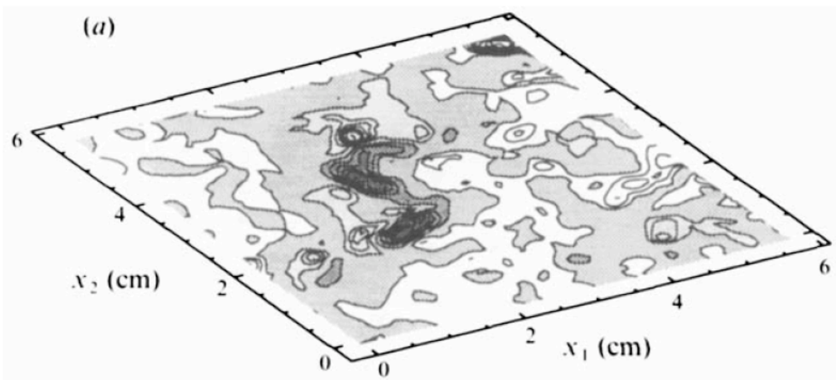
$$\Pi = -\tau_{ij}\tilde{S}_{ij}$$



Top-hat filtered PIV field



Average Π from the wind tunnel experiment compared to molecular dissipation

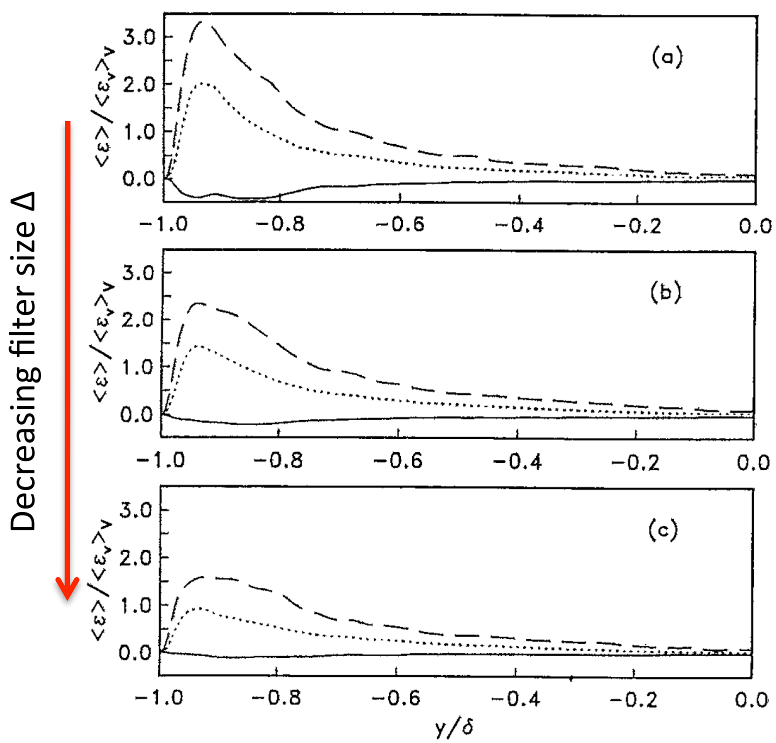


Spatial distribution of Π from PIV

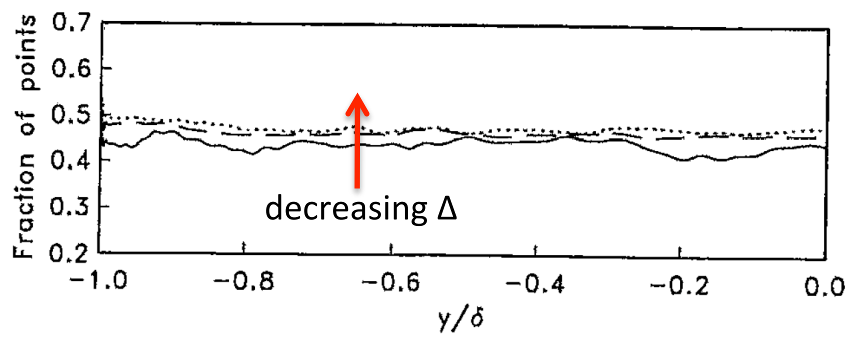
SGS Dissipation

- SGS Energy transfer from DNS of turbulent channel flow Re=3300 (U_c) (Piomelli et al., 1991)

$$\Pi = -\tau_{ij}\tilde{S}_{ij}$$



Π normalized by the total dissipation
___ average; ---- rms and …. backscatter



Fraction of points in channel flow with backscatter for 3 different filter widths

- backscatter increases with Re
- fraction of backscatter points decreases for a Gaussian filter (cutoff results shown) to about 30%.

SGS Model Correlation Coefficients

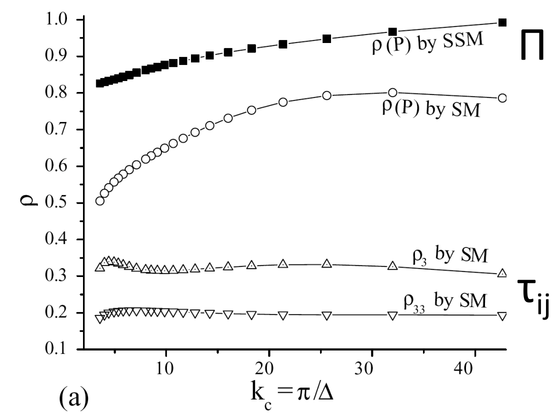
Correlation coefficients from Clark et al, (1979) for different models

Evaluation of subgrid-scale models **13**

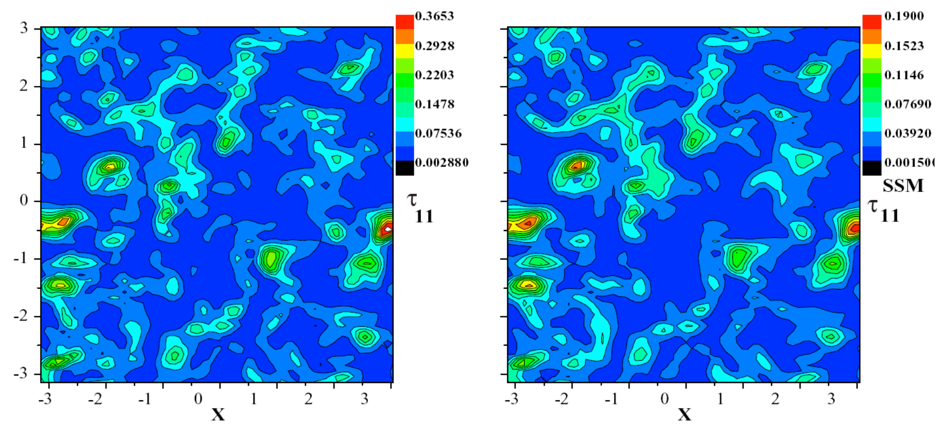
Term	Model	Correlation $\frac{17}{8}\Delta$	Correlation $\frac{9}{8}\Delta$	Model constant $\frac{17}{8}\Delta$	Model constant $\frac{9}{8}\Delta$
τ_{ij} (tensor)	Smagorinsky	0·366	0·277	0·270	0·247
	Vorticity	0·344	0·260	0·294	0·275
	Turbulent kinetic energy	0·363	0·303	0·196	0·175
	Eddy viscosity	0·352	0·295		
$\frac{\partial \tau_{ij}}{\partial x_j}$ (vector)	Smagorinsky	0·425	0·346	0·240	0·264
	Vorticity	0·408	0·327	0·220	0·247
	Turbulent kinetic energy	0·434	0·362	0·138	0·155
	Eddy viscosity	0·426	0·356		
$u_i \frac{\partial \tau_{ij}}{\partial x_j}$ (scalar)	Smagorinsky	0·710	0·580	0·186	0·171
	Vorticity	0·700	0·582	0·202	0·191
	Turbulent kinetic energy	0·723	0·606	0·085	0·095
	Eddy viscosity	0·716	0·605		

TABLE 2. Summary of correlations between exact subgrid-scale Reynolds stresses and models.

- Eddy-viscosity- $\tau_{ij} = -2\nu_T \tilde{S}_{ij}$
- Smagorinsky- $\nu_T = (C_S \Delta)^2 |\tilde{S}|$
- Kinetic energy- $\nu_T = (C_1 \Delta)\, \tilde{k}_r^{1/2}$
- Vorticity- $\nu_T = (C\Delta)^2 (\omega_i \omega_i)^{1/2}$



Correlation coefficients from Lu et al (2007) for Smagorinsky and Similarity models

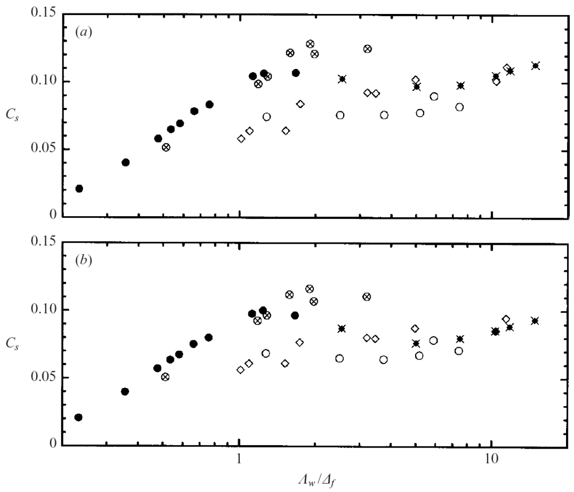


Measured (left) and modeled (right) with the similarity model τ_{11} from Lu et al (2007).

SGS Model Coefficient Estimates

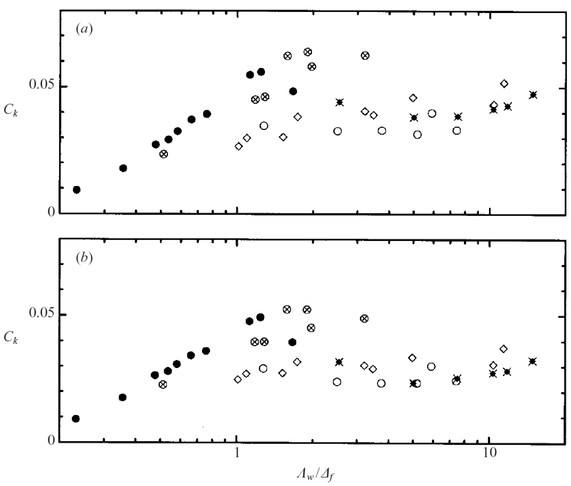
Model coefficients evaluated by matching Π from ABL study of Sullivan et al (2003).

Λ_w is the peak in the w velocity spectra and Λ_f is the filter scale



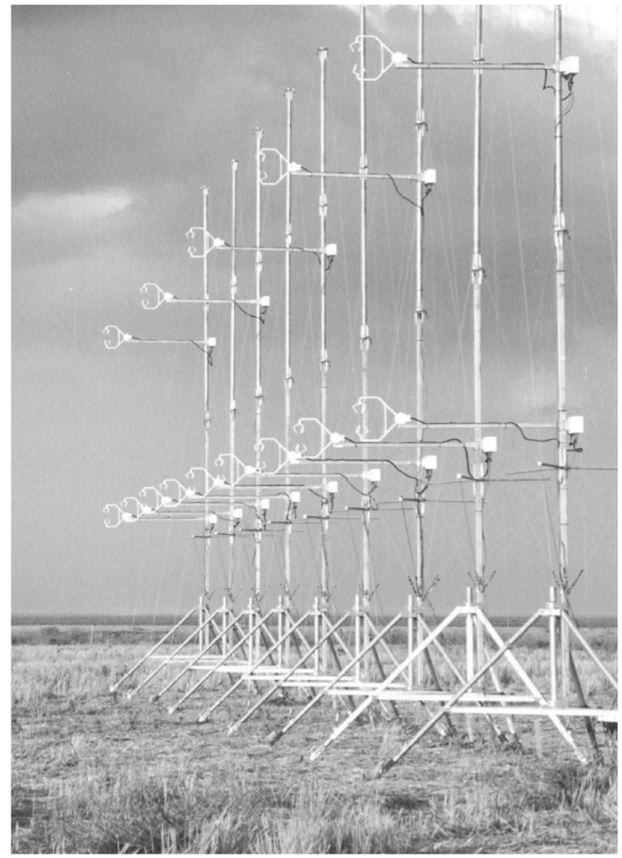
Smagorinsky

Mixed model



Kinetic Energy

Mixed KE



Experimental setup in Colorado

SGS Model Coefficient Estimates

Smagorinsky coefficients with stability (Kleissl et al, 2004)

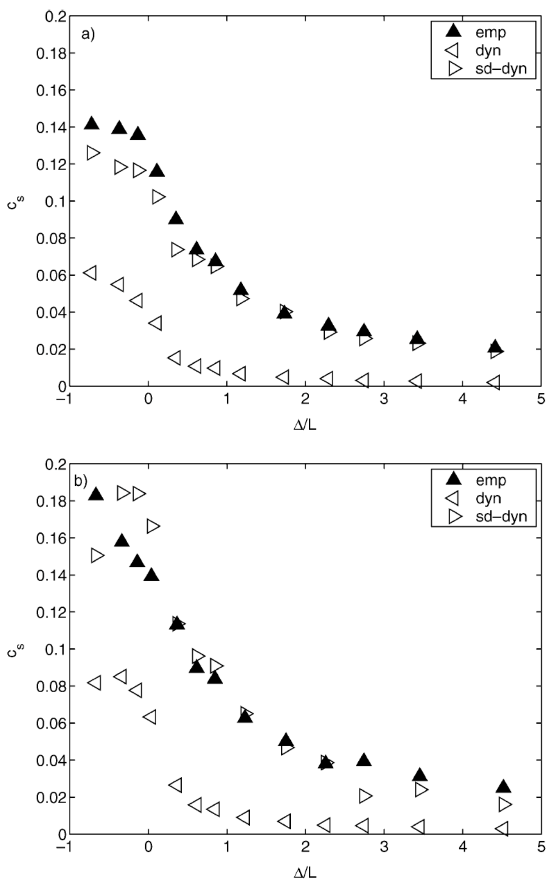
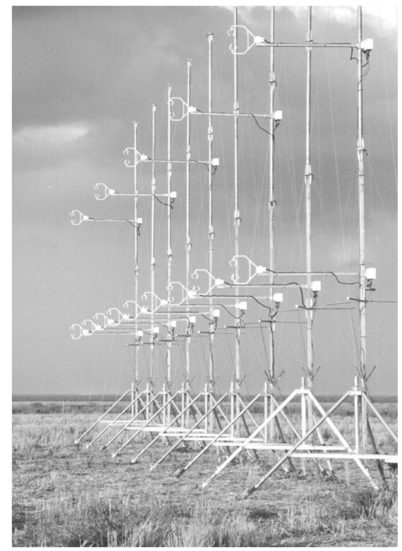
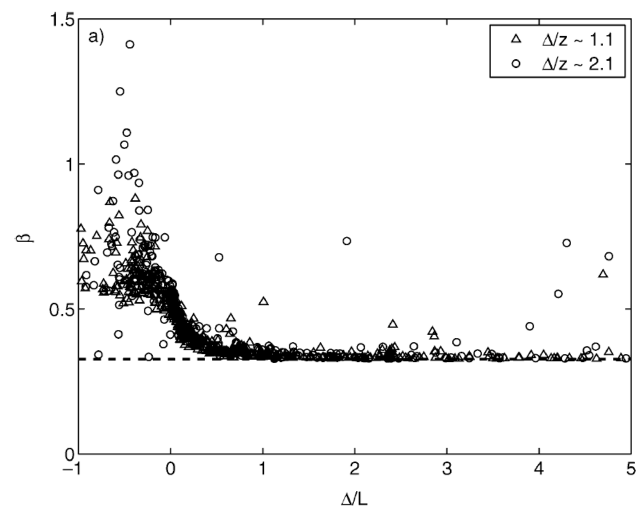


FIG. 14. Smagorinsky coefficient $c_s^{(\Delta)}$ as a function of Δ/L for different SGS models. Variables are averaged over all segments in each stability bin. (a) Array 1, $\Delta/z \sim 2.1$ and (b) array 2, $\Delta/z \sim 1.1$.



Experimental setup in Colorado



$$\frac{C_S^2(2\Delta)}{C_S^2(\Delta)} = \beta$$

SGS Model Coefficient Estimates

Smagorinsky coefficients with stability (Bou-Zeid et al, JFM 2010)

$$q_i^{model} = -k_{SGS}\frac{\partial\widetilde{\theta}}{\partial x_i} = -Pr_{SGS}^{-1}(c_s\Delta)^2\left|\widetilde{S}\right|\frac{\partial\widetilde{\theta}}{\partial x_i}.$$

(*a*) (*b*)

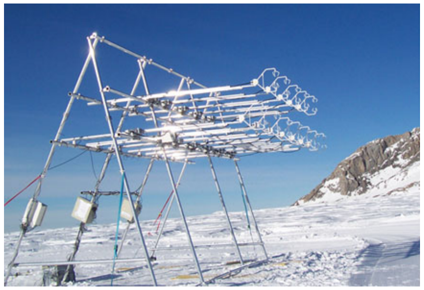


FIGURE 1. SnoHATS: side view of the 12 sonics array (*a*) and the upwind fetch of 1.5 km (*b*).

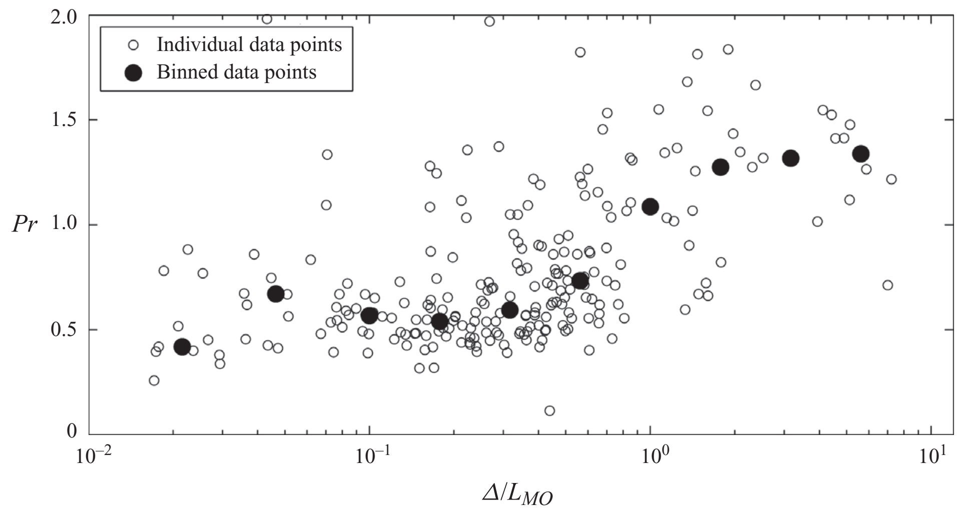


FIGURE 13. Variation of the SGS Prandtl number with the stability parameter based on the Obukhov scale.

SGS Model Coefficient Estimates

Smagorinsky coefficients with stability (Bou-Zeid et al, JFM 2010)

$$q_i^{model} = -k_{SGS}\frac{\partial\tilde{\theta}}{\partial x_i} = -Pr_{SGS}^{-1}(c_s\Delta)^2\left|\tilde{S}\right|\frac{\partial\tilde{\theta}}{\partial x_i}.$$

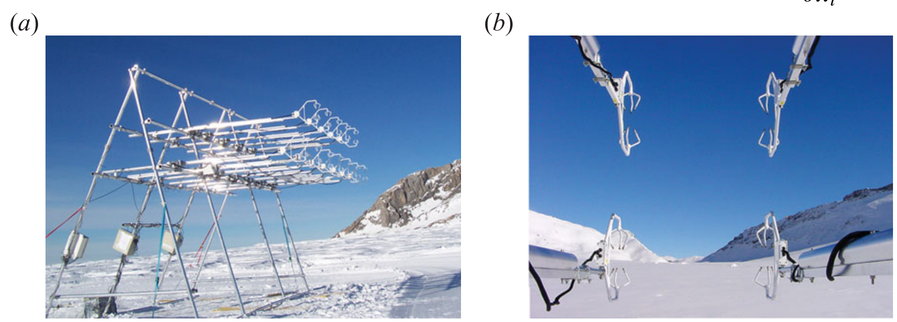


FIGURE 1. SnoHATS: side view of the 12 sonics array (a) and the upwind fetch of 1.5 km (b).

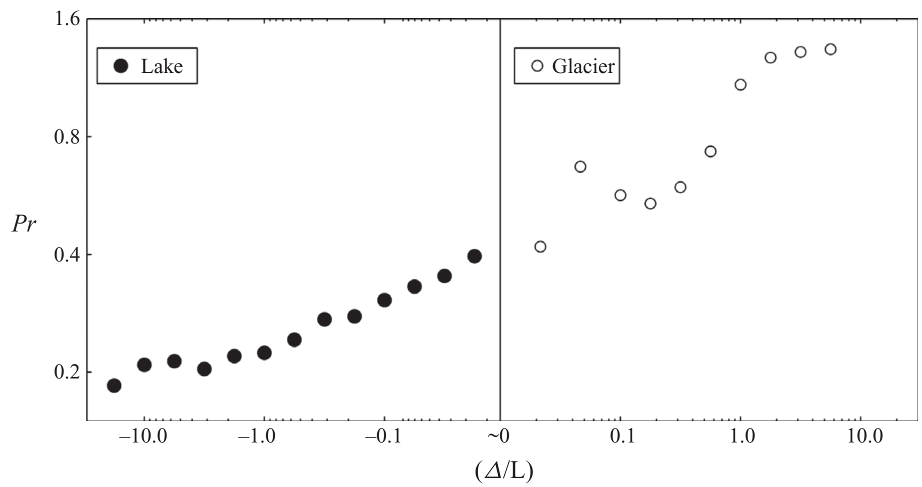
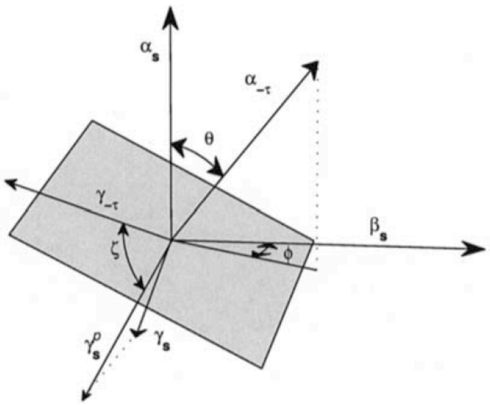


FIGURE 20. Variation of the SGS Prandtl number for unstable and stable conditions (note that both axes are in logarithmic scale).

Geometric Tensor Alignment



Definition of the 3 angles needed to characterize the alignment of 2 tensors (τ_{ij} and S_{ij})

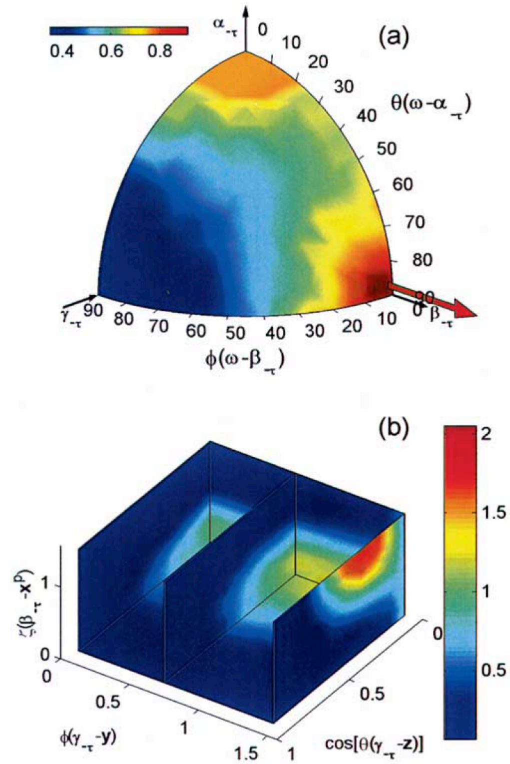
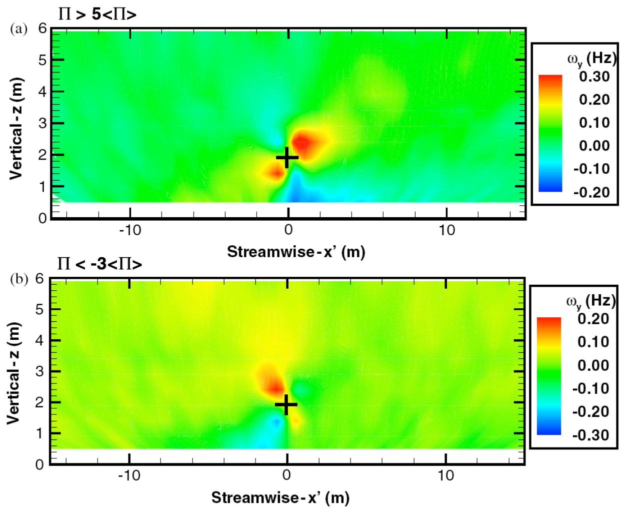
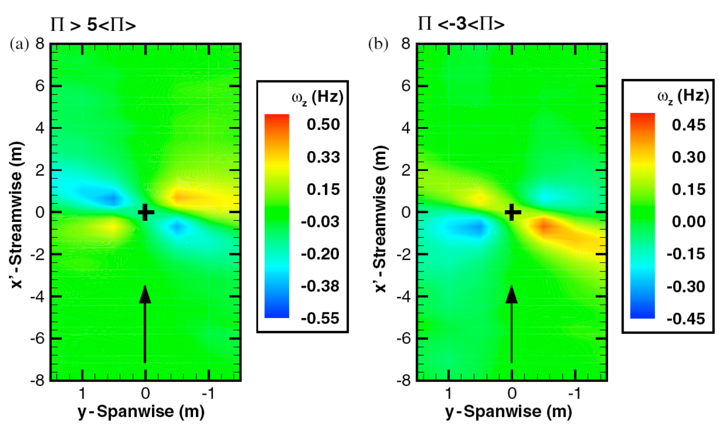


Figure 9. (a) Joint probability density function of two angles describing the orientation of filtered fluctuating vorticity vector in local negative SGS stress-tensor eigensystem. Filter scale is $\Delta = 2$ m. The filtered fluctuating vorticity has a bimodal distribution giving two likely alignment configurations. The primary alignment is between filtered fluctuating vorticity and the intermediate eigendirection of the SGS stress, $\boldsymbol{\beta}_{-\tau}$. The secondary alignment is vorticity and the extensive eigendirection of the negative SGS stress, $\boldsymbol{\alpha}_{-\tau}$. (b) Joint PDF of three angles describing the orientation of SGS stress tensor eigensystem in mean flow frame of reference. Filter scale is $\Delta = 2$ m. The mean flow streamwise direction is x, z is the vertical axis, and y is the transverse horizontal direction. The planes represent slices through the three-dimensional function.

Coherent Structures and SGS models

- SGS and coherent structures in the Utah desert (Carper and Porté-Agel , 2004)





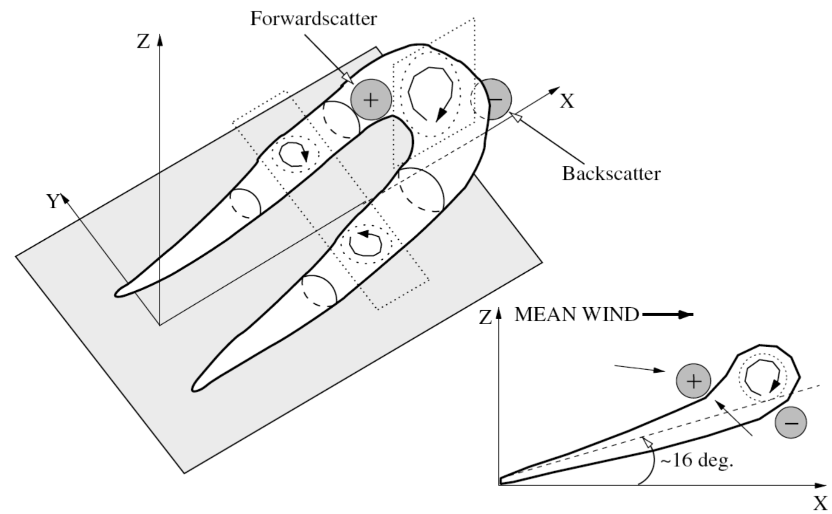


Figure 14. Conceptual model relating strong positive (+) and negative (−) SFS dissipation events to different regions (shaded) around a hairpin-like coherent structure. The solid lines outline an isosurface of vorticity with arrows indicating the direction of rotation. The dotted lines indicate the planes on which the conditionally averaged fields are reported with the key results shown within dashed circles.