LES of Turbulent Flows: Lecture 12
(ME EN 7960-003)

Prof. Rob Stoll
Department of Mechanical Engineering
University of Utah

Fall 2014

More on Similarity models

•Recall from last lecture, Bardina et al. (1980) proposed to use the decomposition of the velocity field into resolved and SFS components to define a new model based on the scales closest to the LES filter scale Δ.

$$\text{using} \quad u_i' \equiv u_i - \tilde{u}_i \Rightarrow \tau_{ij} = \left(\widetilde{\tilde{u}_i \tilde{u}_j} - \tilde{\tilde{u}}_i \tilde{\tilde{u}}_i \right)$$

•Lui et al (JFM, 1994) examined "bands" around Δ and built a scale-similarity model similar to the model of Bardina et al., (1980)

-They argued that energy in the band at one scale larger than Δ (say 2Δ) and one scale smaller (something like ½Δ) would have the largest contribution to τ_{ij} .

-define: $u_i^n = \tilde{u}_i - \bar{u}_i$ where (~) is is a filter at Δ and (–) is a filter at a larger scale 2Δ

- u_i^n can be thought of as the band-pass filtered velocity between Δ and 2Δ. We can do a similar decomposition for u_i^{n+1} and u_i^{n-1}

-With our band-pass filtered decomposition we can build a τ_{ij}^n based on u_i^n and u_i^{n+1} (or any other band)

-For example, the stress one level above *n* can be written using another filter at 4Δ (denoted by a **^**) as:

$$\tau_{ij}^{n-1} = \overline{(\tilde{u}_i - \hat{u}_i)(\tilde{u}_j - \hat{u}_j)} - (\overline{\tilde{u}_i - \hat{u}_i})(\overline{\tilde{u}_j - \hat{u}_j})$$

More on similarity models

•This can be reduced to the following (note that $\hat{u}_i$ is approximately constant to – filter)

$$\tau_{ij}^{n-1} = \left(\overline{\tilde{u}_i\tilde{u}_j} - \bar{\tilde{u}}_i\bar{\tilde{u}}_j\right)$$

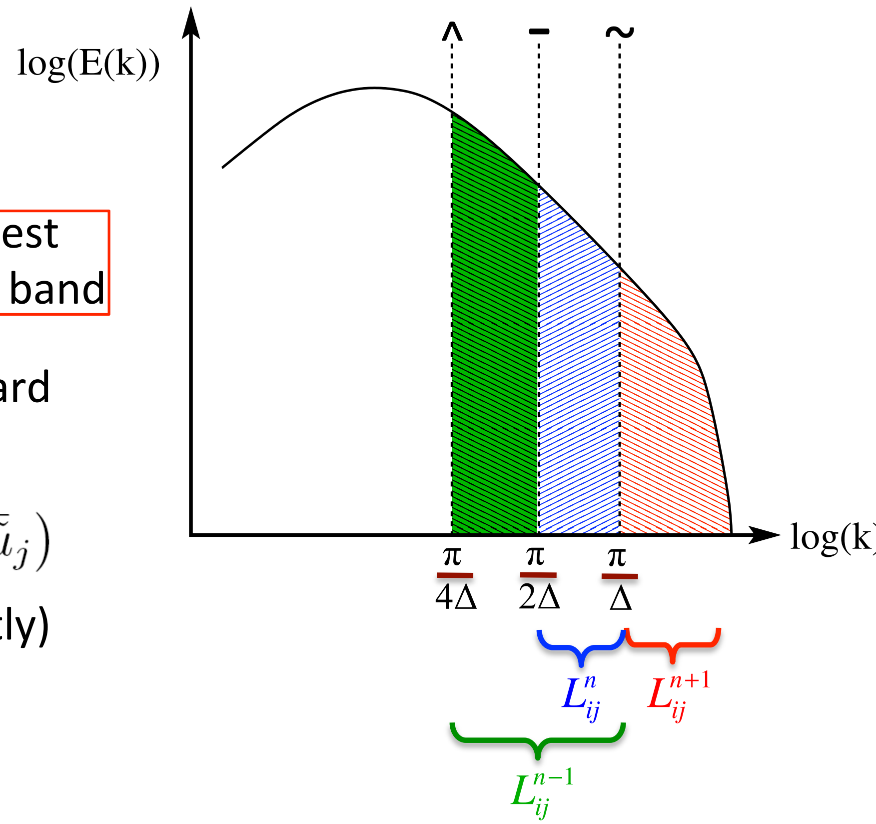
•Lui et al.'s study showed similarity between

$$\tau_{ij}^{n+1} \rightarrow \tau_{ij}^{n} \rightarrow \tau_{ij}^{n-1}$$

1st unresolved band

smallest resolved band

next largest resolved band



• They concluded that because of this the Leonard stress (τ_{ij}^{n-1}) is the best estimate =>

$$\boxed{\tau_{ij} = C_L L_{ij}} \quad \text{where} \quad L_{ij} = \left(\overline{\tilde{u}_i\tilde{u}_j} - \bar{\tilde{u}}_i\bar{\tilde{u}}_j\right)$$

• This is the most commonly used form (currently) of the similarity model.

The Nonlinear Model

• Another form of the similarity model is **the nonlinear model** (also called the Clark model or the gradient model: see Liu et al., JFM 1994)

-idea: approximate $\tilde{u}_i$ by a Taylor series expansion around the "true" mean at a point.

$$\tilde{u}_i(\mathbf{x}) = \bar{\tilde{u}}_i(\mathbf{x_0}) + \tilde{A}_{ik}(\mathbf{x_0})(x_k - x_k^0)$$

where $\tilde{A}_{ik}$ is the filtered gradient tensor $\tilde{A}_{ik} = \dfrac{\partial \tilde{u}_i}{\partial x_k}$

We can use this approximation (Taylor series) to estimate the "resolved" stress $L_{ij} = \overline{\tilde{u}_i \tilde{u}_j} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j$ (more later during discussion of dynamic modeling) and develop another model.

$$\boxed{\tau_{ij} = C_A \Delta^2 \tilde{A}_{ik} \tilde{A}_{jk}}$$

Here we have used the observation that τ_{ij} has a very high correlation with L_{ij}

$$\Rightarrow \tau_{ij} = C_A L_{ij}$$

Mixed Models

- Both the similarity and nonlinear models exhibit a high level of correlation in *a priori* tests with measured values of τ_{ij}^{Δ} but they underestimate the average dissipation and are numerically unstable

- Typically they are combined with an eddy-viscosity model to provide the proper level of dissipation.

    -an example is (Bardina et al, 1980):

$$\tau_{ij} = C_L \left(\overline{\tilde{u}_i \tilde{u}_j} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j \right) - 2 \left(C_S \Delta \right)^2 |\tilde{S}| \tilde{S}_{ij}$$

    -the similarity term has a high level of correlation with τ_{ij} and the eddy-viscosity term provides the appropriate level of dissipation.

- A few notes on mixed models:

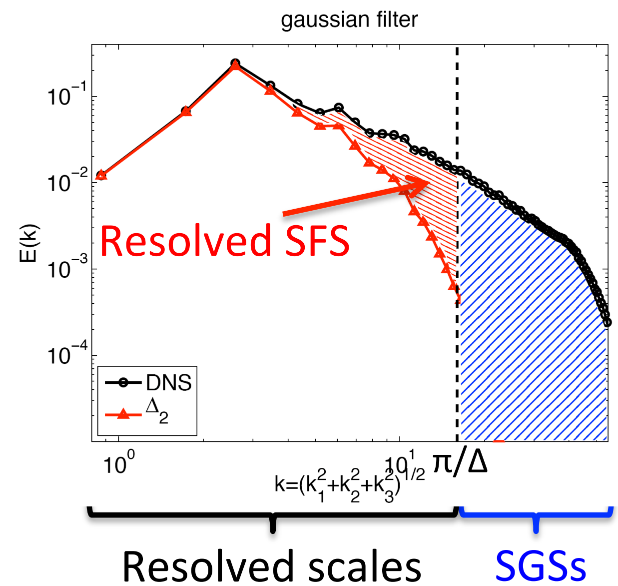
    -Proper justification for the mixed model did not exist at first but a more unified theory has developed in the form of approximate deconvolution or filter reconstruction modeling (Guerts pg 200, Sagaut pg 210)

More on Mixed Models

-Idea: a SGS model should be built from 2 parts, **the first part** accounts for the effect of the filter through an approximate reconstruction of the filter's effect on the velocity field (note the similarity model is a zero-order filter reconstruction). This is the model for the resolved SFS.

-The **second part** accounts for the SGS component of τ_{ij}

-We then assume that we can build τ_{ij} as a linear combination of these two model components.



-A few last notes on Similarity models:

- Bardina et al.'s model is exactly zero for a spectral cutoff filter.

- Lui et al.'s form of the similarity model also fails. This is credited to the nonlocal structure of the cutoff filter. It breaks the central assumption of the similarity model, that the locally τ_{ij} decomposed at different levels is self similar

The Modulated Gradient Model

• A related model of similar form to the nonlinear model is the Modulated Gradient Model (see Lu et al., IJMPC 2008 and Lu and Porté-Agel, BLM 2010)

• We need to improve the magnitude of the estimates of τ_{ij} while keeping the high level of correlation observed for nonlinear (gradient) models.

• Assume: $\tau_{ij} = \tilde{k}_r C_{ij}$ where for consistency $C_{kk} = 2$

• Using our resolved stress $L_{ij} = \overline{\tilde{u}_i \tilde{u}_j} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j$ and the Germano identity (more later) we can show that approximately

$$C_{ij} = 2\left(L_{ij} / L_{kk}\right) \Rightarrow \tau_{ij} = 2\tilde{k}_r \left(L_{ij} / L_{kk}\right)$$

• This model suffers from some drawbacks and (insufficient dissipation at high Re and it isn't material frame indifferent)

• They suggested an improvement by replacing L_{ij} with $\tilde{A}_{ik}$

$$\tau_{ij} = 2\tilde{k}_r \left(\tilde{A}_{ij} / \tilde{A}_{kk}\right) \text{ note we can also use}$$

$$\tilde{G}_{ij} = \frac{\Delta_x^2}{12}\frac{\partial \tilde{u}_i}{\partial x}\frac{\partial \tilde{u}_j}{\partial x} + \frac{\Delta_y^2}{12}\frac{\partial \tilde{u}_i}{\partial y}\frac{\partial \tilde{u}_j}{\partial y} + \frac{\Delta_z^2}{12}\frac{\partial \tilde{u}_i}{\partial z}\frac{\partial \tilde{u}_j}{\partial z}$$