LES of Turbulent Flows: Lecture 11
(ME EN 7960-003)

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1-Equation Eddy viscosity Models

- Evolution of τ_{ij}: Deardorff (ASME J. Fluids Eng., 1973)

  -Deardorff (1973) suggested that the evolution of τ_{ij} should be modeled (see Sagaut pg. 243 for equations)

  -To close this model, equations for the components of τ_{ij} and for $\tilde{k}_r$ (SGS kinetic energy) are both needed (similar to the a 2nd-order RANS closure, see Speziale, ARFM, 1991 for a review of RANS closures including 2nd-order models)

  -Deardorff (BLM, 1980) suggested a simpler 1-equation approach to avoid the need to solve prognostic equations for τ_{ij} (in addition to the N-S equations). This model is equivalent to a k-*l* ,1-equation RANS model (see Speziale, 1991)

  -This was also proposed earlier for the isotropic part of a 2-part eddy-viscosity model by Schumann (J. Comp. Physics, 1975) although the model is usually credited to Deardorff (especially in the atmospheric community).

1-Equation Eddy viscosity Models

- The model takes the eddy-viscosity as: (see Sagaut pg. 128 or Guerts pg. 227)

$$\nu_T = (C_1 \Delta)\, \tilde{k}_r^{1/2}$$

our length scale l^* is $C_1 \Delta$ (same as Smagorinsky) and the velocity scale u^* is $\tilde{k}_r^{1/2}$

so that our model is now: $\boxed{\tau_{ij} = -2\,(C_1 \Delta)\, \tilde{k}_r^{1/2} \tilde{S}_{ij}}$

and $\tilde{k}_r$ is found from an SGS kinetic energy equation of the form:

$$\frac{\partial \tilde{k}_r}{\partial t} = -\frac{\partial \left(\tilde{u}_i \tilde{k}_r\right)}{\partial x_j} + \Pi - \epsilon + \frac{\partial}{\partial x_j}\left(\frac{C_1}{C_2} \Delta \tilde{k}_r^{1/2} \frac{\partial \tilde{k}_r}{\partial x_j}\right) + \frac{1}{Re}\frac{\partial^2 \tilde{k}_r}{\partial x_j^2}$$

-viscous dissipation is typically modeled based on isotropy assumption as $\epsilon \approx \frac{C_k}{\Delta} \tilde{k}_r^{3/2}$

-C_2 is an order 1 constant and C_k is the Kolmogorov constant ≈ 1.7

-Note the transport terms (for pressure and SGS kinetic energy) have been modeled by the 4th term on the RHS of the equation.

-This model is popular in the ABL due to the ability to include SGS transport or energy drain effects as extra parameters in the SGS TKE equation (e.g. for SGS canopy drag, bouyancy forces etc.)

2-point Eddy viscosity Models

- **2-point eddy-viscosity model**: (Metais and Lesieur, JFM, 1992)
 (see Sagaut pg 124 or Lesieur et al., 2005 "Large-Eddy Simulation of Turbulence")

-This model is an attempt to go beyond the Smagorinsky model while keeping, in physical space, the same scaling as the spectral eddy-viscosity model of Kriachnan (JAS, 1976).

-Idea: in physical space build an eddy-viscosity normalized by

$$\sqrt{E_{\vec{x}}(k_c)/k_c} \quad \text{with} \quad k_c = \frac{\pi}{\Delta}$$

and where $E_{\vec{x}}(k_c)$ is the local kinetic energy spectrum at point $\vec{x}$.

-$E_{\vec{x}}(k_c)$ must be evaluated in terms of physical space quantities. The best candidate for this is the **2nd-order structure function**:

$$F^{iso}(r) = \langle [\vec{u}(\vec{x},t) - \vec{u}(\vec{x}+\vec{r},t)]^2 \rangle$$

-Note, the isotropic 2nd-order structure function spectrum (Fourier transform) is equivalent to the the Kolmogorov $k^{-5/3}$ energy spectrum (see Pope sections 6.2 and 6.4 for the relationship between structure functions and energy spectrum).

2-point Eddy viscosity Models

-For the 2-point eddy-viscosity model, the local structure function is used:

$$F_2(\vec{x}, \Delta) = \left\langle \left[\tilde{\vec{u}}(\vec{x}, t) - \tilde{\vec{u}}(\vec{x} + \vec{r}, t)\right]^2 \right\rangle_{||\vec{r}||}$$

where we now have a local statistical average over the nearest 6 points (or 4 points in a boundary layer).

-Assuming a $k^{-5/3}$ spectrum from zero to k_c, we get

$$\nu_T(\vec{x}, \Delta, t) = 0.105 C_k^{-3/2} \Delta \left[F_2(\vec{x}, \Delta)\right]^{1/2}$$

where C_k is the Kolmogorov constant

- **<u>Relating the structure function model to the Smagorinsky model</u>**:

-If we replace the velocity increments by 1st order spatial derivative we can show that

$$\nu_T \approx 0.777 (C_S \Delta)^2 \sqrt{2\tilde{S}_{ij}\tilde{S}_{ij} + \tilde{\omega}_i\tilde{\omega}_i}$$

where $\tilde{\omega} \equiv \text{ filtered vorticity } = \vec{\nabla} \times \tilde{\vec{u}}$

-We can imagine the 2-point (or structure function) model as the Smagorinsky model in a strain/vorticity formulation

Similarity Models

- **<u>Similarity model</u>**: (Bardina et al., AIAA, 1980)

  -Bardina et al., (1980) proposed an alternative model to the eddy-viscosity model. They were motivated by the low correlations between $\tau_{ij}^{\Delta}(\vec{x},t)$ and $\tau_{ij}^{\Delta,M}(\vec{x},t)$ in *a priori* studies (more on *a priori* studies in later lectures).

  -Based on their analysis (and general reasoning) they hypothesized that near grid-scale energy transfers were the most important (and the most active).

  -Recall: $u'_i \equiv u_i - \tilde{u}_i$

  and the filtered SFS velocity is: $\widetilde{u'}_i = \tilde{u}_i - \tilde{\tilde{u}}_i$

  Recall Leonard's decomposition of τ_{ij} is:

$$\tau_{ij} = L_{ij} + C_{ij} + R_{ij}$$

$$= \left(\widetilde{\tilde{u}_i\tilde{u}_j} - \tilde{u}_i\tilde{u}_j\right) + \left(\widetilde{\tilde{u}_i u'_j} + \widetilde{u'_i\tilde{u}_j}\right) + \widetilde{u'_i u'_j}$$

"Resolved" + "Cross" + "Reynolds" stresses

Similarity Models

-using our definition of the filtered velocity fluctuations, and the following assumption shown for R_{ij}, we can write each of our terms as follows:

$$R_{ij} = \widetilde{(u_i - \tilde{u}_i)(u_j - \tilde{u}_j)} \approx (\tilde{u}_i - \tilde{\tilde{u}}_i)(\tilde{u}_j - \tilde{\tilde{u}}_j)$$

$$C_{ij} \approx \tilde{\tilde{u}}_i(\tilde{u}_j - \tilde{\tilde{u}}_j) + \tilde{\tilde{u}}_j(\tilde{u}_i - \tilde{\tilde{u}}_i)$$

$$L_{ij} = \left(\widetilde{\tilde{u}_i \tilde{u}_j} - \tilde{u}_i \tilde{u}_j\right)$$

-if we put all the terms together and do some algebra this reduces to:

$$\tau_{ij} = \left(\widetilde{\tilde{u}_i \tilde{u}_j} - \tilde{\tilde{u}}_i \tilde{\tilde{u}}_j\right)$$

-giving us an estimate for the SGS stress (see Saguat pg 231 for similarity models)

Similarity models

•Lui et al (JFM, 1994) examined "bands" around Δ and built a scale-similarity model similar to the model of Bardina et al., (1980)

-They argued that energy in the band at one scale larger than Δ (say 2Δ) and one scale smaller (something like ½Δ) would have the largest contribution to τ_{ij} .

-define: $u_i^n = \tilde{u}_i - \bar{u}_i$ where (~) is is a filter at Δ and (**–**) is a filter at a larger scale 2Δ

- u_i^n can be thought of as the band-pass filtered velocity between Δ and 2Δ. We can do a similar decomposition for u_i^{n+1} and u_i^{n-1}

-With our band-pass filtered decomposition we can build a τ_{ij}^n based on u_i^n and u_i^{n+1} (or any other band)

-For example, the stress one level above *n* can be written using another filter at 4Δ (denoted by a **^**) as:

$$\tau_{ij}^{n-1} = \overline{(\tilde{u}_i - \hat{u}_i)(\tilde{u}_j - \hat{u}_j)} - (\overline{\tilde{u}_i - \hat{u}_i})(\overline{\tilde{u}_j - \hat{u}_j})$$

More on similarity models

•This can be reduced to the following (note that $\hat{u}_i$ is approximately a constant with respect to the – filter)

$$\tau_{ij}^{n-1} = (\overline{\tilde{u}_i \tilde{u}_j} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j)$$

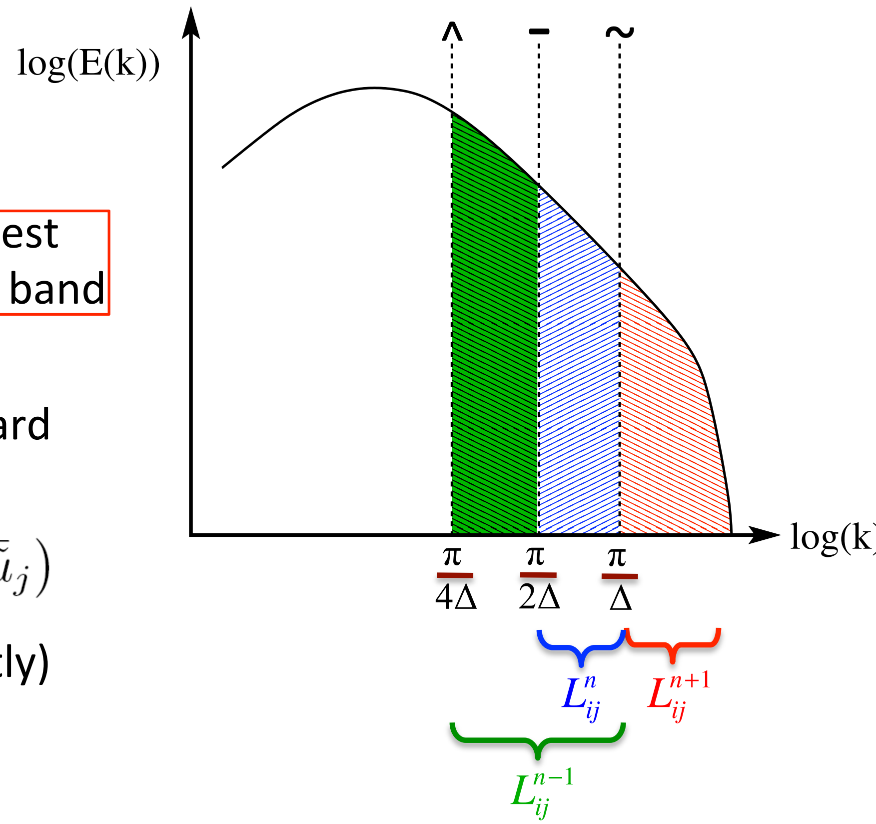
•Lui et al.'s study showed similarity between

$$\tau_{ij}^{n+1} \rightarrow \tau_{ij}^{n} \rightarrow \tau_{ij}^{n-1}$$

1st unresolved band

smallest resolved band

next largest resolved band



• They concluded that because of this the Leonard stress (τ_{ij}^{n-1}) is the best estimate =>

$$\tau_{ij} = C_L L_{ij} \quad \text{where} \quad L_{ij} = (\overline{\tilde{u}_i \tilde{u}_j} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j)$$

• This is the most commonly used form (currently) of the similarity model.