LES of Turbulent Flows: Lecture 9
(ME EN 7960-003)

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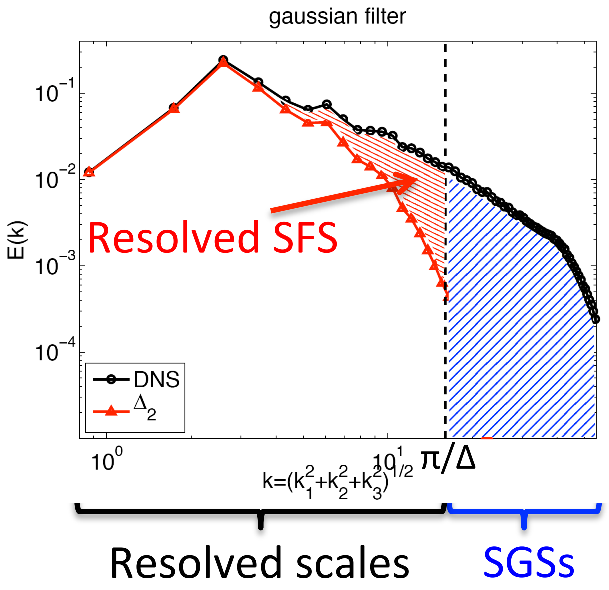
Subgrid-Scale Modeling

- One of the major hurdles to making LES a reliable tool for engineering and environmental applications is the formulation of SGS models and the specification of model coefficients.
- **Recall:** we can define 3 different "scale regions" in LES depicted in the figure to the right => **Resolved Scales**, **Resolved SFS**, **SGSs**
- We can also decompose a general variable as:

$$\phi = \widetilde{\phi} + \phi'$$

- When we talk about SGS models we are specifically talking about the scales below Δ **NOT** the Resolved SFS.



Modeling τ_{ij}

- see Pope pgs. 582-583 or Sagaut pgs. 49-50, 59-60 (this will mostly follow Sagaut)
- we can decompose the nonlinear term as follows (using $\phi = \widetilde{\phi} + \phi'$):

$$\widetilde{u_i u_j} = \widetilde{\left(\tilde{u}_i + u_i'\right)\left(\tilde{u}_j + u_j'\right)}$$

$$= \widetilde{\tilde{u}_i \tilde{u}_j} + \widetilde{\tilde{u}_i u_j'} + \widetilde{\tilde{u}_j u_i'} + \widetilde{u_i' u_j'}$$

- we now have the nonlinear term as a function of $\tilde{u}_i$ and u_i' .
- two different basic forms of the decomposition (based on the above equation are prevalent)

-The 1st one is based on the idea that all terms appearing in the evolution of a filtered quantity should be filtered:

$$\boxed{\tau_{ij} = C_{ij} + R_{ij}} = \widetilde{u_i u_j} - \widetilde{\tilde{u}_i \tilde{u}_j}$$

where $C_{ij} = \widetilde{\tilde{u}_i u_j'} + \widetilde{\tilde{u}_j u_i'} \;\Rightarrow\;$ interaction between resolved and SFSs

and $R_{ij} = \widetilde{u_i' u_j'} \;\Rightarrow\;$ SFS “Reynold’s” stress

Modeling τ_{ij} (continued)

-A 2nd definition can be obtained by further decomposition of $\widetilde{\tilde{u}_i\tilde{u}_j}$ =>

$$\widetilde{\tilde{u}_i\tilde{u}_j} = \underbrace{\left(\widetilde{\tilde{u}_i\tilde{u}_j} - \tilde{u}_i\tilde{u}_j\right)} + \tilde{u}_i\tilde{u}_j$$

L_{ij} => Leonard stress (the interaction among the smallest resolved scales)

Our total decomposition is now:

$$\boxed{\tau_{ij} = L_{ij} + C_{ij} + R_{ij}} = \widetilde{u_i u_j} - \tilde{u}_i\tilde{u}_j$$

(Leonard, Adv. in Geo., 1974)

- If our filter is a Reynolds operator (e.g., cutoff filter) **C_{ij} and L_{ij} vanish!**
- **NOTE:** while τ_{ij} is invariant to Galilean transformations, L_{ij} and C_{ij} are NOT. Because of this, the decomposition given above (for the most part) is not used anymore (although we will see similar terms again in our SGS models)
- **A more rigorous decomposition was proposed by Germano into generalized filtered moments.** Under this framework the generalized moments look just like "Reynolds" moments, our original τ_{ij}! (see Germano, JFM, 1992 in the handouts section. This is also recommended reading for a discussion of filtering and the relationship between LES filters and Reynolds operators)

Classes of LES Models (for τ_{ij})

1. **Eddy-Viscosity Models:**
   - The largest and most commonly used class of SGS models
   - The general idea is that turbulent diffusion at SGSs (i.e. how SGS remove energy) is analogous to molecular diffusion. It is very similar in form to K-theory for Reynolds stresses.
   - The deviatoric part of τ_{ij} is modeled as:

$$\boxed{\tau_{ij} - \frac{1}{3}\tau_{kk}\delta_{ij} = -2\nu_T \tilde{S}_{ij}}$$

where ν_T is the SGS eddy-viscosity and $\tilde{S}_{ij} = \frac{1}{2}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right)$

Within this class of models we have many different ways of determining ν_T

a) Smagorinsky model (Smagorinsky, Mon. Weath. Rev., 1963)
b) "Kolmolgorov" eddy-viscosity (Wong and Lilly, Phys. of Fluids, 1994)
c) Two-point closure models (based on Kraichan's (JAS, 1974) spectral eddy-viscosity model and developed by Lesiour (see Lesiour et al., 2005)
d) One-equation models that use the filtered KE equation (see Lecture 6 for the KE equation and Deardorff (BLM, 1980) for an early application of the model)

Classes of LES Models (for τ_{ij})

2. **Similarity Models:**
   - Based on the idea that the most active SGS are those close to the cutoff wavenumber (or scale) these models were first introduced by Bardina et al. (AIAA, 1980). A subclass under this type of model is the nonlinear model. Many times these models are paired with an eddy-viscosity model.
3. **Stochastic SGS models:**
   - In these models a random (stochastic) component is incorporated into the SGS model. Usually the nonlinear term is combined with an eddy-viscosity model in a similar manner to the similarity models (Mason and Thomson, JFM, 1992).
4. **Subgrid Velocity Reconstruction Models:**
   - These models seek to approximate the SGS stress through a direct reconstruction of the SGS velocity or scalar fields. Two examples are:
     - fractal models (Scotti and Meneveau, Physica D, 1999)
     - Linear-eddy and ODT models (Kerstein, Comb. Sci. Tech, 1988)
5. **Dynamic Models:**
   - Actually more of a procedure that can be applied with any base model, first developed by Germano (Phys of Fluids, 1991).
   - One of the biggest and most influential ideas in LES SGS modeling.

Testing SGS Models

How do we test SGS models?

- ***a posteriori*** testing (term can be credited to Piomelli et al., Phys of Fluids, 1988)
  - Run "full" simulations with a particular SGS model and compare the results (statistically) to DNS, experiments and turbulence theory.
  - A "complete" test of the model (including dynamics and feedback) but is has the disadvantage of including numerics and **we can't gain insight into the physics of SGSs**.

- ***a priori*** testing (term can be credited to Piomelli et al., Phys of Fluids, 1988)
  - Use DNS or high resolution experimental data to test SGS models "offline"
  - **Goal:** directly compare $\tau_{ij}^{\Delta}(\vec{x},t)$ with $\tau_{ij}^{\Delta,M}(\vec{x},t)$ by:
    - Filter DNS (or experimental) data at Δ and compute the exact $\tau_{ij}^{\Delta}(\vec{x},t)$ and other relevant parameters (e.g., *Π*)
    - Use the filtered *u*,*v* and *w* to compute $\tau_{ij}^{\Delta,M}(\vec{x},t)$ from the model (and stats) and compare with above results (this can also be used to calculate unknown model coefficients)
    - Allows us to look specifically at how the model reproduces SGS properties and the physics associated with those properties
    - Drawback, it doesn't include dynamic feedback and numerics!

Eddy viscosity Models

- **<u>Eddy Viscosity Models:</u>** (Guerts pg 225; Pope pg 587)

-Above, we noted that eddy-viscosity models are of the form:

momentum: $\tau_{ij} = -2\nu_T \tilde{S}_{ij}$ filtered strain rate

eddy-viscosity

SGS Prandtl number

scalars: $q_i = -D_T \dfrac{\partial \tilde{\theta}}{\partial x_i}$ where $D_T = \dfrac{\nu_T}{Pr_{sgs}}$

eddy-diffusivity

-This is the LES equivalent to 1st order RANS closure (k-theory or gradient transport theory) and is an analogy to molecular viscosity (see Pope Ch. 10 for a review)

-Turbulent fluxes are assumed to be proportional to the local velocity or scalar gradients

-In LES this is the assumption that stress is proportional to strain: $\tau_{ij} \sim \tilde{S}_{ij}$

-The SGS eddy-viscosity ν_T must still be parameterized.

Eddy viscosity Models

-Dimensionally =>

$$\nu_T = \left[\frac{L^2}{T}\right]$$

-In almost all SGS eddy-viscosity models $\nu_T \sim u^* l^*$

velocity scale length scale

-Different models use different u^* and l^*

-Recall the filtered N-S equations:

$$\frac{\partial \tilde{u}_i}{\partial t} + \frac{\partial \tilde{u}_i \tilde{u}_j}{\partial x_j} = -\frac{\partial \tilde{p}}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 \tilde{u}_i}{\partial x_j^2} - \frac{\partial \tau_{ij}}{\partial x_j} + F_i$$

-Note, for an eddy-viscosity model we can write the viscous and SGS stress terms as:

$$\frac{1}{Re}\frac{\partial}{\partial x_j}\tilde{S}_{ij} \quad \text{and} \quad \tau_{ij} = -2\nu_T \tilde{S}_{ij}$$

We can combine these two and come up with a new (approximate) viscous term:

$$\frac{\partial}{\partial x_j}\left[\left(\nu_T + \frac{1}{Re}\right)\tilde{S}_{ij}\right]$$

What does the model do? We can see it effectively lowers the Reynolds number of the flow and for high Re (when 1/Re=>0), it provides all of the energy dissipation.