LES of Turbulent Flows: Lecture 8 (ME EN 7960-003)

Prof. Rob Stoll
Department of Mechanical Engineering
University of Utah

Fall 2014

LES and numerical methods

• LES requires that the filtered equations of motion (see Lectures 5 and 6) be solved on a numerical grid.

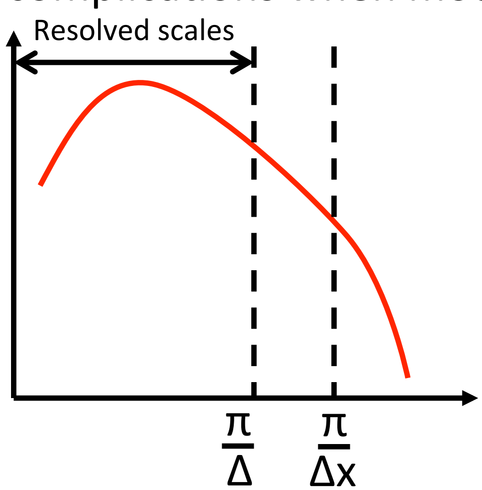
• **In LES we need to accurately represent high wavenumber turbulent fluctuations** (small scale turbulence) this means either:

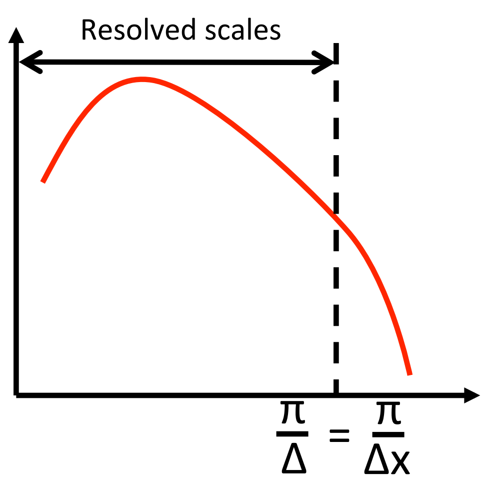
-we use high-order schemes (e.g., spectral methods)

-we use fine grids with low-order schemes (e.g., 2nd order central differences)

• High-order schemes are more expensive but for a given mesh they are more accurate (see Pope 571-579 for a discussion of resolving filtered fields)

• Low-order finite difference (or volume) schemes provide flexibility of geometry but give rise to complications when modeling small scales motions





LES and numerical methods

- The filter applied in LES can be either **implicit** or **explicit**

  -Implicit filtering: The grid (or numerical basis) is assumed to be the LES low-pass filter

  - **Pro:** takes full advantage of the numerical grid resolution
  - **Cons**: for some methods it is helpful to know the shape of the LES filter (this can be difficult to determine for some numerical methods). Truncation error (see above) can also become an issue.

  -**Explicit filtering**: A filter (typically box or Gaussian) is applied to the numerical grid (i.e., explicitly to the discretized N-S equations)

  - **Pros**: truncation error is reduced and the filter shape is well defined
  - **Cons**: loss of resolution. The total simulation time goes up as Δ_g^4 (where Δ_g is the grid spacing) so maintaining the same space resolution as an implicit filter with $\Delta_g/\Delta=½$ will take $2^4=16$ more grid points.

- For reviews of LES and numerics see Guerts chapter 5 and Sagaut chapter 8.2 and 8.3

LES and numerical methods

• For **low-order FD schemes truncation errors can be on the order of SGS contributions** unless Δ is considerably larger than the grid spacing (see Ghosal J. Comp. Phys, 1996)

- Key findings: given a Von Karman spectrum (e.g. $E(k) = \frac{ak^4}{\left(b+k^2\right)^{17/6}}$)

  The discretization error exceeds the subgrid error for 2nd – 8th order schemes. Additionally, for spectral schemes, aliasing error (if 3/2 rule isn't used) will dominate subgrid errors.

- Reducing the ratio C=Δx/Δ to values less than 1 reduces the error faster (by a factor of $C^{-3/4}$ than increasing the order of accuracy (a factor of 2 reduction from 2nd to 8th).

• Note that although 2nd order schemes may have undesirable truncation errors (with respect to SGS model terms), **even order schemes are non-dissipative while SGS models (on average) are purely dissipative**, therefore, all hope is not lost!

• **The same is not true for dissipative schemes** common in compressible flow solutions For example in upwind schemes and TVD or FLS schemes (see Leveque 1992 for a review of this type of numerics). For these schemes,

LES and numerical methods

- **The same is not true for dissipative schemes** common in compressible flow solutions For example in upwind schemes and TVD or FLS schemes (see Leveque 1992 for a review of this type of numerics).

- Using these types of schemes with LES is somewhat controversial since their dissipative nature introduces an eddy viscosity like term to the solution (more later).

- The total numerical dissipation in many cases introduced by upwind schemes is grater than that of SGS viscosity models (if no prefiltering is performed) even for 7^{th} order schemes (Beaudan and Moin, 1994).

- For the Smagorinsky model (more later) Garnier et al. (J.Comp.Phys., 1999) found that up to 5^{th} order upwind schemes in decaying isotropic turbulence are more dissipative.

Subgrid-Scale Modeling

• One of the major hurdles to making LES a reliable tool for engineering and environmental applications is the formulation of SGS models and the specification of model coefficients.

• **Recall:** we can define 3 different "scale regions" in LES depicted in the figure to the right =>

-**Resolved Scales**, **Resolved SFS**, **SGSs**

• We can also decompose a general variable as:

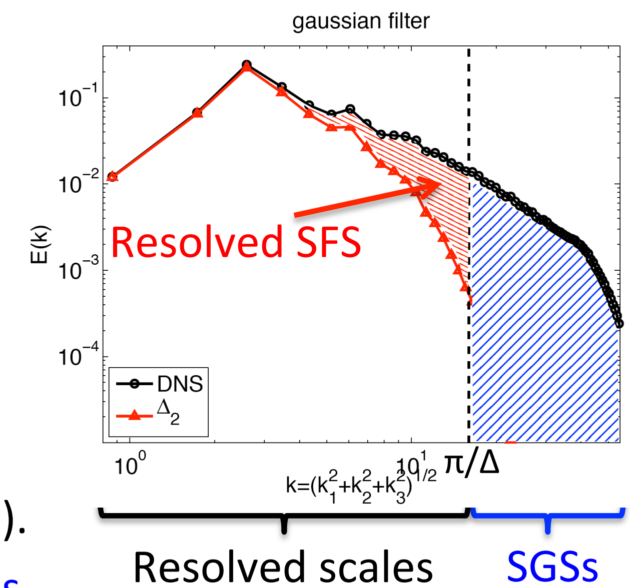
$$\phi = \widetilde{\phi} + \phi'$$

• When we talk about SGS models we are specifically talking about the scales below Δ **NOT** the Resolved SFS.

• We will specifically discuss the Resolved SFSs when we talk about filter reconstruction later on (time permitting).

• Also, here we will focus on LES with explicit SGS models.

• A class of LES referred to as Implicit LES (**ILES**) also exists.



• **ILES** was 1st developed for compressible flow. It assumes the SGSs are purely dissipative and act in a similar way to dissipative numerical schemes (in general ILES uses monotinicity preserving numerical schemes). See handout Grinstein_etal_2007_ch2.pdf for details.

Modeling τ_{ij}

- see Pope pgs. 582-583 or Sagaut pgs. 49-50, 59-60 (this will mostly follow Sagaut)
- we can decompose the nonlinear term as follows (using $\phi = \widetilde{\phi} + \phi'$):

$$\widetilde{u_i u_j} = \widetilde{\left(\tilde{u}_i + u_i'\right)\left(\tilde{u}_j + u_j'\right)}$$
$$= \widetilde{\tilde{u}_i \tilde{u}_j} + \widetilde{\tilde{u}_i u_j'} + \widetilde{\tilde{u}_j u_i'} + \widetilde{u_i' u_j'}$$

- we now have the nonlinear term as a function of $\tilde{u}_i$ and u_i'.
- two different basic forms of the decomposition (based on the above equation are prevalent)

  -The 1st one is based on the idea that all terms appearing in the evolution of a filtered quantity should be filtered:

$$\tau_{ij} = C_{ij} + R_{ij} = \widetilde{u_i u_j} - \widetilde{\tilde{u}_i \tilde{u}_j}$$

where $C_{ij} = \widetilde{\tilde{u}_i u_j'} + \widetilde{\tilde{u}_j u_i'} \Rightarrow$ interaction between resolved and SFSs

and $R_{ij} = \widetilde{u_i' u_j'} \Rightarrow$ SFS "Reynold's" stress

Modeling τ_{ij} (continued)

-A 2nd definition can be obtained by further decomposition of $\widetilde{\tilde{u}_i\tilde{u}_j}$ =>

$$\widetilde{\tilde{u}_i\tilde{u}_j} = \underbrace{\left(\widetilde{\tilde{u}_i\tilde{u}_j} - \tilde{u}_i\tilde{u}_j\right)}_{} + \tilde{u}_i\tilde{u}_j$$

L_{ij} => Leonard stress (the interaction among the smallest resolved scales)

Our total decomposition is now:

$$\boxed{\tau_{ij} = L_{ij} + C_{ij} + R_{ij}} = \widetilde{u_i u_j} - \tilde{u}_i\tilde{u}_j$$

(Leonard, Adv. in Geo., 1974)

- If our filter is a Reynolds operator (e.g., cutoff filter) **C_{ij} and L_{ij} vanish!**
- **NOTE:** while τ_{ij} is invariant to Galilean transformations, L_{ij} and C_{ij} are NOT. Because of this, the decomposition given above (for the most part) is not used anymore (although we will see similar terms again in our SGS models)
- **A more rigorous decomposition was proposed by Germano into generalized filtered moments.** Under this framework the generalized moments look just like "Reynolds" moments, our original τ_{ij}! (see Germano, JFM, 1992 in the handouts section. This is also recommended reading for a discussion of filtering and the relationship between LES filters and Reynolds operators)