LES of Turbulent Flows: Lecture 7
(ME EN 7960-003)

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Turbulence modeling (alternative strategies)

• So far our discussion of turbulence modeling has centered around separating the flow into resolved and SFSs using a low-pass filtering operation with the goal of reducing the # of degrees of freedom in our numerical solution.

• This is not the only way to accomplish complexity reduction in a turbulent flow. Here we briefly review a few other methods

• **Coherent Vortex Simulations (CVS)**: Farge and Schneider, Flow Turb. Comb. 2001
The turbulent flow field is decomposed into coherent and random components using either a continuous or orthonormal wavelet filter (see the next page for a very brief review of wavelets)

-In the original formulation the separation between coherent and random motions is assumed to be complete with the random part mimicking viscous dissipation.
-Goldstein and Vasilyev (Phys of Fluids, 2004) introduced "**stochastic coherent adaptive LES**", a variation on CVS

- they use the CVS wavelet decomposition but do not assume that the wavelet filter completely eliminates all the coherent motions from the SFSs => the SFS components themselves contain coherent and random components.

Wavelet Decomposition (a brief overview)

- The Coherent Vortex Simulations (CVS) method uses wavelet decomposition.
- Here is a brief overview of wavelets. For a more detailed view see:
  - -Daubechies, 1992 (most recent printing is 2006)
  - -Mallat, 2009 (3rd edition)
  - -Farge, Ann. Rev. Fluid Mech., 1992 (specific to turbulence research)

First: Windowed Fourier Transforms (or Gabor transform)

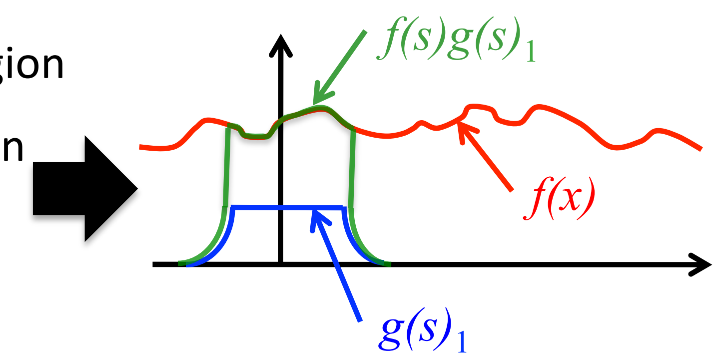
- recall: $$f_k = \frac{1}{2\pi}\int f(x)e^{ikx}\,dx$$

- a windowed Fourier transform can be expressed as:

$$f_{k,s} = \frac{1}{2\pi}\int f(s)g(s-x)e^{iks}\,ds$$

-where s is the position over a localized region

-the windowed transform is our convolution with a filter function in Fourier space



Wavelet decomposition (a brief overview)

- Wavelets offer an optimal space frequency decomposition, in 1D:

$$W_f(a,b) = |a|^{-\frac{1}{2}} \int f(x)\Psi\left(\frac{x-b}{a}\right)dx$$

where ***Ψ*** **is the basis function** (sometimes called the "Mother Wavelet"), ***b*** **translates** the basis function (location) and ***a*** **scales** the basis function (dilatation).

- A few different types of Wavelets exist, the main general classes (similar to Fourier) are:
  - -continuous transforms (what we wrote above)
  - -discrete transforms
    - Redundant discrete systems
    - Orthonormal systems (this is used in CVS)

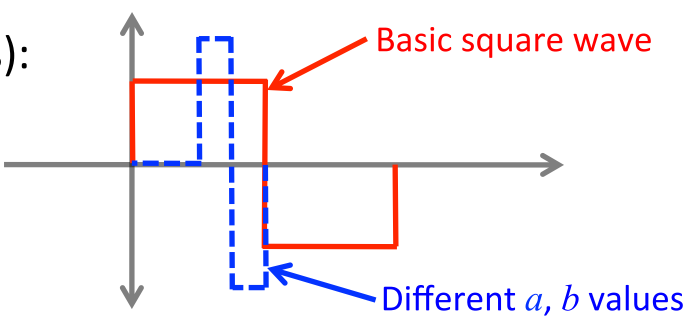
Wavelet decomposition (a brief overview)

- Common Properties of Wavelets (Burrus et al., 1998):
  - A wavelet transform is a set of building blocks to construct or represent a signal (function). It is a 2D expansion set (usually a basis) for a 1D signal.
  - A wavelet expansion gives a time-frequency localization of a signal
  - The calculation of coefficients can be done efficiently
  - Wavelet systems are generated from a single scaling function (ie wavelet) by simple scaling and translation
  - Most useful wavelet systems satisfy the multiresolution condition. If the basic expansion signals (the wavelets) are made half as wide and translated in steps half as wide, they will represent a larger class of signals exactly or give a better approximation of any signal.
  - The lower resolution coefficients can be calculated from the higher resolution coefficients by a tree-structured algorithm.

•Example of a common wavelet (see listed refs for others):

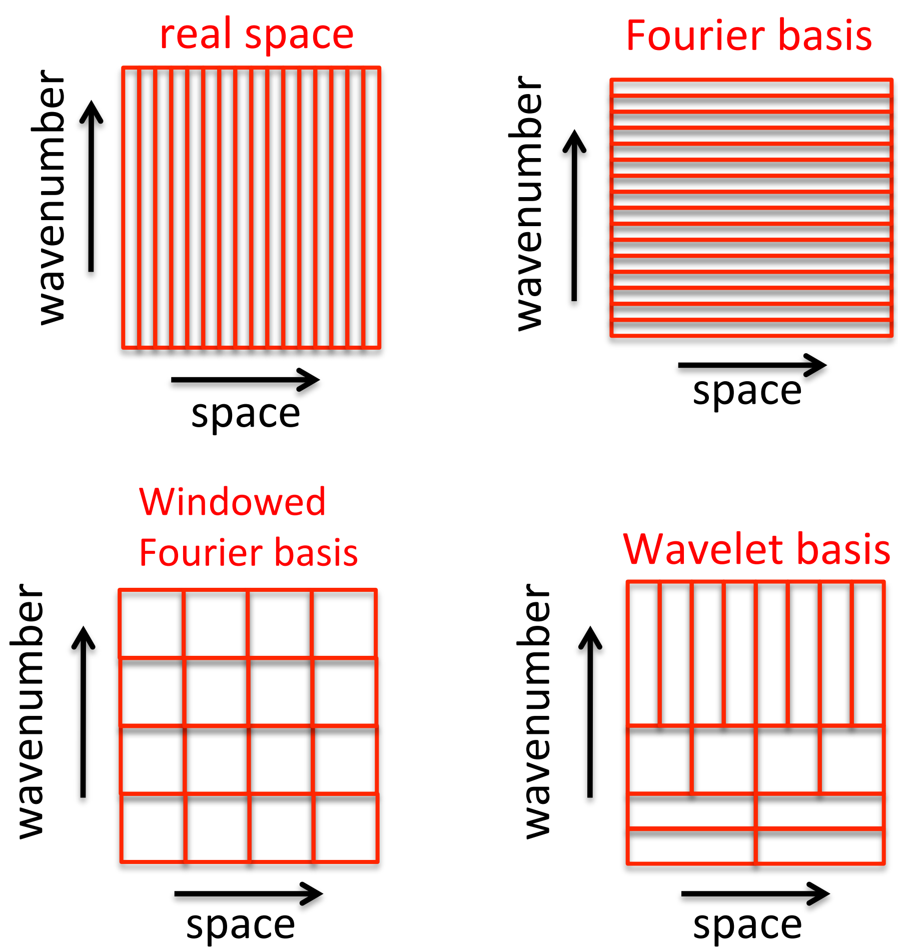
- **<u>Haar Wavelet (Haar, 1910)</u>**:

$$\Psi(x) = \begin{cases} 1 & 0 \le x < \frac{1}{2} \\ -1 & \frac{1}{2} \le x < 1 \end{cases}$$



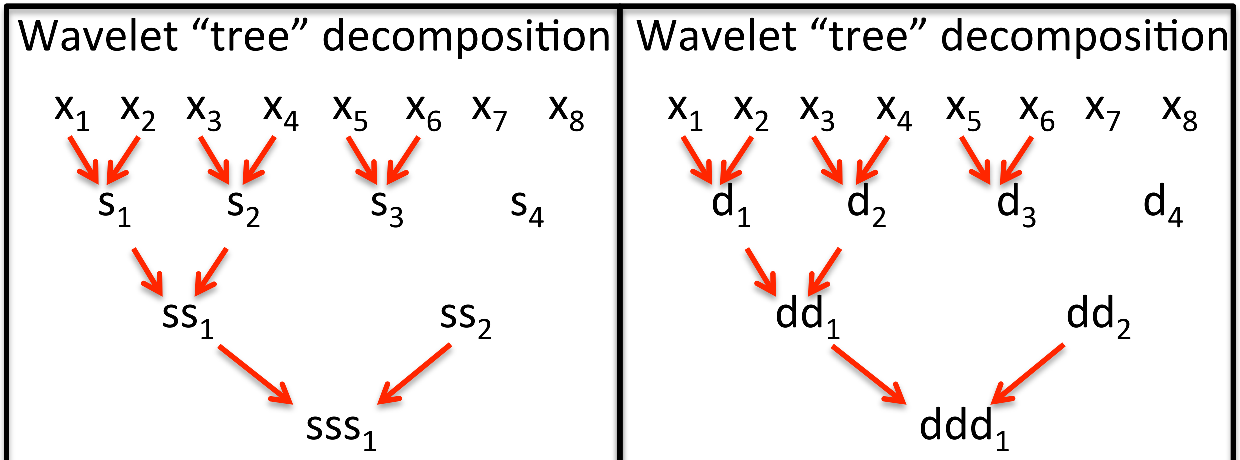
Wavelet decomposition (a brief overview)

- How does wavelet decomposition break down a signal in space and time?



Wavelet decomposition (a brief overview)

- Example: Haar Wavelet decomposition



For the Haar wavelet	For the Haar wavelet
$s_1=(x_1+x_2)/2^{½}$	$d_1=(x_2-x_1)/2^{½}$

- The signal is reconstructed by combining s_i and d_i at the desired level (s, ss, sss, etc).
  - 1st level – s_i and d_i
  - 2nd level – ss_i, dd_i, d_i
  - …

Turbulence modeling (alternative strategies)

- **Filtered Density Functions (FDF)**: Colucci et al., (Phys. of Fluids, 1998)

  -In this method, the **evolution of the filtered probability density functions** is solved for (i.e., we solve for the evolution of the SFS general moments)

  -Similar to general PDF transport methods 1st introduced by Lundgren (Phys. of Fluid, 1969) and outlined in detail in Pope chapter 12.

  -Many applications use FDF for scalars in turbulent reacting flows while traditional (low-pass filtered N-S) equations are solved for momentum. For a more extensive discussion see Fox, 2002.

  -This type of method is often employed for LES with Lagrangian particle models and for chemically reactive flows. In **Lagrangian particle models it leads to** a form of the **Langevin equations for SFS particle evolution** and in **chemically reactive flows** it has the advantage that the **reactions occur in closed form**

  -We will return to these type of methods later in the class when we discuss combing LES with particle models.