LES of Turbulent Flows: Lecture 2
(ME EN 7960-003)

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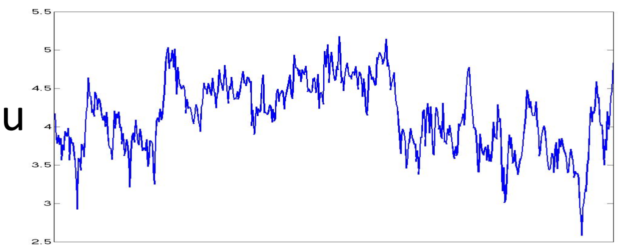
Turbulent Flow Properties

- Review from Previous

Properties of Turbulent Flows:

1. <u>Unsteadiness:</u>

u

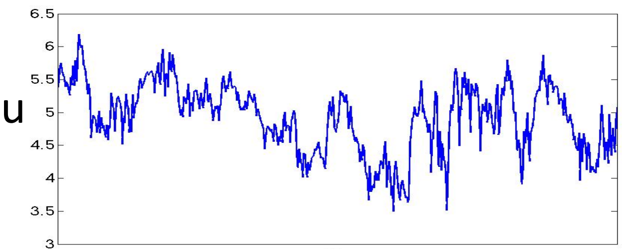


u=f(**x**,t)

time

2. <u>3D:</u>

u



contains random-like variability in space

x_i (all 3 directions)

3. <u>High vorticity:</u>

Vortex stretching ➡ mechanism to increase the intensity of turbulence

(we can measure the intensity of turbulence with the turbulence intensity => $\frac{\sigma_u}{\langle u \rangle}$)

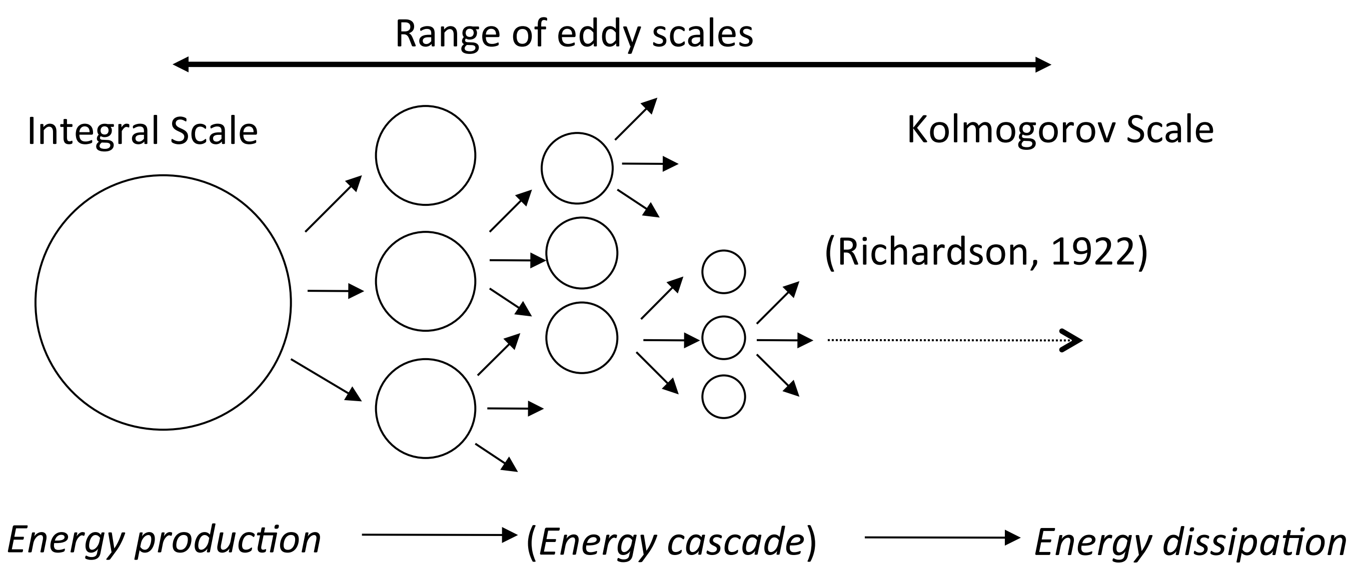
Vorticity: $\omega = \vec{\nabla} \times \vec{u}$ or $\omega_k = \epsilon_{ijk} \frac{\partial}{\partial x_i} u_j \hat{e}_k$

Turbulent Flow Properties (cont.)

Properties of Turbulent Flows:

4. Mixing effect:
   Turbulence mixes quantities with the result that gradients are reduced (e.g. pollutants, chemicals, velocity components, etc.). This lowers the concentration of harmful scalars but increases drag.

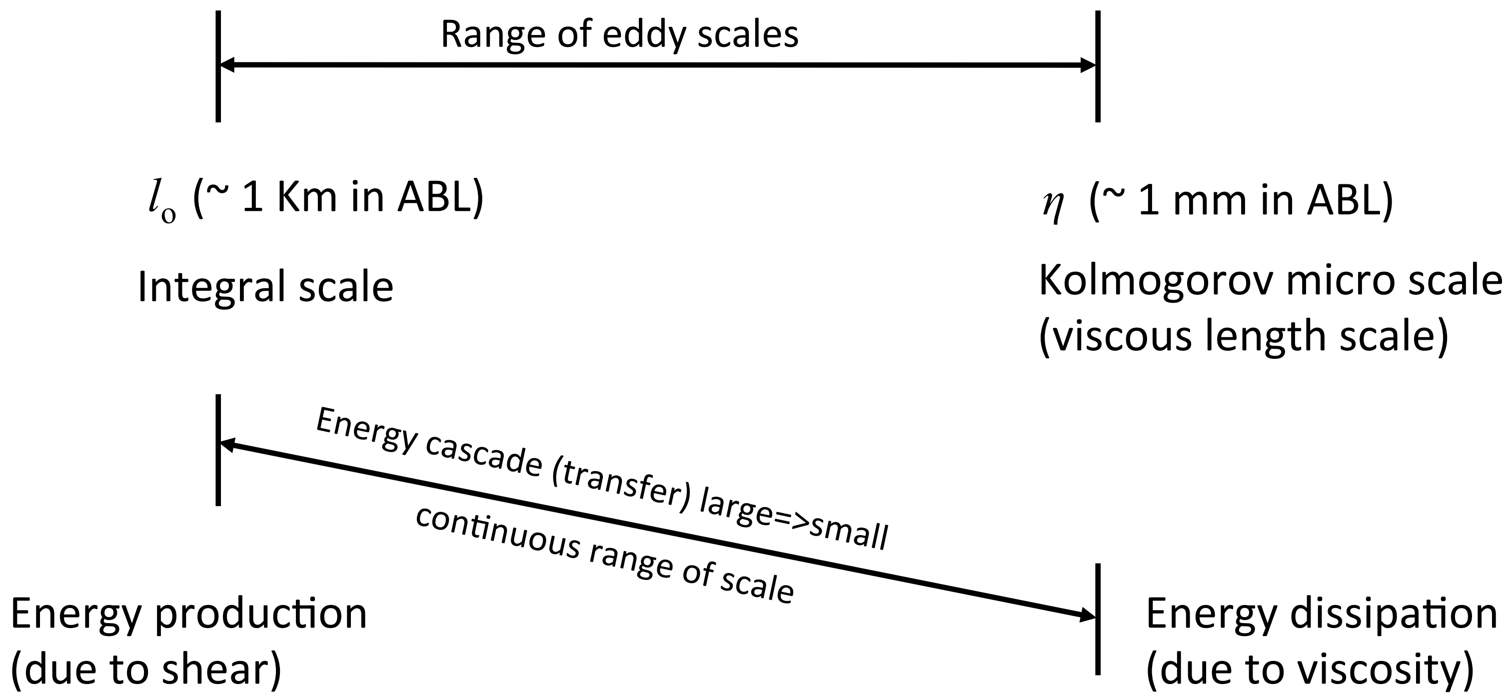
5. A continuous spectrum (range) of scales:



Turbulence Scales

- The largest scale is referred to as the Integral scale (l_o). It is on the order of the autocorrelation length.

- In a boundary layer, the integral scale is comparable to the boundary layer height.



Kolmogorov's Similarity hypothesis (1941)

Kolmogorov's 1st Hypothesis:

• smallest scales receive energy at a rate proportional to the dissipation of energy rate.

• motion of the very smallest scales in a flow depend only on:

a) rate of energy transfer from small scales: $\epsilon \left[\frac{L^2}{T^3}\right]$

b) kinematic viscosity: $\nu \left[\frac{L^2}{T}\right]$

With this he defined the Kolmogorov scales (dissipation scales):

- length scale: $\eta = \left(\frac{\nu^3}{\epsilon}\right)^{\frac{1}{4}}$
- time scale: $\tau = \left(\frac{\nu}{\epsilon}\right)^{\frac{1}{2}}$
- velocity scale: $v = (\nu\epsilon)^{\frac{1}{4}}$

Re based on the Kolmolgorov scales => Re=1

Kolmogorov's Similarity hypothesis (1941)

From our scales we can also form the ratios of the largest to smallest scales in the flow (using $\ell_o,\ U_o,\ t_o$).
Note: dissipation at large scales => $\epsilon \sim \frac{U_o^3}{\ell_o}$

- length scale:

$$\eta = \left(\frac{\nu^3}{\epsilon}\right)^{\frac{1}{4}} \sim \left(\frac{\nu^3 \ell_o}{U_o^3}\right)^{\frac{1}{4}} \Rightarrow \frac{\eta}{\ell_o^{1/4}} \sim \frac{\nu^{3/4}}{U_o^{3/4}} \Rightarrow \boxed{\frac{\eta}{\ell_o} \sim \frac{\nu^{3/4}}{U_o^{3/4}\ell_o^{3/4}} \sim Re^{-3/4}}$$

- velocity scale:

$$v = (\nu\epsilon)^{\frac{1}{4}} \sim \left(\frac{\nu U_o^3}{\ell_o}\right)^{\frac{1}{4}} \Rightarrow \frac{v}{U_o^{3/4}} \sim \frac{\nu^{1/4}}{\ell_o^{1/4}} \Rightarrow \boxed{\frac{v}{U_o} \sim Re^{-1/4}}$$

- time scale:

$$\tau = \frac{\eta}{v} \Rightarrow \boxed{\frac{\tau}{t_o} \sim Re^{-1/2}}$$

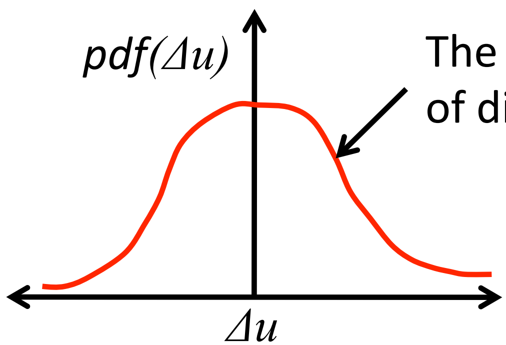
For very high-Re flows (e.g., Atmosphere) we have a range of scales that is small compared to ℓ_o but large compared to η. As *Re* goes up, η / ℓ_o goes down and we have a larger separation between large and small scales.

Kolmogorov's Similarity hypothesis (1941)

<u>Kolmolgorov's 2nd Hypothesis</u>:

In Turbulent flow, a range of scales exists at very high Re where statistics of motion in a range ℓ (for $\ell_o >> \ell >> \eta$) have a universal form that is determined only by ϵ (dissipation) and independent of ν (kinematic viscosity).

• Kolmogorov formed his hypothesis and examined it by looking at the pdf of velocity increments Δu.



The moments of this pdf are the structure functions of different order (e.g., 2nd, 3rd, 4th, etc.)

variance   skewness   kurtosis

- What are structure functions ???

Important single point stats for joint variables

- **covariance**:

$$\mathrm{cov}(U_1,U_2)=\langle u_1u_2\rangle=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(V_1-\langle U_1\rangle)(V_2-\langle U_2\rangle)f_{12}(V_1,V_2)dV_2dV_1$$

- Or for discrete data

$$\mathrm{cov}(U_1,U_2)=\langle u_1u_2\rangle=\frac{1}{N-1}\sum_{j=1}^{N}(V_{1j}-\langle U_1\rangle)(V_{2j}-\langle U_2\rangle)$$

- We can also define the correlation coefficient (non dimensional)

$$\rho_{12}=\frac{\langle u_1u_2\rangle}{\left[\langle u_1^2\rangle\langle u_2^2\rangle\right]^{1/2}}$$

  - Note that -1 ≤ ρ_{12} ≤ 1 and negative value mean the variables are anti-correlated with positive values indicating a correlation

- **Practically speaking**, we find the PDF of a time (or space) series by:
  1. Create a histogram of the series (group values into bins)
  2. Normalize the bin weights by the total # of points

Two-point statistical measures

- **autocovariance:** measures how a variable changes (or the correlation) with different lags

$$R(s) \equiv \langle u(t)u(t+s) \rangle$$

- or the autocorrelation function

$$\rho(s) \equiv \langle u(t)u(t+s) \rangle / \langle u(t)^2 \rangle$$

  - These are very similar to the covariance and correlation coefficient
  - The difference is that we are now looking at the linear correlation of a signal with itself but at two different times (or spatial points), i.e. we lag the series.

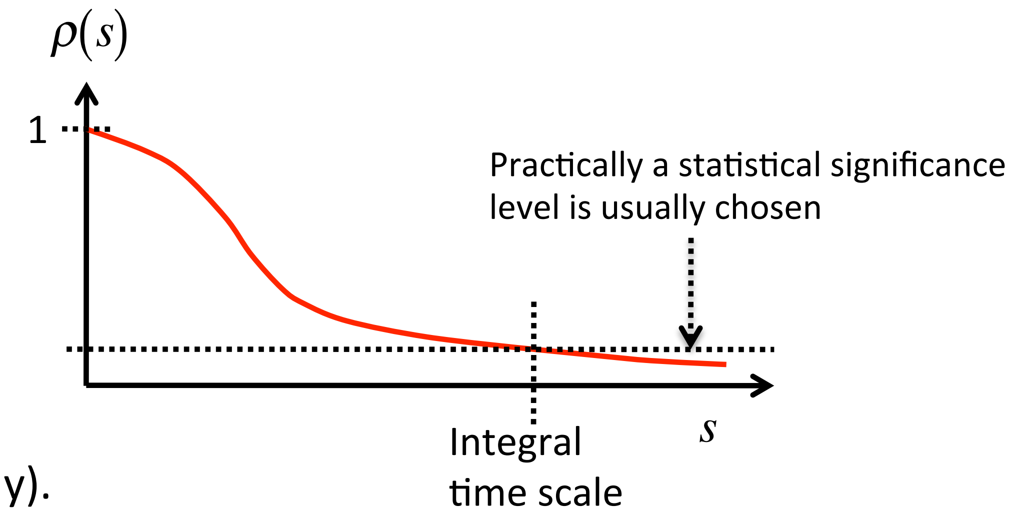
- Discrete form of autocorrelation:

$$\rho(s_j) = \frac{\sum_{k=0}^{N-j-1} (u_k u_{k+j})}{\sum_{k=0}^{N-1} (u_k^2)}$$

- We could also look at the cross correlations in the same manner (between two different variables with a lag).
- Note that: $\rho(0) = 1$ and $|\rho(\mathrm{s})| \leq 1$

Two-point statistical measures

- In turbulent flows, we expect the correlation to diminish with increasing time (or distance) between points:



- We can use this to define an Integral time scale (or space). It is defined as the time lag where the integral $\int_0^\infty \rho(s)ds$ converges. and can be used to define the largest scales of motion (statistically).

- Another important 2 point statistic is the **structure function**:

$$D_n(r) \equiv \left\langle \left[U_1(x+r,t) - U_1(x,t)\right]^n \right\rangle$$

This gives us the average difference between two points separated by a distance r raised to a power n. In some sense it is a measure of the moments of the velocity increment PDF. Note the difference between this and the **autocorrelation which is statistical linear correlation** (ie multiplication) of the two points.

Fourier Transforms

Alternatively, we can also look at turbulence in wave (frequency) space:

Fourier Transforms are a common tool in fluid dynamics (see Pope, Appendix D-G, Stull handouts online)

Some uses:

- Analysis of turbulent flow
- Numerical simulations of N-S equations
- Analysis of numerical schemes (modified wavenumbers)

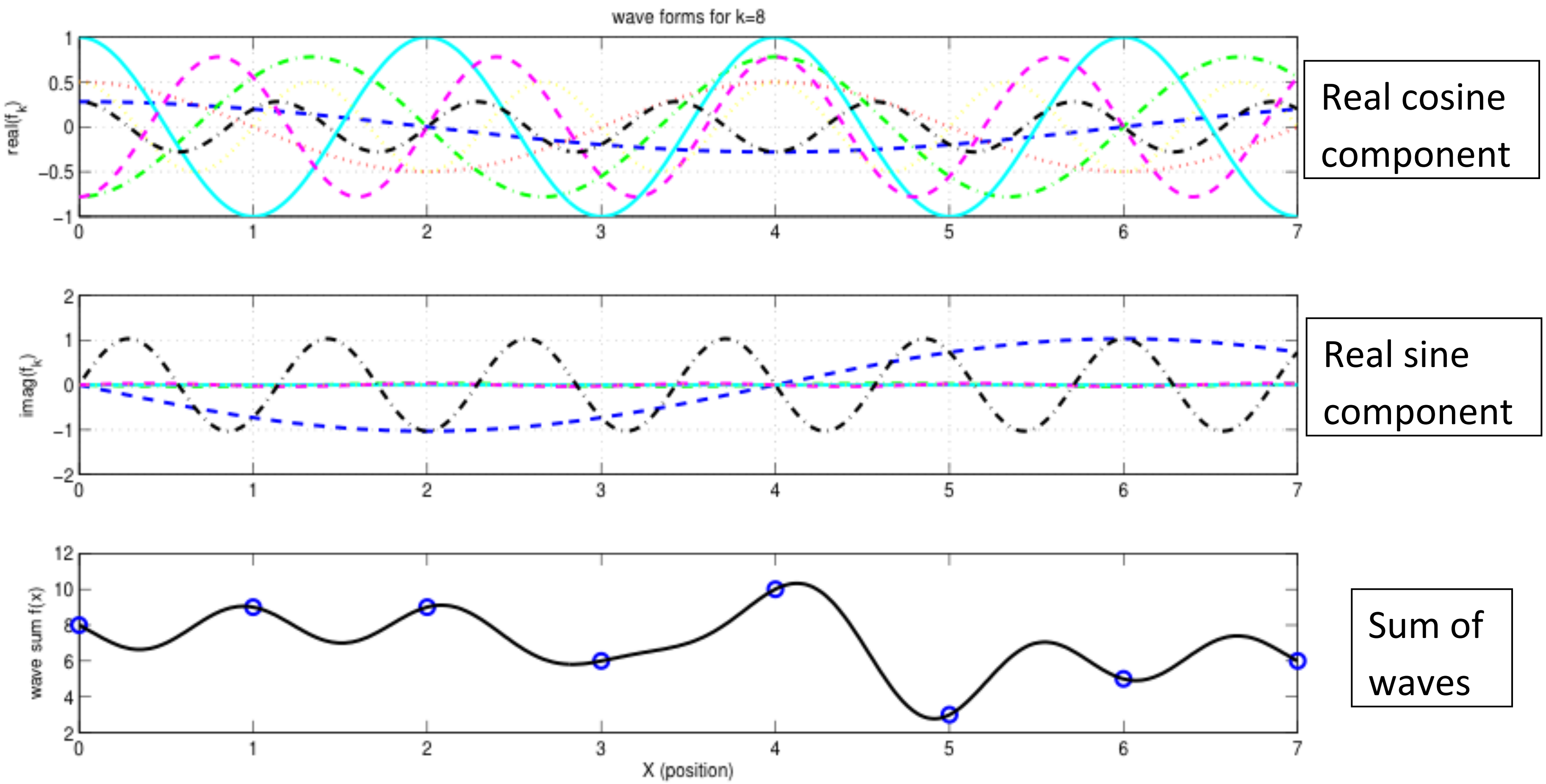
- consider a periodic function $f(x)$ (could also be $f(t)$) on a domain of length 2π
- The Fourier representation of this function (or a general signal) is:

$$f(x) = \sum_{k=-\infty}^{k=\infty} \hat{f}_k e^{ikx} \qquad (*)$$

- where k is the wavenumber (frequency if $f(t)$)
- $\hat{f}_k$ are the Fourier coefficients which in general are complex

Fourier Transforms

- Fourier Transform example (from Stull, 88 see example: FourierTransDemo.m)



Fourier Transform Applications

Energy Spectrum: (power spectrum, energy spectral density)

- If we look at specific k values from we can define:

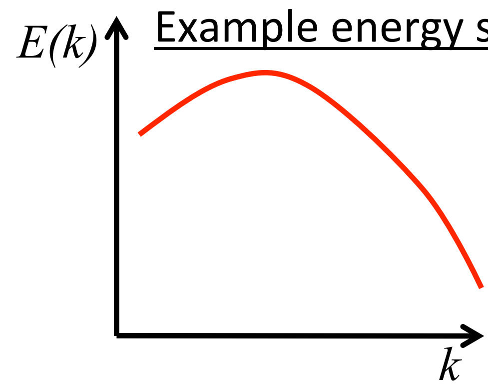
$$E(k) = N\left|\hat{f}_k\right|^2$$

The square of the Fourier coefficients is the contribution to the variance by fluctuations of scale k (wavenumber or equivalently frequency)

where $E(k)$ is the energy spectral density

- Typically (when written as) $E(k)$ we mean the contribution to the turbulent kinetic energy (tke) = $\frac{1}{2}(u^2+v^2+w^2)$ and we would say that $E(k)$ is the contribution to tke for motions of the scale (or size) k . For a single velocity component in one direction we would write $E_{11}(k_1)$.
- See supplement for more on Fourier Transforms

Example energy spectrum



Kolmogorov's Similarity Hypothesis (1941)

- Another way to look at this (equivalent to structure functions) is to examine what it means for *E(k)* where $E(k)dk = \text{t.k.e. contained between } k \text{ and } k + dk$
- What are the implications of Kolmolgorov's hypothesis for E(*k*)? $\text{K41} \Rightarrow E(k) = f(k, \epsilon)$

By dimensional analysis we can find that: $E(k) = c_k \epsilon^{2/3} k^{-5/3}$

• This expression is valid for the range of length scales ℓ where $\ell_o >> \ell >> \eta$ and is usually called the inertial subrange of turbulence.

• graphically: