ME EN 7960-003 Large-Eddy Simulation of Turbulent Flows

Homework #3: SFS modeling, Due Tuesday Nov. 24th

1. Using the basic properties of a convolution filters and starting with the scalar conservation equation given by

$$\frac{\partial \theta}{\partial t} + u_i \frac{\partial \theta}{\partial x_i} = \frac{1}{ScRe} \frac{\partial^2 \theta}{\partial x_i^2} + Q :$$

   (a) derive the filtered scalar conservation equation. Clearly show each step in the process and clearly define the subfilter scale term.

   (b) Use Leonard's decomposition (Leonard, Adv. Geophys. 1974) to decompose your answer to (a) and label terms that indicate the, SFS Reynolds flux, interaction between resolved and unresolved scales, and the interaction amongst the smallest resolved scales (Leonard term).

2. Using the filtered conservation of momentum equation given by:

$$\frac{\partial \tilde{u}_i}{\partial t} + \frac{\partial \tilde{u}_i \tilde{u}_j}{\partial x_j} = -\frac{\partial \tilde{P}}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 \tilde{u}_i}{\partial x_j^2} - \frac{\partial \tau_{ij}}{\partial x_j} + F_i,$$

   derive an equation for the residual kinetic energy $k_r = \frac{1}{2}\tau_{ii}$. Clearly identify the following terms in the equation: all transport terms (i.e., terms that don't create or destroy SFS energy), SFS energy transfer (i.e., Π) and viscous dissipation of energy.

3. Derive an equation for the SFS scalar flux bulk coefficient $C_s^2 Sc_{sfs}^{-1}$ that appears in the Smagorinsky eddy-diffusivity model given by:

$$q_i = -\Delta^2 C_s^2 Sc_{sfs}^{-1} |\tilde{S}| \frac{\partial \tilde{\theta}}{\partial x_i},$$

   using the dynamic procedure (assume scale invariance). Clearly list any assumptions that you make along the way.

4. Starting with SFS scalar flux given by:

$$q_i = \widetilde{u_i \theta} - \tilde{u}_i \tilde{\theta},$$

   derive a scale similarity model following Lui et al., (J. Fluid Mech. 1994). Clearly state all assumptions.

5. Derive a scale-dependent dynamic model for the SFS stress based on the Wong-Lilly model (Wong, Lilly; Phys. Fluids, 1994) for the SFS stress given by:

$$\tau_{ij} - \frac{1}{3}\tau_{kk} = -2C_\epsilon \Delta^{4/3} \tilde{S}_{ij}$$

   You will need to use Germano's identity (Germano et al., Phys. Fluids, 1991) at two different scales to do this and you should end up with an algebraic expression for C_ϵ where C_ϵ is a function of Δ (and the resolved velocity field). Clearly state all assumptions that you make along the way.