As will be discussed in the next chapter, both the abscissa and ordinate are often made dimensionless by normalizing with respect to scaling variables (see Fig 8.9g). The scaling variables used in this example are listed in Table 8-1.

8.7 Spectral Characteristics

Instead of discussing spectral behavior theoretically, this section demonstrates spectral behavior for a single variable through a series of examples with synthetic data. In each of the following cases, an artificial time series of 20 data points is plotted, along with the spectrum computed with an FFT program. The spectrum shows E(n) normalized by the total biased variance, and shows the fraction of the total variance explained by each frequency. The Nyquist frequency is n=10 for all cases.

Case A (Fig 8.10a): Simple waves of one frequency. All of these examples show a wave having four cycles per time period. The first four examples in this case show that the spectrum is independent of the phase of the original time series. A single simple wave in physical space produces a single spike in the spectrum at n=4 that explains all the variance. The fifth example shows that if the spectrum is normalized by the total variance, we still have a single spike that explains 100% of the variance. If the spectrum had not been normalized, the spike for this fifth case would have been twice as large as the spikes for the other four cases, because the time series for the fifth case consisted of a wave with twice the amplitude.

Case B (Fig 8.10b): Simple waves of different frequencies. The first example shows a time series filled by one wave, resulting in a spectrum with a spike at n = 1. The next three examples show waves with 4, 8, and 10 cycles per period in the time series, resulting in spectra with frequency spikes at n = 4, 8, and 10 respectively. The fifth example shows a time series with a wave having 12 cycles per period, but the aliasing problem causes this signal to be folded back to n = 8, where it appears as a spike on the spectrum.

Case C (Fig 8.10c): Frequencies between resolvable frequencies. The FFT consists of waves of the fundamental frequency (n = 1) and only the exact harmonics (n = 2, 3, 4,...). But what happens if the real signal has a frequency of n = 4.2 or 4.5? These examples show that a wave of n = 4.5 appears as two large spikes at n = 4 and n = 5. The closer the signal is to an exact harmonic, the greater the spectral energy at that harmonic and the smaller the energy at the next nearest neighbor. Notice that for a signal with n = 4.5, the spectrum not only has the two large spikes described above, but there is also a leakage of some small amount of spectral energy to all the other frequencies. We might expect that a real turbulence signal consisting of a multitude of frequencies, many of which are not exact harmonics of the fundamental frequency, will result in a spectrum with a lot of leakage, making it difficult to separate the true signals from the underlying noise.

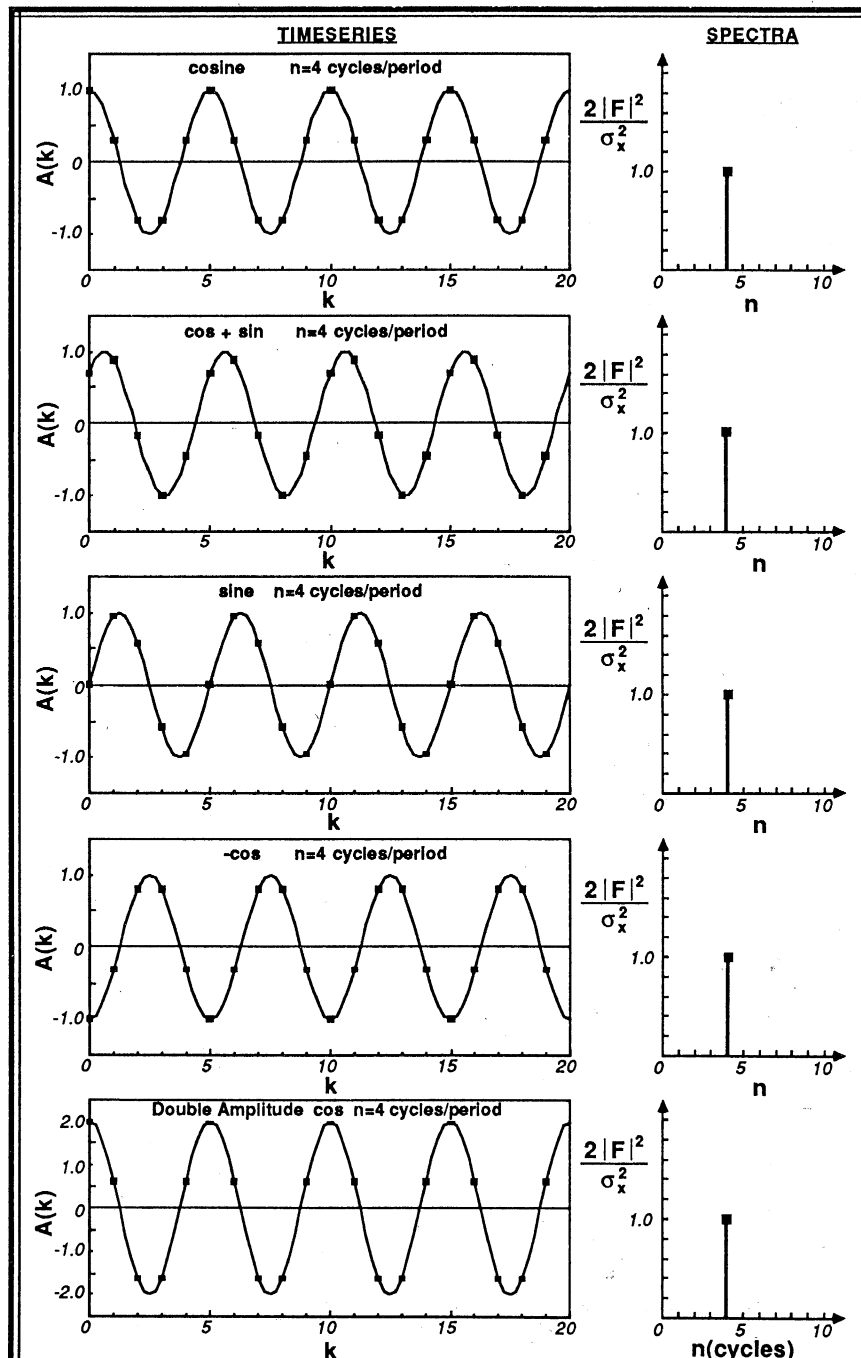

Fig. 8.10a Simple waves.

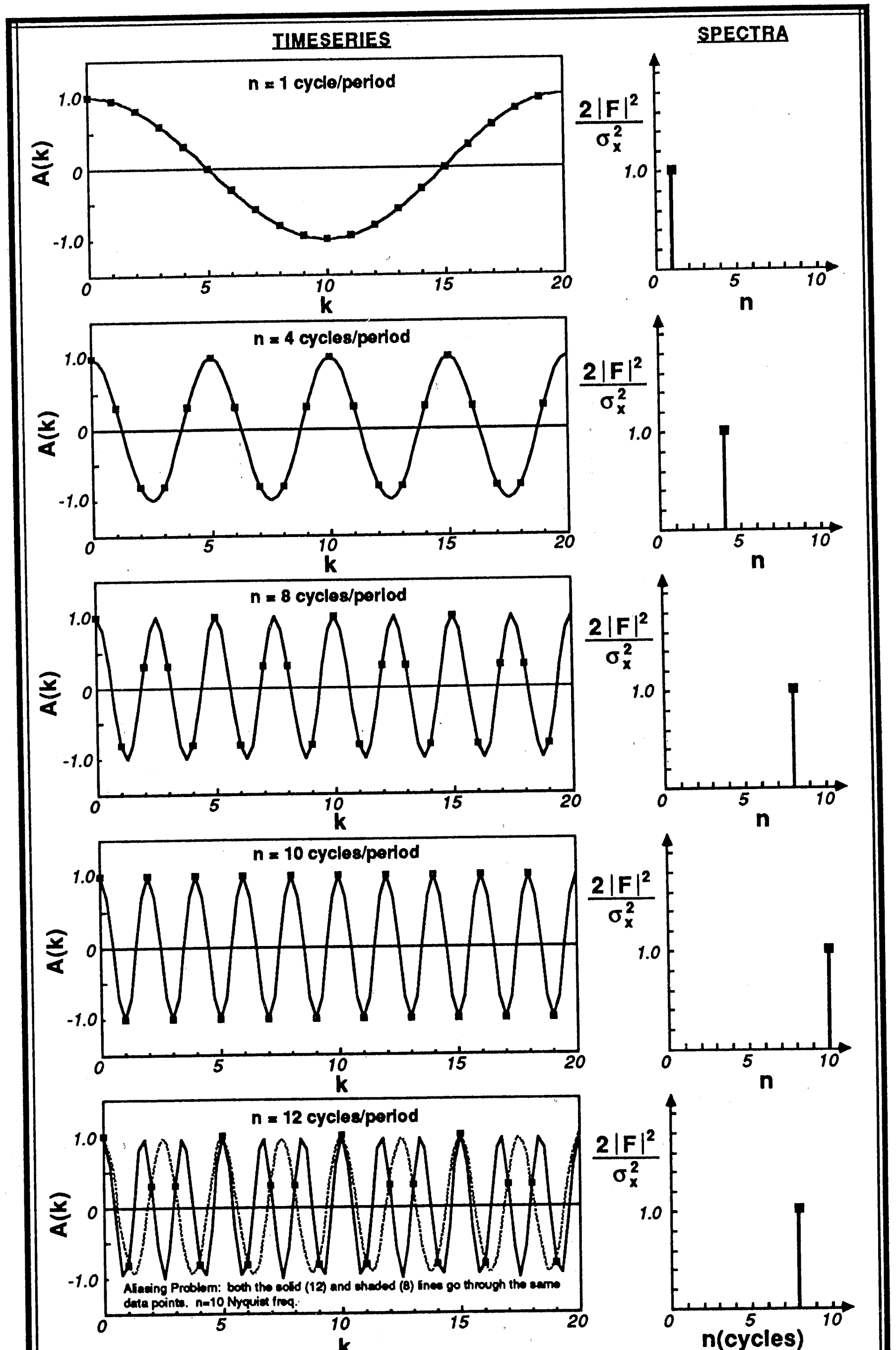

Fig. 8.10b Simple waves.

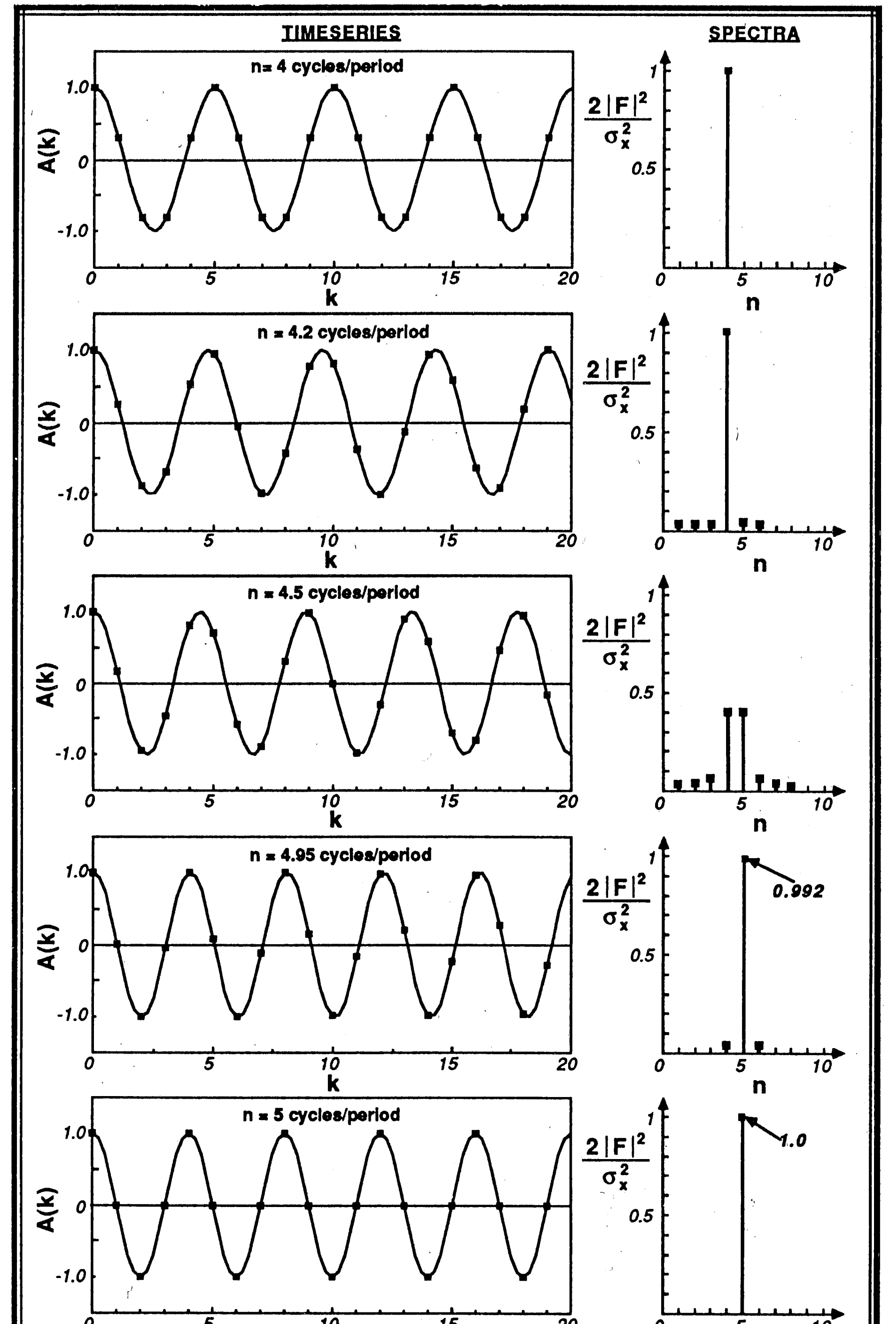

Fig. 8.10c Unresolved waves.

Case D (Fig 8.10d): Unresolvable high and low frequencies. The first example shows that if the real signal has n = 0.5, then the computed spectrum has a spike at n = 1 with a significant amount of leakage to higher frequencies. This is called ***red noise*** and will be discussed in Case E. At n = 1.5, there is still significant red noise. It is as if the leakage from the left side of the spike is folded back around n = 0 to larger n values. At n = 8.5, the leakage off the right of the peak appears to fold back to the left, creating a ***blue noise*** signal.

Case E (Fig 8.10e): Red noise. When signal with time period longer than the sampling period is truncated to fit within the sample window, the resulting periodic shape is fit by waves of the fundamental period and shorter. These waves are largest at the low-frequency end of the spectrum. As the wave period increases, this becomes more apparent. In the extreme case of a linear trend (which acts like an infinite period or wavelength wave), we find a purely red noise spectrum. Its name comes from the fact that the spectrum shows energy at the incorrect frequencies (i.e., error or noise), and that most of this noise is at the low frequencies (analogous to the red portion of the visible light spectrum). We see why it is important to detrend raw signals before computing the FFT.

Because of unresolvable low frequencies in general, and red noise in particular, most meteorologists do not consider frequencies of 3 or less as being reliable. Some use n=5 or n = 10 as the cut off. In any case, we look for at least three waves per sampling period before we are confident that the spectral results are telling us about the physics of the boundary layer. Often, these low frequencies are not even plotted on spectra that are presented in the literature.

Case F (Fig 8.10f): Red, white, and blue noise. White noise consists of approximately equal amplitude spectral energies across the whole range of frequencies. This can be produced by a spike in the time series, or by completely random "hash" signal. If we could hear white noise (e.g., the audio analogy), it would sound like a hiss, like many leaves rustling or many waves breaking.

Blue noise is associated with larger spectral amplitudes at the higher frequencies. A constant signal, shown in the fourth example, consists of just a mean value (i.e., at n = 0), and hence has zero variance and no spectral energy. A square wave yields a spectrum with many peaks and zeros.

Case G (Fig 8.10g): Leakage. The shorter a signal lasts within a record, the more difficult it is to resolve it. Each of these examples shows a signal with five cycles per period. In the first example, the spectrum shows the desired spike at n = 5 with no energy at other frequencies. However, as the signal is cut shorter and shorter, the energy from the spike at n = 5 leaks more and more into the neighboring frequencies. In the last example with just one wave left in the time series, the spectrum shows a nearly Gaussian spread. Hence, even though certain signals in the time series may be evident to the eye, the FFT can have difficulty detecting it.

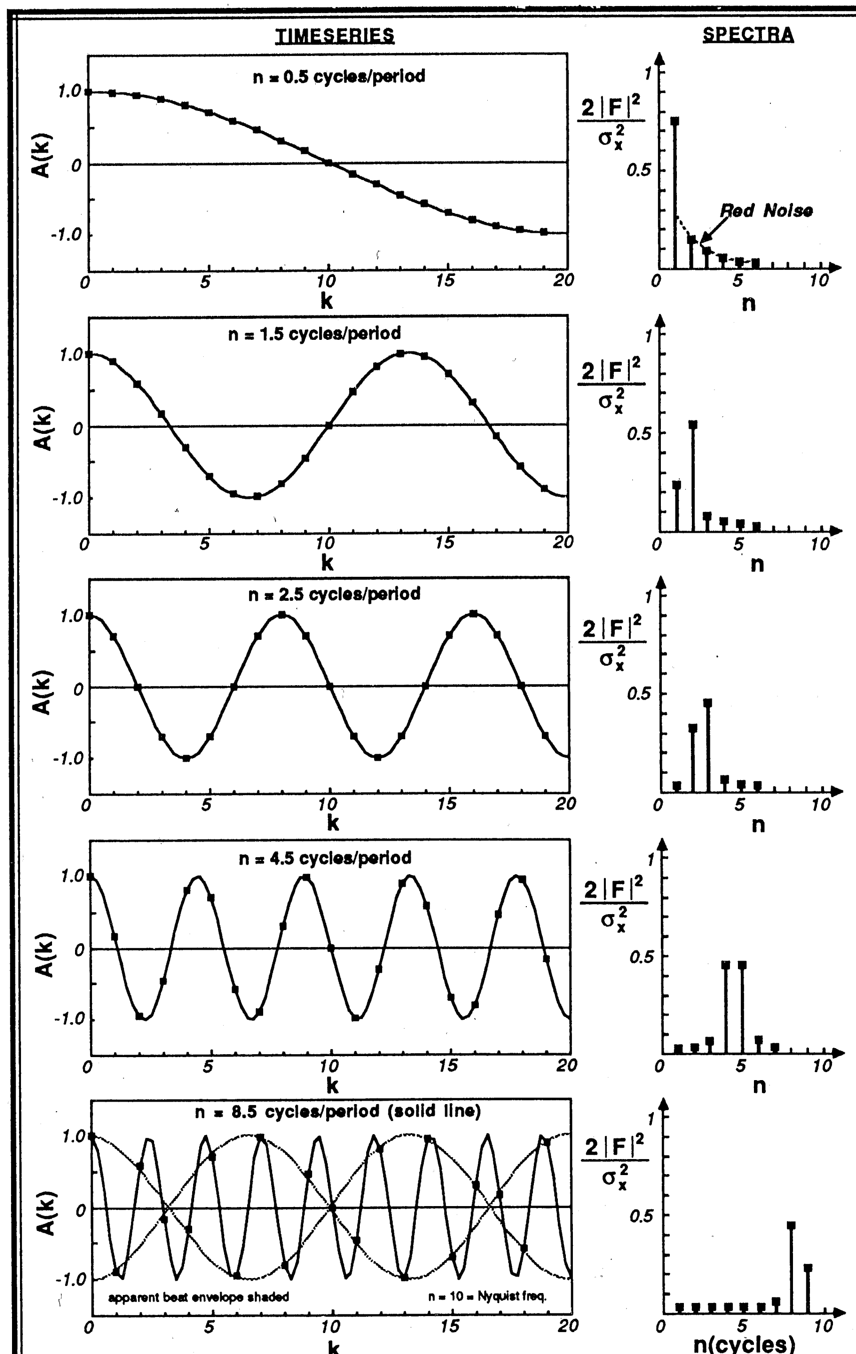

Fig. 8.10d Unresolved waves.

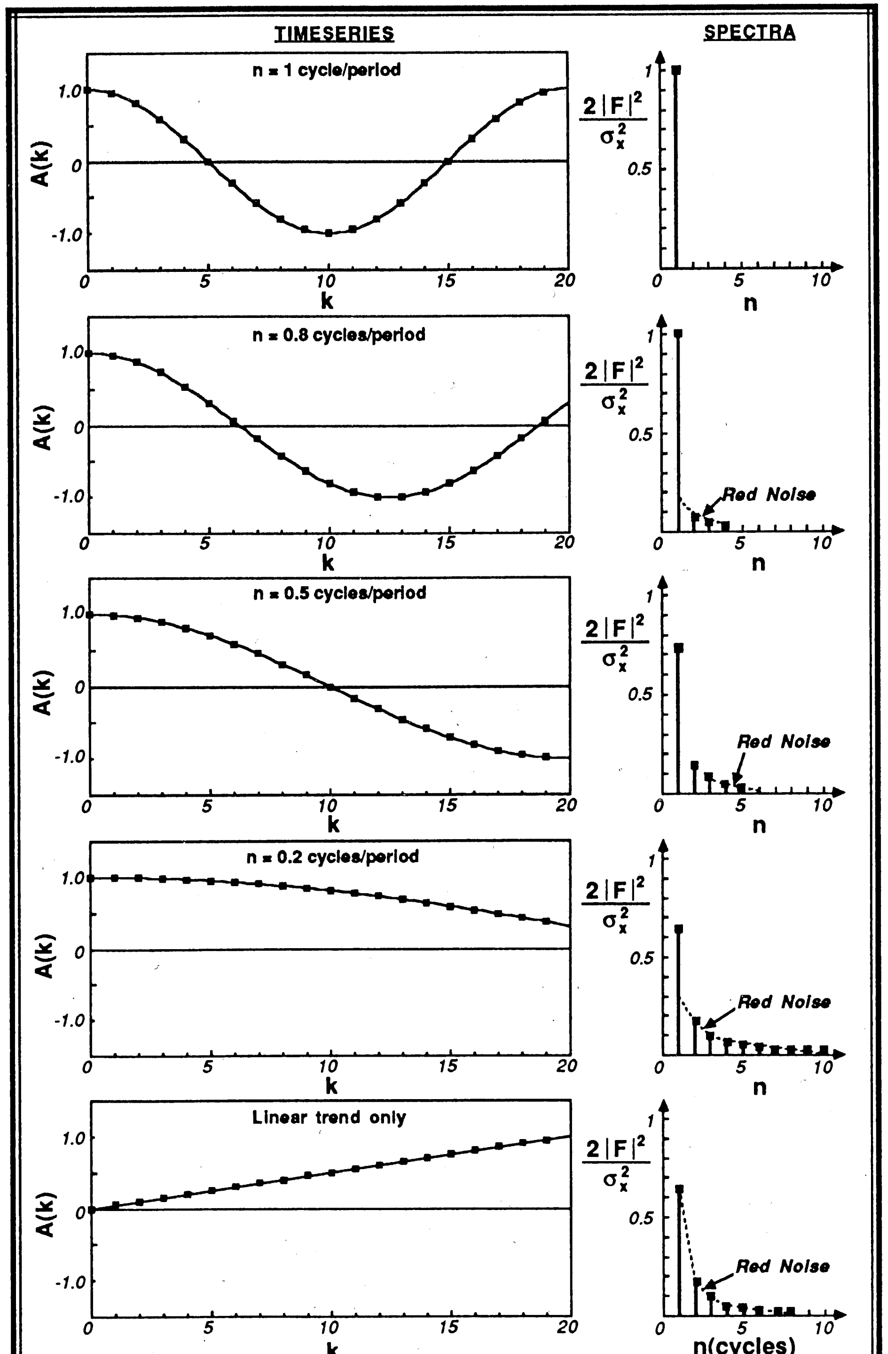

Fig. 8.10e Red noise.

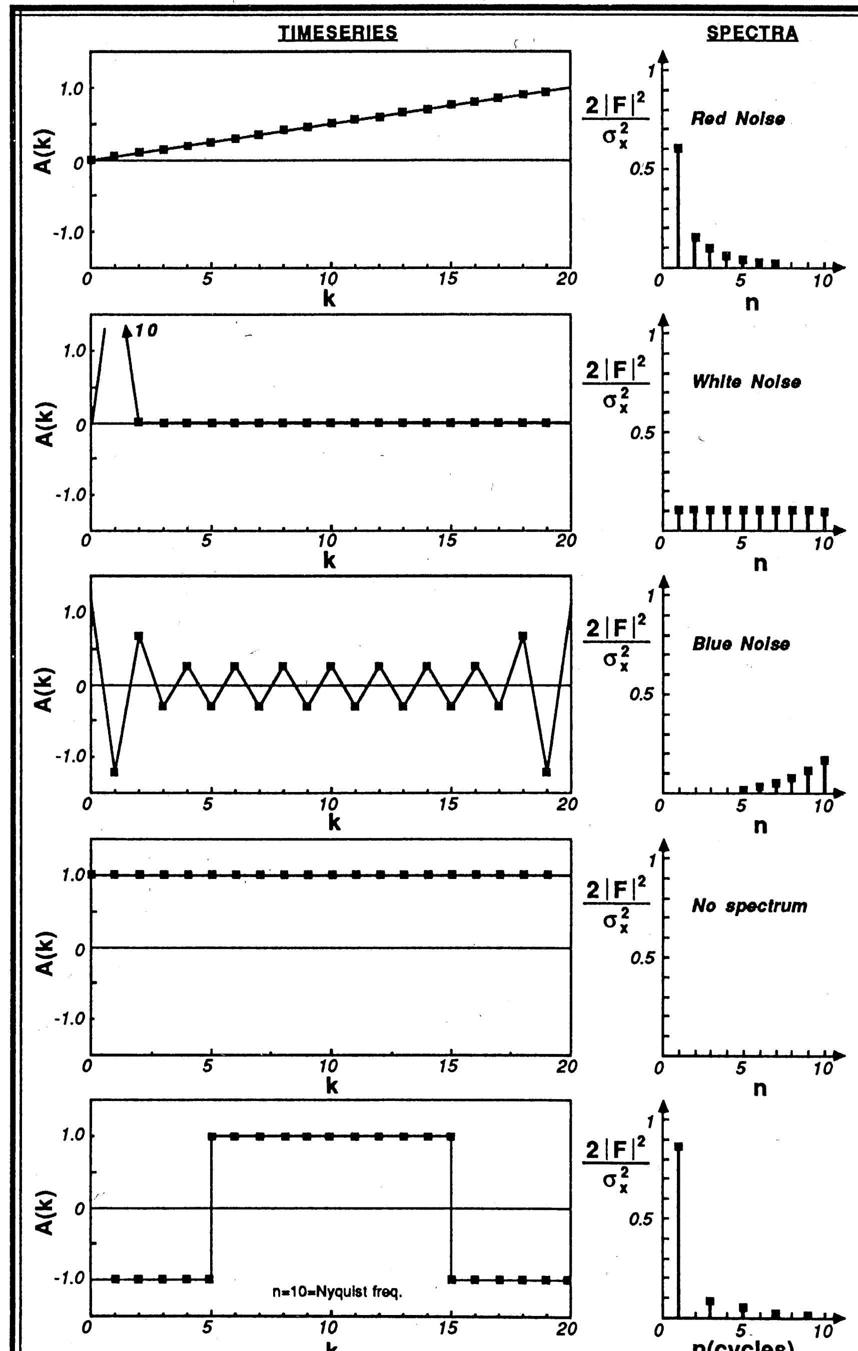

Fig. 8.10f Red, white and blue noise.

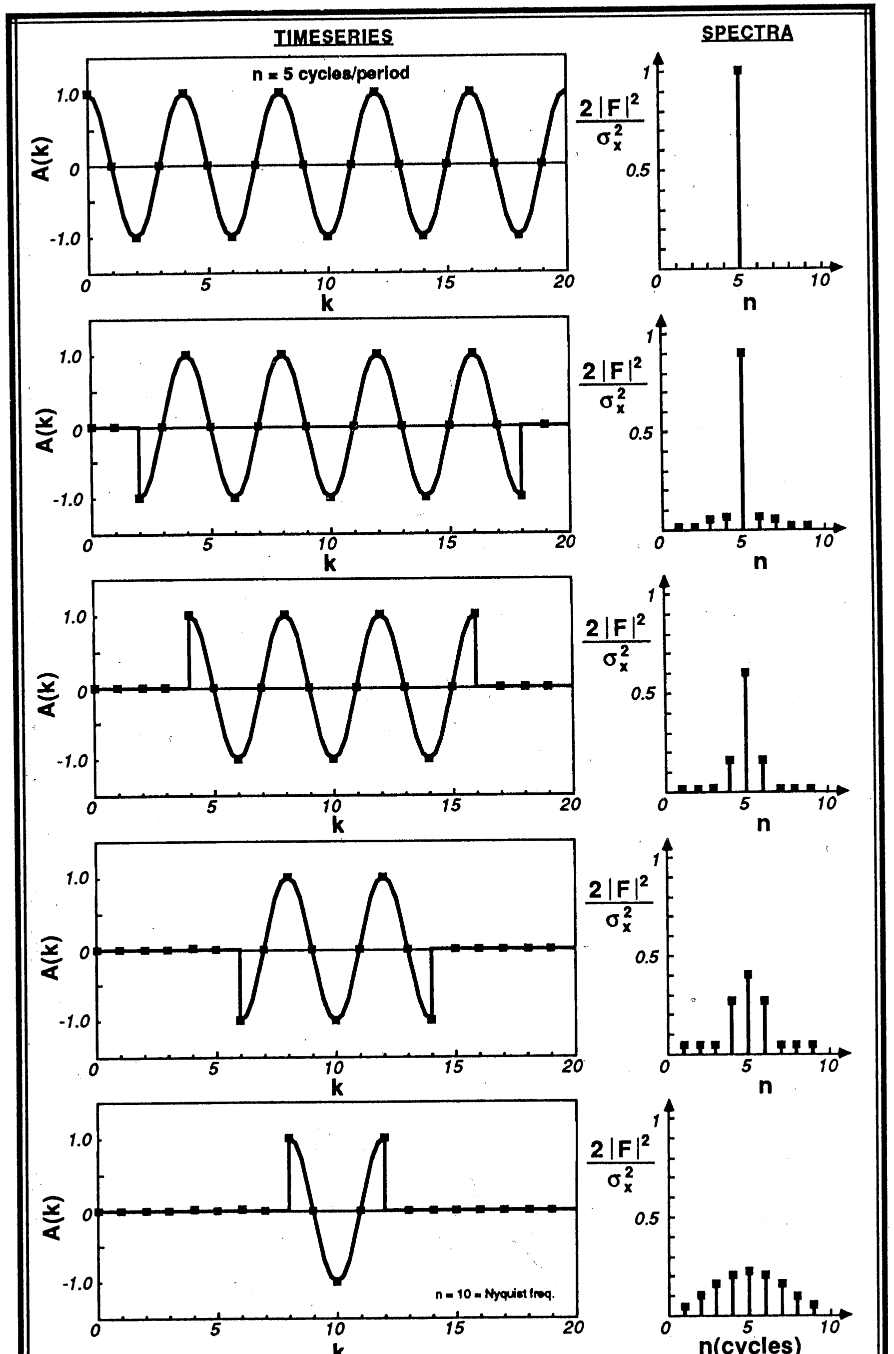

Fig. 8.10g Line broadening (leakage).

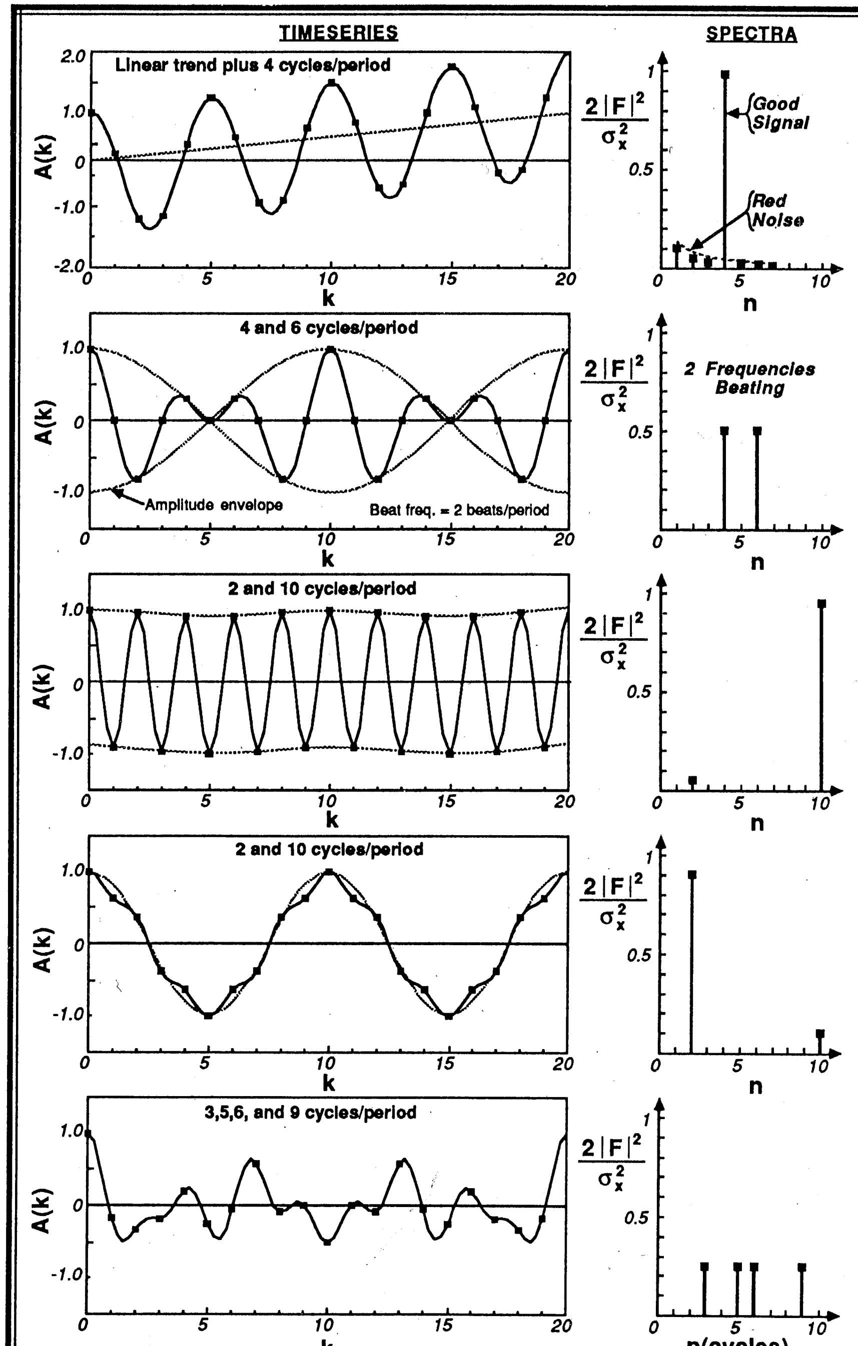

Fig. 8.10h Multiple waves.

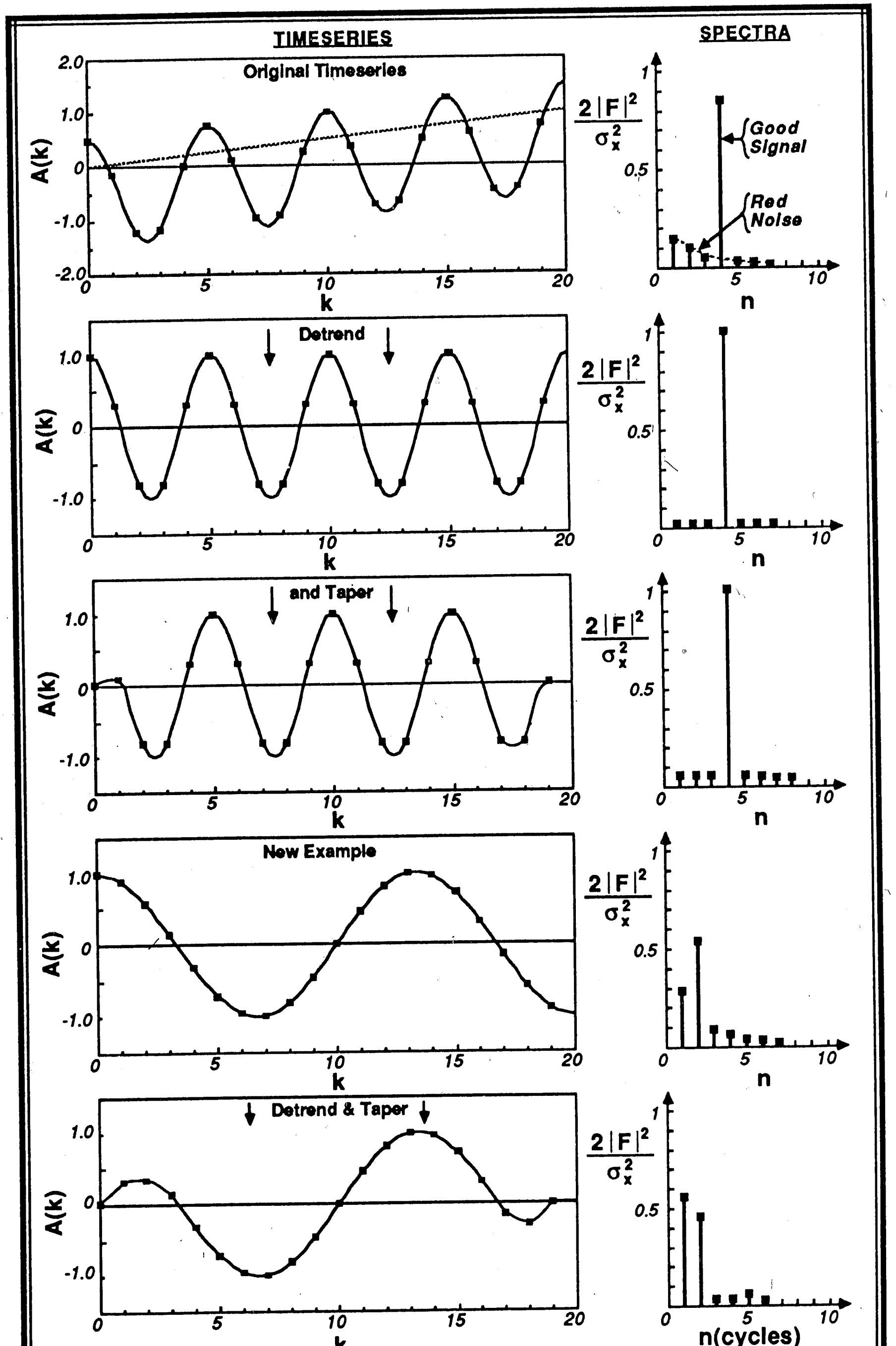

Fig. 8.10i Characteristics of DFT and FFT.

Case H (Fig 8.10h): Multiple Waves. These examples were constructed in reverse, where the spectrum was specified and the time series was generated with an inverse FFT. When the time series consists of the superposition of a number of different wave periods or wavelengths, the spectrum shows a number of spikes. If some of these waves are between harmonics or longer than the fundamental frequency, then the problems of spreading, leakage, and red noise are superimposed on the other resolvable signals. Also, for the second example, the two frequencies at n = 4 and 6 result in a beat frequency of 2, causing the amplitude envelope of the original time series to oscillate as shown.

Case I (Fig 8.10i): Conditioning. The first three examples show one situation of an original time series that is superimposed on a trend. Detrending the time series eliminates the red noise in the spectrum, and tapering the ends has little effect after that. The last two examples show a wave with n = 1.5, causing a significant amount of noise in the spectrum. However, after detrending and tapering, the spectrum yields the desired spikes at n = 1 and n = 2.

8.8 Spectra of Two Variables

Just as we can find the spectrum for a single variable, we can also find a spectrum for a product of two variables. For example, given observations of w'(t) and θ'(t), we can create a new time series w'θ'(t) on which we can perform routine spectral analyses using an FFT. Occasionally it is useful to get more information about the spectrum of w'θ', such as how the phase of the w' fluctuations relate to the phase of the θ' fluctuations as a function of frequency. ***Cross-spectrum analysis*** relates the spectra of two variables.

8.8.1 Phase and Phase Shift

Phase refers to the position within one wave, such as at the crest or the trough (Fig 8.11a). It is often given as an angle. For example, the crest of a sine wave occurs at 90°, or at π/2 radians. ***Phase shift*** refers to the angle between one part of a wave like the crest and some reference point like a "start time" or the crest of another wave. For example, in Fig 8.11b the phase of the second wave is shifted 90° to the right of the first wave.

The equation for a single sine wave of amplitude C that is shifted by angle Φ to the right is:

$$A(k,n) = C(n)\cdot\sin\left(\frac{2\pi kn}{N} - \Phi(n)\right) \qquad (8.8.1a)$$

Through trigonometric identities, we can show that the same wave described above can also be written as the sum of one sine wave and one cosine wave: