CS 6640: IMAGE PROCESSING
Spring 2009
Practice Final

Name: ______________________________
Student ID Number: ______________________________

Rules:

- Closed book.
- One page of notes (front and back).
- No calculators.

Hints:

- The term "describe" does not mean complete sentences and paragraphs or essays. If it's easier you may use simple bullets and meaningful phrases to answer such questions.
- If you split answers across pages (or on the backs of pages) make a clear note on the page where the question is posed to indicate you have done so. Clearly note the question number (and part) on the separate page.
- There are **six** questions for a total of 100 points. Point values are roughly correlated with the amount of time you should devote to each question.

1. **(15 pts.)** Suppose that a digital image is A transformed by a histogram equalization to produce B. Prove that a second application of histogram equalization produces the same exact image, B.

2. **(20 pts.)** Give the Fourier transform of the following functions (you may derive them any way you wish — show relevant work). Hint: They are quick to derive if you use known identities of the Fourier transform.

   (a)
$$\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-x^2}{2\sigma^2}\right) \cos\left(\frac{2\pi x}{k}\right) \quad -\text{where k}, \sigma \text{ are constants .}$$

   (b)
$$f(x,y) = \begin{cases} (1-|x|)(1-|y|) & |x| \leq 1 \text{ and } |\text{y}| \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

3. **(15 pts.)** Consider an even function, $f(x)$. Prove that the following statements hold for all integers $n \geq 0$:

   (a)
$$\frac{d^{(2n)} f}{dx^{(2n)}} \text{ is even}$$

   (b)
$$\frac{d^{(2n+1)} f}{dx^{(2n+1)}} \text{ is odd}$$

4. **(20 pts.)** The Canny edge, as originally proposed, computes a binary map E, which is the zero crossings of second derivatives—as we discussed in class. Rather than a simple threshold, however, it incorporated a special thresholding mechanism called *hysteresis thresholding*.

   Hysteresis thresholding of the gradient image entails two thresholds, T_{lo} and T_{hi}. A pixel p from E makes it into the final result if one of the following two criteria hold:

   (a) p is marked as a zero crossing in E and has gradient (magnitude) above T_{hi}.

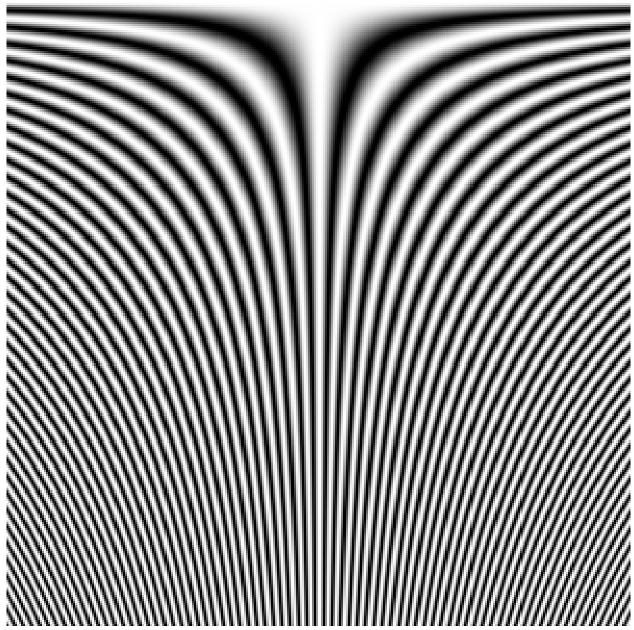
   (b) p is marked as a zero crossing in E, has gradient above T_{lo}, and is connected to pixel meeting the first criterion along a path of pixels this criterion (i.e. above the lower threshold and in E).

   The idea is that a pixel with high gradient is an edge, and it brings with it all those pixels of lower gradient which are connected to it.

   Write psuedo code (high level, e.g. Matlab-like) to perform hysteresis thresholding. You can assume as input: the image zero crossings of second derivatives E, the gradient magnitude image G, and the thresholds. You may also assume that you can call some appropriate flood-fill function or equivalent low-level operations.

5. **(15 pts.)** Suppose you have an edge detector (such as Canny) that produces binary images that indicate thin edges. Suppose you want to save many such images to disk. Given two (good) methods by which you could take advantage of the special structure of such an image to do lossless image compression.

6. **(15 pts.)** Consider the image below.



Along each row is a sinusoid, and the frequency of the sinusoid increases (linearly) as we move from the top (zero frequency) of the image to the bottom. The image has M rows and M columns, and if the pixel location is (x, y) (x going across) the intensity of the image is given by the equation

$$\cos\left(\pi \frac{y}{M}(x - \frac{M}{2})\right).$$

Thus, the frequency of each horizontal cosine is proportional to y/M and the cosines are centered in the image.

The four images below have been convolved with functions of the following form

$$h(x, y) = a(x)\delta(y)$$

Thus, there is essentially a 1D convolution across each row with a 1D function $a(x)$. We have used four different $a(x)$ functions: a *narrow-box* (rectangle), a *wider-box*, a *sinc-function*, and a *Gaussian*.

Identify which of the images below corresponds to each of the four functions above and justify your choices using the Fourier transform. Note, there is a small amount of error in the numerical calculations and the printing, so the images are not perfect.