CS 6640: IMAGE PROCESSING
Spring 2009
Practice Exam — Test #1

Name: _____
Student ID Number: _____

Rules:

- Closed book.
- One page of notes (front and back).
- No calculators.

Hints:

- The term "describe" does not mean complete sentences and paragraphs or essays. If it's easier you may use simple bullets and meaningful phrases to answer such questions.
- If you split answers across pages (or on the backs of pages) make a clear note on the page where the question is posed to indicate you have done so. Clearly note the question number (and part) on the separate page.
- There are **five** questions for a total of 100 points. Point values are roughly correlated with the amount of time you should devote to each question.

1. **(25 pts.)**

   Give the fourier transform of the following functions (you may derive them any way you wish — show relevant work). Hint: They are quick to derive if you think about each one.

   (a)

$$f(t) = \begin{cases} -1 & -1 \leq t \leq 0 \\ 1 & 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   (b)

$$f(t) = \begin{cases} -t-2 & -2 \leq t \leq -1 \\ t & -1 \leq t \leq 1 \\ -t+2 & 1 \leq t \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

2. **(20 pts.)** Consider an input image I with a intensity histogram $A(u)$, where $0 \leq u \leq 1$. Suppose we would like to modify the grey levels of I so that the resulting image has the histogram

$$C(u) = \begin{cases} 2 - 4u & 0 \leq u \leq \frac{1}{2} \\ -2 + 4u & \frac{1}{2} \leq u \leq 1 \end{cases}$$

   (a) Give a complete description (with equations) of how you would construct the new image.

   (b) What is the cumulative histogram associated with $C(u)$? Given an equation and sketch it.

   (c) What qualitative affect would you expect this transformation to have on most images?

3. **(15 pts.)** Compute the discrete Fourier transform (DFT) of the following 2D function. Show all work, equations, and coefficients. Hint: you can compute the coefficients for a four-point DFT, and use them over and over again. Also answer the question—how could you have known what the solution would be without computing it.

$$f(i,j) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

4. **(15 pts.)** Suppose we have an $N \times N$ image and we want to do local histogram equalization with an $M \times M$ window.

   (a) Assume we transform each point by its own histogram. What is the run time using a naive algorithm?

   (b) What is an improved algorithm that produces the same result and what is the run time of this improved algorithm?

   (c) What is the approximation that most people do for an even faster speed up? Describe the run time of the approximat algorithm in terms of whatever relevant parameters you can think of.

5. **(25 pts.)** Suppose you are given a set of N correspondences $\{(\bar{C}_1, \bar{C}'_1), (\bar{C}_2, \bar{C}'_2), \ldots, (\bar{C}_N, \bar{C}'_N)\}$ and you are told that they represent an image warping which is adequately modeled by a second-order polynomial in x and y, which are the image coordinates. Hint: a second-order polynomial in x and y is the sum of monomials in x and y of order two or less.

   (a) How would you go about finding that second-order polynomial?

   (b) Given all of the relevant equations (e.g. expression for linear system) that you would use to find that polynomial.

   (c) How many control points would you need to solve this problem?