CS 6640: Introduction to Image Processing
Fall 2013
Test 1 — Takehome — Due 11/18/2013

Name: ________________________________
Student ID Number: ________________________________

Rules:

- Closed book.
- One page of notes (front and back).
- No calculators.

Hints:

- The term "describe" does not mean complete sentences and paragraphs or essays. If it's easier you may use simple bullets and <u>meaningful</u> phrases to answer such questions.
- If you split answers across pages (or on the backs of pages) make a clear note on the page where the question is posed to indicate you have done so. Clearly note the question number (and part) on the separate page.
- There are **six** questions for a total of 100 points. Point values are roughly correlated with the amount of time you should devote to each question.

1. **[15 pts.]** Suppose we have correspondences $(x_1, y_1), (x_2, y_2), \ldots (x_N, y_N)$ in one image, and $(x'_1, y'_1), (x'_2, y'_2), \ldots (x'_N, y'_N)$ in another.

   (a) We wish to find a simple translation (t_x, t_y), and we wish to overconstrain the system. What penalty function would we minimize (to get a good answer) and what linear system would we use to solve the problem? Give the equations/matrices.

   (b) If we wish to find both a translation and an isotropic scaling of one of the images, what linear system would we use? Give equations/matrices.

   (c) Suppose we wish to find the angle of rotation between two images that best describes these correspondences. Show that the problem is no longer linear. Give the equation (hint: summation) for the penalty function in terms of the unknowns and describe (briefly) one strategy for finding the best rotation.

2. **[25 pts.]** This question deals with the topic of *bilateral filtering*. The general form of bilateral filter is

$$f_i \leftarrow k_i^{-1} \sum_{j \in N} f_j G(\bar{x}_i - \bar{x}_j) H(f_i - f_j)$$

where N is the set of pixels in the neighborhood, $\bar{x}_i$ and $\bar{x}_j$ are pixel locations for the pixel being updated and those pixels in the neighborhood (respectively), and

$$k_i = \sum_{j \in N} G(\bar{x}_i - \bar{x}_j) H(f_i - f_j),$$

is a normalization factor for each pixel i in the image.

(a) Write a pseudocode implementation of the bilateral filter that runs over the whole image for a specified $M \times M$ window size.

(b) What are typical choices of G and why?

(c) What are typical choices of H and why?

(d) There are typically two parameters in this filter, one that controls the shape of G (call it α) and one that controls the shape of H (call it β). What aspect of the filter do each of these parameters affect and how (briefly) should they be chosen?

(e) What is the purpose of including the normalization k_i in the filter? What artifacts could occur if you were to exclude this division by k_i?

3. **[20 pts]**

(a) Prove that

$$\mathrm{sinc(x)} \otimes \mathrm{sinc(x)} = \mathrm{sinc(x)}$$

(b) Prove that for any signal that is band limited to have only nonzero energy below frequency s_0, that $2s_0\mathrm{sinc}(2\mathrm{s}_0\mathrm{x})$ is the identity operator for convolution.

4. **[15 pts.]** Give a continuous Fourier transforms for the following. It is recommended that you use identities where ever possible, rather than derive them from scratch.

(a)

$$f(x) = \begin{cases} \frac{1}{2}\left(\cos(\pi x) + 1.0\right) & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(b)

$$\cos(\omega x) e^{-\frac{-x^2}{2\sigma^2}}$$

(c)

$$\cos(ax)\sin(bx)$$

5. [**15 pts.**]

(a) Find and plot the Fourier transform of

$$f(t) = \cos(4\pi t)\cos(6\pi t)$$

Hint: you may use the convolution theorem.

(b) If we were to sample $f(t)$, what would be the largest sampling interval (ΔT) that still avoids aliasing?