CS/BioENG 6640: IMAGE PROCESSING
Fall 2013
Final Exam—Take Home

Name: ________________________________
Student ID Number: ________________________________

Rules:

- Open book and other noninteractive resources.
- You may not talk, consult, or interact with anyone about this test, either electronically or in person.
- Show all derivations and all work
- Cite/explain any online or written resources you have used to answer the question (it's allowed, but you should document it).
- For your answers, clearly mark each one with the appropriate question number.
- For each question, there is a page limit that you must stay within for your answer.

Please sign the following:
I have read and understand the rules of this test. I understand that I am not allowed to work with, talk with, or talk to any other person about this exam, either directly or electronically. I understand that for any outside resources that I refer to, such as web pages or books, I should give a citation for that resource with my answer.

Signed ______________________________________

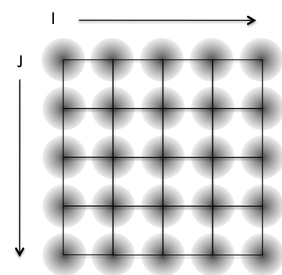
1. **[10 pts.]** (1 page) Consider histogram equalization $\mathcal{H}$ as an operator that takes one image as input and produces another image:

$$g = \mathcal{H} \circ f$$

Prove that $\mathcal{H}$ is *idempotent*. Use graph, equations, and/or carefully worded arguments. (1 page).

2. **[25 pts.]** (2 pages) This question deals with the problem of representing and estimating coordinate transformations based on sets of correspondences in 2D images. Here we consider a set of M correspondences $\bar{C}_1, \dots \bar{C}_M$ in one image and $\bar{C}'_1, \dots \bar{C}'_M$ in the other. In class, we used radial basis functions (RBFs) at the locations of correspondence points, but in practice we can put the RBFs anywhere. For the transformation in this question, we will locate RBFs on a coarse, $K \times K$ grid, and denote the grid positions x_{IJ} for $I, J = 1, \dots, K$, as shown in the figure below for $K = 5$. Notice, the basis functions are fixed in number and location. Thus the coordinate transformation $T(\bar{x})$ is:

$$T(\bar{x}) = \sum_{I=1}^{K} \sum_{J=1}^{K} \bar{\beta}_{IJ} G_\sigma(|\bar{x}_{IJ} - \bar{x}|),$$



where G_σ is a Gaussian and $\bar{\beta}_{IJ}$ are the coefficients (the bar indicates they are vector valued to represent both the x and y components, i.e. $\bar{\beta} = (\beta_x, \beta_y)$) that describe the transformation.

   (a) For a $K \times K$ control grid with vector-valued coefficients (displacements) how many degrees of freedom are there in this tranformation, and how many correspondence points are needed to uniquely define this transformation?

   (b) If you were to overconstrain this problem, with lots of correspondences, what penality function would you use to define the best transformation (write down the equation).

   (c) You can set this problem up as the solution of a linear system. Write down the form of the system and give the equation for the matrices/vectors.

   (d) How would you solve this linear system for the overconstrained case mentioned above?

   (e) What is the impact of the standard deviation of the Gaussian, σ, in the nature of the solution?

3. **(15 pts.)** (1 page) We define two functions:

$$f(x) = \frac{\sin(\pi x)}{\pi x}$$

$$g(x) = \frac{\sin(\pi x)\cos(2\pi x)}{\pi x}$$

Recall that their inner product is

$$< f, g >= \int f(x)g(x)dx$$

Use the Fourier to show that g is *orthogonal* to f (inner product of zero).

4. **[15 pts.]** (1 page) Consider an even function, $f(x)$. Prove that the following statements hold for all integers $n \geq 0$:

   (a)
   $$\frac{d^{(2n)} f}{dx^{(2n)}} \text{ is even}$$

   (b)
   $$\frac{d^{(2n+1)} f}{dx^{(2n+1)}} \text{ is odd}$$

5. **[10 pts.]** (1 page) Give (compute or derive) the DFT of the following matrix

$$f(i, j) = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

6. **[25 pts.]** (2 pages) This question deals with the issue of building continuous interpolations of discretely sampled, greyscale images.

   (a) Consider linear interpolation in one dimension, in which we wish to represent the continuous function $f(x)$ using discrete samples $f_i = f(x_i)$, which are on a regular grid. This interpolation process can be described as a convolution.

   - Give the mathematical equations for this (interpolation by convolution).
   - Give the kernel associated with linear interpolation.

   (b) Give a mathematical expression for the kernel associated with bilinear interpolation, in which we would construct $f(x, y)$ from $f_{ij} = f(x_i, y_j)$.

   (c) Give a mathematical expression for 2D kernel associated with nearest-neighbor interpolation (in which we assign a value to (x, y) based on the closest discrete sample (i, j)).

   (d) If we knew the signal were limited by a bandwidth (in the Fourier domain) of s, what interpolation kernel would we use for a perfect or "ideal" reconstruction? Give the equations.

   (e) Discuss (using diagrams) the frequency characteristics of nearest-neighbor, bilinear interpolation, and "ideal" interpolation and explain how these would affect the resulting interpolated images. (Be brief. Use bullet points. This discussion should probably be less than 1/2 page — not too long).