## Practice Midterm Exam 1

CS 5510, Fall 2017
September 26

Instructions: You have eighty minutes to complete this open-book, open-note, closed-interpreter exam. Please write all answers in the provided space, plus the back of the exam if necessary.
Note on actual exam: The exam may refer to the env.rkt, lambda.rkt, and store-with.rkt interpreters. If you need the interpreters for reference to answer the questions, please bring a copy (paper or electronic) with you.

1) [15 pts] Given the following grammar:

$$
\begin{aligned}
\langle\text { weed }\rangle= & \text { leaf } \\
& (\text { branch }\langle\text { weed }\rangle\langle\text { weed }\rangle) \\
& (\text { stem }\langle\text { weed }\rangle)
\end{aligned}
$$

Provide a define-type declaration for Weed that is a suitable representation for 〈weed〉s.
2) [25 pts] Implement the function weed-forks, takes a Weed and returns the number of branches that it contains. Your implementation must follow the shape of the data definition and include tests.
3) [20 pts] For each of the following expressions, a show the store that would be returned with the program's value when using the store-with.rkt interpreter. Instead of nested override-stores, you can show the store as a list of cells. Recall that locations are allocated starting at 1.
a) $\{\operatorname{box}\{+12\}\}$
b) $\quad\{l e t\{[b$ box $\{+12\}\}]\}$

## \{begin

\{set-box! b 4\} \{box 5\}\}\}
c) $\quad$ let $\{[f$ \{lambda $\{x\}$ \{box $x\}\}]\}$
\{set-box! \{f 0\} \{f 1\}\}\}
d) $\quad\{1 e t\{[f$ \{lambda $\{x\}$
\{box x\}\}]\}
\{let $\{[\mathrm{b}\{\mathrm{f} 0\}]\}$ \{set-box! b b\}\}\}
4) [40 pts] The following expression is evaluated using the lambda.rkt interpreter:

```
{let {[g {lambda {x} {lambda {y} {+ y x}}}]}
    {let {[x 13]}
        {let {[f {g 6}]}
            {f x}}}}
```

(Note: the actual exam will also use lambda.rkt.) Describe a trace of the evalaution in terms of arguments to an interp function for every call. (There will be 15 calls.) The interp function takes two arguments - an expression and an environment - so show both for each call. For each number, variable, and lambda expression, show the result value, which is immediate. (To simplify your job, you do not need to show results for other expressions, but you can show results if you prefer.) Use the back of the exam for additional space, and use the following abbreviations to save time:

```
\(X_{0}=\) the whole expression
\(X_{1}=\{\) lambda \(\{\mathrm{x}\}\) \{lambda \(\left.\{\mathrm{y}\}\{+\mathrm{y} \mathrm{x}\}\}\right\}\)
\(X_{2}=\{\) let \(\{[\mathrm{x} 13]\}\{\) let \(\{[\mathrm{f}\{\mathrm{g} 6\}]\}\{\mathrm{f} x\}\}\)
\(X_{3}=\{\) let \(\{[f\{g 6\}]\}\{f x\}\}\)
```


## Answers

1) $[15 \mathrm{pts}]$
```
    (define-type Weed
        [leaf]
        [stem (rest : Weed)]
        [branch (left : Weed)
            (right : Weed)])
```

2) $[25 \mathrm{pts}]$
(define (weed-forks [w : Weed]) : number (type-case Weed w [leaf () 0] [stem (rest) (weed-forks rest)] [branch (l r) (+ 1
(+ (weed-forks l)
(weed-forks r)))]))
(test (weed-forks (leaf)) 0)
(test (weed-forks (stem (leaf))) 0)
(test (weed-forks (stem (branch (leaf) (leaf)))) 1)
(test (weed-forks (branch (branch (leaf) (leaf)) (leaf))) 2)
3) $[20 \mathrm{pts}]$
a) (list (cell 1 (numV 3)))
b) (list (cell $2(n u m V)$ ) ( cell 1 (numV 4)) (cell 1 (numV 3)))
c) (list (cell 1 (boxV 2)) ( cell $2(n u m V 1))(\operatorname{cell} 1(n u m V 10))$
c) (list (cell 1 (boxV 1)) (cell 1 (numV 0)))
4) $[40 \mathrm{pts}]$

$$
\begin{aligned}
\operatorname{expr} & =X_{0} \\
\text { env } & =\text { mt-env } \\
\text { expr } & =X_{1} \\
\text { env } & =\text { mt-env } \\
\text { result } & =\text { (closV 'x } \mathrm{x} \text { \{lambda }\{\mathrm{y}\}\{+\mathrm{y} \mathrm{x}\}\} \\
& \mathrm{mt} \text {-env) }=C_{1} \\
\text { expr } & =X_{2} \\
\text { env } & =\text { (extend-env (bind 'g } C_{1} \text { ) mt-env) }=E_{1} \\
\text { expr } & =13 \\
\text { env } & =E_{1} \\
\text { result } & =\text { (numV 13) } \\
& \\
\text { expr } & =X_{3} \\
\text { env } & =\text { (extend-env (bind 'x (numV 13)) } E_{1} \text { ) }=E_{2}
\end{aligned}
$$

$$
\begin{aligned}
& \text { expr }=\left\{\begin{array}{l}
\mathrm{g} 6\} \\
\hline
\end{array}\right. \\
& \text { env }=E_{2} \\
& \text { expr }=\mathrm{g} \\
& \text { env }=\overrightarrow{E_{2}} \\
& \text { result }=C_{1} \\
& \text { expr }=6 \\
& \text { env }=E_{2} \\
& \text { result }=\text { (numV 6) } \\
& \operatorname{expr}=\{\text { lambda }\{\mathrm{y}\}\{+\mathrm{y} x\}\} \\
& \text { env }=\left(\text { extend-env (bind ' } \mathrm { x } \left(\text { numV 6)) mt-env) }=E_{3}\right.\right. \\
& \text { result }=\left(\mathrm{closV} \text { 'y }\{+\mathrm{y} \mathrm{x}\} E_{3}\right)=C_{2} \\
& \operatorname{expr}=\{\mathrm{f} \text { x }\} \\
& \text { env }=\left(\text { extend-env (bind ' } f C_{2}\right) E_{2} \text { ) }=E_{4} \\
& \operatorname{expr}=\mathrm{f} \\
& \text { env }=E_{4} \\
& \text { result }=C_{2} \\
& \begin{array}{ll}
\text { expr } & =\mathbf{x} \\
\text { env } & =E_{4}
\end{array} \\
& \text { result }=\text { (numV 13) } \\
& \operatorname{expr}=\{+\mathrm{y} \mathrm{x}\} \\
& \text { env } \quad=\left(\text { extend-env (bind 'y (numV 13)) } E_{3}\right)=E_{5} \\
& \operatorname{expr}=\mathrm{y} \\
& \text { env }=E_{5} \\
& \text { result }=\text { (numV 13) } \\
& \operatorname{expr}=\mathrm{x} \\
& \text { env }=E_{5} \\
& \text { result }=\text { (numV 6) }
\end{aligned}
$$

